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P.S.S.C. PHYSICS

TEACHER'S RESOURCE BOOK AND GUIDE

PART ONE

D. C. HEATH AND COMPANY

INDIAN
EDITION

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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P.S.S.C. PHYSICS

TEACHER'S RESOURCE BOOK AND GUIDE

INSTITUTE OF EDUCATION FOR WOMEN
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PART ONE

INDIAN
EDITION

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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ACKNOWLEDGEMENTS

Many people have collaborated in preparing the Teacher's Resource Book and Guide for PSSC Physics. The authors of the Guide have worked closely with those who wrote the textbook for students, and have drawn heavily from the experience of many secondary school teachers who have taught the PSSC course. So many individuals have contributed in such varied ways, that it is not possible to acknowledge adequately all of the contributions.

From his position of responsibility for the PSSC textbook, Professor Francis L. Friedman of the Massachusetts Institute of Technology has been the key individual in achieving integrated form and substance in all of the PSSC materials. In the preparation of the Guide, Professor Friedman has been an unfailing source of information and advice.

The yellow pages of the Guide (laboratory suggestions) were prepared by the PSSC group working at Cambridge, Massachusetts on the development of the laboratory experiments and apparatus. This work was under the direction of Professor Uri Haber-Schaim of the Physics Department of the Massachusetts Institute of Technology. This group is acknowledged in the introduction to the yellow pages, Teacher's Guide for Laboratory Experiments.

The white pages of the Guide (general discussion of teaching the PSSC course) and the green pages (problem solutions) were prepared by a group working at the University of Illinois. During the first stages of its work, this group was organized under the guidance of Professor F. Wheeler Loomis, who has since retired from the University. During the latter stages, the work of the participating physicists was organized by Professor Edwin L. Goldwasser of the Physics Department of the University of Illinois.

In different years, David A. Page and Kenneth B. Hunt contributed to the Guide, and taught the PSSC course in the University High School (with the back row of seats reserved for Guide writers!). James Connell and Alice Summerbell Culbert assisted in this work.

Throughout the development of the Guide, Professor Gilbert C. Finlay of the College of Education of the University of Illinois provided general guidance, final editing, and liaison with the Massachusetts Institute of Technology group and with teachers who used the course.

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INTRODUCTION TO THE TEACHER'S GUIDE

The PSSC course is more than a textbook. It is a coordinated set of resources for the teaching of secondary school physics. It includes a text, laboratory guide, laboratory apparatus, closely-related films, tests, and a growing collection of supplementary reading material – the Science Study Series. To help teachers become familiar in detail with the content and purposes of the PSSC course, a number of colleges and universities have developed teacher institute programs based on the PSSC materials. Good materials can contribute to making a good course, but the most important element of a successful PSSC course is the teacher. The teacher provides the indispensable services of interpretation and clarification of the substance of the course. His presentation can be tailored to the needs and abilities of his individual students. Only he can provide the day-by-day personal stimulation and inspiration which can come from a competent and enthusiastic advocate who is sensitive to individual student's interests and learning problems.

The purpose of this Guide is to help teachers translate the PSSC materials into class and laboratory programs. To this end, the Guide provides background information on the over-all and the detailed structure of the materials, makes concrete suggestions for class and laboratory activities, and discusses selected sections of the course at a level directed to teachers rather than students. The Guide does not presume to lay down hard and fast rules for teaching the course, but rather to provide suggestions which some will find directly useful and which some will find stimulates the development of other approaches. Much of the Guide is based on the experience of many teachers who have found effective ways of presenting the PSSC course. The authors will welcome suggestions from teachers who use the Guide.

In greater detail, the functions of the Guide, as seen by the authors, are:

1) To put the individual topics in the perspective of the purposes and content of the entire course. Beyond the development in the text, it often is helpful for teachers to understand reasons for a particular argument or sequence of ideas.

2) To present certain of the course topics to the teacher at a somewhat higher level than in the text for students. Through discussions that capitalize on the teacher's maturity, experience, and training, the Guide hopes to help the teacher save time in gaining an increased understanding of the details of the course structure and content.

3) To suggest appropriate use of the related materials relevant to each part of the text: laboratory experiments; films; Home, Desk and Lab problems; demonstrations; books in the Science Study Series; and occasional other materials.

4) To provide detailed information on the use of laboratory experiments and apparatus. The laboratory Teacher's Guide (yellow pages) includes information on the purposes and optimum time for the various experiments, answers to the questions asked in the student's Laboratory Guide, and concrete suggestions for manipulating apparatus and conducting experiments.

5) To suggest possible schedules for teaching the course. For a teacher using the course for the first time, it is important to have some indication of a realistic schedule which will carry his class through Part IV by the end of the school year. For teachers who have taught the course, suggested schedules are less important. However, they serve for all as a reminder of the relative emphasis with which, experience has shown, different parts of the course can be treated to achieve both completeness and adequate depth. Scheduling the course is discussed in greater detail in the next section, Teaching The PSSC Course.

6) To provide solutions to the problems in the Home, Desk and Lab sections of the text. For those problems which can be answered briefly or numerically, short answers are given at the beginning of the green pages for each chapter. Following the short answers, detailed solutions are given for each problem. The detailed solutions are not particularly intended for quick reference. Neither are they intended as models for students' solutions.

They are intended to help the teacher prepare for class discussion of problems. Because of this, the detailed solutions approach a problem much as a teacher might discuss it in class. They include much more "talking" than should be expected in a student's homework assignment.

7) To suggest occasional supplementary problems. Extra problems can be used for homework assignments, but, considering the large number of problems in the Home, Desk and Lab sections of the textbook, such problems probably are more appropriate for quizzes or class discussion.

The Guide is divided into four parts, corresponding to Parts I, II, III, and IV of the text. A general section, Teaching the PSSC Course, is included in this part of the Guide.

Each part of the Guide includes an introductory section discussing the salient features of the content of that Part of the course and suggesting appropriate schedules for the various chapters. Then each Part of the course is discussed chapter by chapter. Each chapter includes a summary, suggestions for scheduling the various sections, and a listing of related materials relevant to that chapter. Then each section, or a convenient group of sections, is discussed. In general, these discussions are organized in paragraphs headed PURPOSE, EMPHASIS, CONTENT, DEVELOPMENT, and COMMENTS.

TEACHING THE PSSC COURSE

The field of physics has advanced with ever-increasing rapidity during the past fifty years. Despite this progress, little attention has been given to revising the introductory teaching of basic ideas in the light of modern developments. Simply updating existing course structures may result, especially for a student who is not going on to specialize in the field, in a disjointed collection of ideas which falls short of giving an accurate perspective of the nature and role of physics in today's world.

The PSSC course essays the task of providing, at the introductory level, a conceptual framework of contemporary physics, and of showing how physical knowledge is acquired experimentally and woven into physical theory – how theory in turn directs and illuminates experimentation. The subject is presented not as a static codification of physical ideas, but as an integrated picture of contemporary physics – as a model of man's intellectual activity, human, and therefore fallible, but a purposeful mode of inquiry.

The content of the PSSC course has been chosen, not simply to "cover" physics, but to display the structure of the field. The course is not as broad topically as some, but the topics that have been selected are explored more fully than in other beginning courses. The pattern evolved in the course is one in which the earliest work is cast in terms of the over-all picture that is sought. It is a pattern in which central ideas recur, each time to be carried further in a higher synthesis of ideas. It is a pattern which, as an alternative to authoritative assertion of principles followed by illustration or example, works from phenomena to theory. The frequent analysis of experiments in the text and films and the carefully integrated laboratory work strive to give meaning to physical laws and theories and an understanding of how they are formulated.

This approach to secondary school physics has been taken with the conviction that it is appropriate both for students who expect to continue in physics or other sciences, and for students who will have no further formal contact with physics. Those who have worked on the PSSC course would be the last to claim that it is the only satisfactory approach to secondary school physics. The course is simply offered as an approach which is appropriate to the requirements of education in science for our times.

Introducing a New Course.

With all of the many details that must be taken care of in getting a new course started, it is sometimes easy to forget the importance of keeping parents, other faculty members, and the general student body aware of the general nature of the new work.

Many teachers have found it helpful, early in the semester, to invite students' parents to an out-of-school meeting for a discussion of the purposes and nature of the new course. In addition to a general discussion, you might show, with a well-chosen demonstration or two, how laboratory work fits into the picture. If available, you might screen one of the films and show how it fits in.

Some teachers have arranged similar programs during lunch hours or after school for students and faculty. Quite often they have asked some of the physics students to participate in demonstrations of laboratory work.

In getting your students started, there is probably no substitute for rolling up your sleeves and plunging in. A great deal of general talk about what is expected and about the nature of the course will probably be like "water on a duck's back". For background, you might suggest that students read the preface to the text.

Almost certainly some students will find the course and its materials quite different from what they expected. Students who are habituated to an authoritative approach to science will be rudely shocked. Your discriminating perception of such reactions and on-the-spot handling of them will be important. With some students you may have to work hard to get them to see the satisfaction of studying and working for understanding rather than a capsulated summary of the "facts". Early laboratory work can help.

Using the Text.

When the student reads or studies the text to follow a line of thought rather than to pick up factual conclusions, the text is a powerful teaching tool. Its care in moving from the concrete phenomenon to the abstract concept will often serve the careful student without a great deal of explanatory discussion. What we hope to combat is a student approach of skimming (or skipping) the text in the belief that the teacher will go over it all in class. When your students come to class having studied the text for the major line of argument, with such supporting details as they have been able to glean, you are ready to sail! You will be in a position to nail down understanding rather than scramble for a minimum grasp of the subject.

Use of Home, Desk and Lab Problems.

If your students use the text in the vein indicated above, you will be able to use appreciable class time in analyzing problems. This will help you uncover misunderstandings, summarize the pertinent ideas and more broadly show their applicability.

In general, the Home, Desk and Lab sections of the text include considerably more problems than a teacher will be able to use with any one class. To provide some guidance in selecting problems, the Guide frequently indicates those problems that are regarded as being among those which are most instructive.

The order in which the problems appear in the Home, Desk and Lab sections of the text has been chosen with considerable care. While the order in general follows the conceptual development of the chapter, sometimes easy problems have been inserted ahead of their "logical" place to provide leads toward new ideas. Sometimes problems that are topically related to early parts of a chapter are placed near the end of a set of problems. One reason for this is because the problem is complex and a little maturity with the ideas will help. Sometimes it is because a problem is particularly valuable for summarizing and tying together the work of a chapter.

In the summary section for each chapter, the Guide includes tables (repeated at the beginning of the green pages) which indicate the sections to which the various problems are conceptually related. The tables also classify problems according to their estimated level of difficulty and identify those problems which are particularly suited to class discussion.

Two questions on the handling of problems come up so frequently that they deserve comment at this point. These questions relate to the use of significant figures and handling dimension in working through a problem.

Significant figures are discussed in Chapter 3 of Part I. The authors of the text and the guide feel that extended emphasis is not warranted in the classroom development of this material. The main purpose of the presentation is to demonstrate to students, for example, that if an object has a mass of 7.5 grams and a volume of 3.3 cubic centimeters, its density can meaningfully be specified to be 2.3 grams per cubic centimeter. There is neither point nor meaning in carrying out the long division further to get 2.2727...... gms/cm^3. If this idea is communicated successfully to students, that is all that is intended. An introductory course is surely not the place to develop a theory of errors. Neither should rigor be attributed to any of the acceptable significant figure recipes, none of which is truly precise. Try not to "lose sight of the forest for the trees". Use the material on significant figures only as it is intended – to help students in finding a rationale for cutting off their arithmetical computation at a reasonable place.

In presenting solutions to HDL problems, the authors of the Guide have tried to maintain a consistent significant figure practice. If a problem statement gives, for instance, the mass of an object as 3 grams, it is assumed, where reasonable, that this is an exact number and not the result of a measurement that is limited in accuracy to one significant figure.

Many questions are asked about the explicit handling of dimensions in following through the work of a problem.

For example, suppose that we wish to know the number of atoms in a cubic centimeter of a substance. The data given might be:

$N = \text{Avogadro's number} = 6.0 \times 10^{23}$ atoms/mole

$A = \text{atomic mass} = 200$ gms/mole

$D = \text{Density} = 20\ \text{gm/cm}^3$

The problem could be solved, writing,

$$n = \frac{ND}{A} = \frac{6.0 \times 10^{23} \times 20}{200} = 6 \times 10^{22}\ \text{atoms/cm}^3$$

Some teachers prefer:

$$n = \frac{ND}{A} = \frac{6.0 \times 10^{23}\ \text{atoms/mole} \times 20\ \text{gm/cm}^3}{200\ \text{gm/mole}} = 6 \times 10^{22}\ \text{atoms/cm}^3.$$

The latter procedure is certainly the more rigorous of the two. Furthermore, by actually following through the dimensional cancellation, a student can check to see that he put the proper numbers in the numerator and in the denominator. This is a tool that is often helpful in the initial phases of learning a new relation and which every student should learn to use when he needs it. However the authors feel that the proper time to penalize a student for shortcutting an explicit following of dimensions is when it causes him to have trouble. For instance, when a student, in doing the above problem, writes:

$$n = \frac{NA}{D} = \frac{6.0 \times 10^{23} \times 200}{20} = 6.0 \times 10^{24}\ \text{atoms/cm}^3,$$

this is the time to correct his procedure. Unvarying insistence on writing out all dimensions, long after a student clearly understands what he is doing, may lead to dampening the enthusiasm which often accompanies the newly-found power to solve quantitative problems.

Using the Laboratory.

The laboratory can make unique and key contributions to the success of the course. In certain parts of the course the laboratory should carry the main burden of developing the subject. For a statement on the role of the laboratory in the PSSC course, and general suggestions on handling the laboratory work, see the introduction to the Teacher's Guide for Laboratory Experiments (yellow pages).

Films.

In designing the PSSC films careful attention has been given to the aims and structure of the course as a whole and to the related development of the topics in the text and laboratory.

While many experiments are shown in the films, rarely are experiments included which can and should be done in the laboratory. Such experiments are only included where it is necessary to the logic of the film. In such cases it is usually done briefly and allusively. Some films include experiments suitable for the school laboratory but done with sophisticated apparatus; in such cases the film is intended to help tie ideas together and clearly should be shown after the student has done the lab work.

Because of the close correlation of films, text, and laboratory, proper scheduling of the films is extremely important. The Guide will make suggestions to help you meet these urgent scheduling problems.

The films are not intended to usurp the teacher's function. The various general roles they play – introducing a subject, providing data through an experiment difficult for students to perform, helping to summarize a subject – are all intended to provide take-off points for teacher and class. By directing attention to key ideas they are meant to help teachers see that their class develops a proper perspective.

Beyond their content the films are intended to show real scientists in action, presenting them not as disembodied intellects, but as normal, active, occasionally fallible human beings, dealing with real problems and deriving satisfaction and at times excitement from their work. In this quietly implicit way the films attempt to show the nature of the scientist and the scientific way of life.

Finally, the films are scrupulously honest. Filmed experiments were carefully performed and are accurately reported. The films contain no high gloss; they are frankly teaching films.

Tests.

Ten achievement tests, two for each of the four Parts of the course, and two comprehensive tests covering the first and the second half of the course, have been prepared with the cooperation of the Educational Testing Service. These are objective tests. Because of their emphasis on the application of knowledge rather than the recall of facts, the tests are valuable as teaching devices as well as for testing purposes. Class discussion of tests following their completion and scoring can be a useful teaching technique.

While these tests can contribute to the evaluation of students' progress, they are not intended as sole criteria for judging student accomplishment. For this purpose the tests should be supplemented by teacher-devised procedures for considering various other aspects of a student's work – such as laboratory and homework. For examination purposes most teachers will want to supplement these tests with their own quizzes and examinations.

The Science Study Series.

"The Science Study Series offers to students and to the general public the writing of distinguished authors on the most stirring and fundamental topics of physics, from the smallest known particles to the whole universe. Some of the books tell of the role of physics in the world of man, his technology and civilization. Others are biographical in nature, telling the fascinating stories of the great discoverers and their discoveries. All the authors have been selected both for expertness in the fields they discuss and for ability to communicate their special knowledge and their own views in an interesting way. The primary purpose of these books is to provide a survey of physics within the grasp of the young student or the layman. Many of the books, it is hoped, will encourage the reader to make his own investigations of natural phenomena."*

The Science Study Series is being developed to provide supplementary material for secondary school physics students as well as to provide science resource material for other students and laymen. At this writing, eleven books have been published, about sixty more are in some phase of preparation, and more are planned.

In developing the books of the Science Study Series, the PSSC does not intend them as formal parts of the course. However, they will be valuable resources for any secondary school physics teacher. You will have many occasions to refer students to appropriate books. Sets of these books will be a valuable addition to your classroom and to the school library. Your school's English teachers should find them a useful resource for science-oriented students.

Published Information on PSSC.

For teachers who are interested in reading some of the published information and commentary on the PSSC course, the following sources are listed.

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*Quoted from the introduction to each volume of the Science Study Series.

Physics Today, "Physical Science Study Committee, A Planning Conference Report." Vol. 10, No. 3, 28-29, March, 1957

This article reports the proceedings of the December, 1956 fifty-member planning conference which initially outlined the PSSC work.

The Science Teacher, 24:316-327. November, 1957. Elbert P. Little, "From these Beginnings...." Francis L. Friedman, "A Blueprint...." Jerrold R. Zacharias, "Into the Laboratory...." Gilbert C. Finlay, "What Are the Questions?"

These articles were prepared near the end of the summer of 1957, just after the first large group of physicists, teachers and other specialists had begun intensive work on the preparation of materials, and before the materials were used in any schools. The articles describe the background and purposes of the Committee's work.

Physical Science Study Committee "First Annual Report of the Physical Science Study Committee", The Committee, 1958, 42 p.

This report is out of print, but a great many were distributed to teachers and schools. The report described the work of the Committee from its beginnings through June 1958.

"A Symposium: The Physical Science Study Committee". Harvard Educational Review, 29:1-36. Winter 1959

These articles provide commentary on the work of the PSSC both by members of the Committee and by others.

Gilbert C. Finlay and Frederick L. Ferris Jr., "Physical Science Study Committee: A Status Report and an Achievement Test Report" The Science Teacher, 26:574 - 581, December, 1959.

These articles describe the work of the Committee between September 1957 and September 1959, and report the results of the physics achievement testing program carried out with secondary school students who studied the PSSC course in 1958-59. A brief resume is given on the opinions of teachers who taught the course.

Gilbert C. Finlay "Secondary School Physics: The Physical Science Study Committee", The American Journal of Physics, 28:286-293, March, 1960

The content of the PSSC course is outlined in some detail. Examples of laboratory work and films are described to show how these parts of the course are integrated with the text.

Frederick L. Ferris Jr. "Testing for Physics Achievement", The American Journal of Physics, 28:269-278, March, 1960

While not dealing specifically with the PSSC program, this article includes many illustrations drawn from the PSSC achievement testing program, and reflects the point of view behind the development of the PSSC tests.

Educational Services Incorporated, "1959 Progress Report", The Corporation, 1959, 47 p

This report reviews the work of the PSSC through 1959.

Scheduling the Course.

Because of the investigative, questioning nature of the text, laboratory, and films; and because the main theme of the course is found in the fundamental ideas that bind together a wide range of physical phenomena into an integrated picture; the course provides many temptations for students (and teachers) to go beyond the material presented, both in depth and breadth of coverage.

If a teacher shares the aim of the authors to convey a sense of the range and power of the fundamental physical ideas, he will want to schedule his course so as to be able to spend enough time on the latter chapters of Part IV to show how these ideas reach to the domain of inside the atom. While the open-endedness of the course is tempting, over-extending the treatment of earlier material will force a sacrifice near the end. Ways must be found of preserving the spirit of inquiry without responding to every invitation to a side excursion. One of the purposes of the Guide is to spell this out as clearly as possible, principally for the benefit of new teachers.

In scheduling and teaching the PSSC course most of the "adjustments" that can be made to accomodate a specific schedule must come from changes in depth of treatment. This is because the course material has been selected and organized so as to build toward an integrated picture. If the course consisted of weakly-connected units, it would be easy to select an appropriate number to fit a particular schedule. But this is not the case. The course carefully builds new ideas in the context of the preceding material. This continuity is one of the course's chief pedagogical virtues, however it means that understanding of the later parts would be only superficial without the foundation of the preceding material. Thus, for the teacher who needs to adapt the course to a short schedule, time can be saved, with one or two exceptions, only by controlling the depth with which certain topics are covered.

To help the teacher avoid the frustration of finding himself in the middle of Part III at the beginning of June, the Guide is presumptuous in that it suggests quite detailed sets of schedules. These are not given with the thought that they are the only feasible schedules. Final decisions on scheduling obviously must rest with the teacher. The scheduling suggestions are made in the belief that they will be helpful as a general guide.

In order to put numbers in a suggested schedule, some assumptions are required about the classroom time that is available. This may vary from something like five 45 minute periods per week for a 35 week school year to seven 55 minute periods per week for a 54 week (one and one half year) course. With all kinds of variations in local schedule requirements it is not possible to make precise recommendations. Two rough schedules are suggested. One schedule is for a relative time division which is consistent with covering the entire course in one year. The longer schedule is included to suggest the parts of the course that might be expanded most profitably if time is more plentiful. Each teacher must wrestle with the suggested emphasis and his own teaching calendar.

The suggested schedules do not include the ten to fifteen days which some teachers will use for giving and discussing tests. Neither do they provide for the days your class is cancelled or your period is shortened for other events!

It will be noted that the shorter schedule provides for about 6 1/2 weeks on Part I, and 9 weeks on each of the other Parts. This totals to fewer weeks than are provided by many school terms. This was done to make room for some of the interruptions that frequently eat into class time, testing, and the inevitability of some "slippage" in keeping to a schedule.

Adhering to such a schedule will be especially difficult in Part I. However, most teachers who have taught the course and come to the end of the year with insufficient time to cover the latter parts of the course, report that the next time through, they will try to spend less time on Part I.

Part I, as well as introducing some of the key ideas that will be used and reused throughout the course, marks out the broad domain of physics. With its scope, it could easily be the subject of a half year of study. However, it is not intended that Part I be elaborated extensively. While you will want to seek quantitative facility at some points (the Guide will make suggestions), at many other points it will be desirable simply to get the ideas implanted through making them reasonable, letting them mature through reuse in later Parts. Beyond this, some of Part I, while having quantitative underpinning, can be treated qualitatively in a secondary school course. A first year teacher, especially, will do well to keep his schedule as close to suggested schedules as is consistent with his own and his students' satisfaction with the class rate of progress and depth of understanding. Enough teachers have taught the course to make it clear that these factors vary widely from one class to another, and that they can be influenced strongly by the teacher.

Suggested Schedules for the Course

Principal Subject	Relevant Text	36-week schedule (weeks)	54-week schedule (weeks)
Time, Space, Functions	Part I, Chaps. 1-4	2.5	3
Kinematics	Part I, Chaps. 5-6	2.5	3
Matter	Part I, Chaps. 7-10	1.5	4
Total	Part I	6.5	10
Geometrical Optics	Part II, Chaps. 11-15	4.5	7
Waves	Part II, Chaps. 16-19	4.5	7
Total	Part II	9	14
Motion and Gravitation	Part III, Chaps. 20-22	3.5	6.5
Momentum	Part III, Chap. 23	2	3
Energy	Part III, Chaps. 24-26	3.5	5.5
Total	Part III	9	15
Electricity and Magnetism	Part IV, Chaps. 27-31	4	8
Atomic Structure	Part IV, Chaps. 32-34	5	7
Total	Part IV	9	15

INTRODUCTION TO PART I

Part I is a general introduction to the fundamental physical ideas of time, space and matter. At this introductory stage, these ideas are not pursued deeply, but we are concerned that students begin to understand the process of physical inquiry. They should learn not just the results of measurement (these can be looked up when the occasion requires), but how measurement, associated instrumentation and mathematical tools extend our ability to comprehend the physical world. For this purpose, while the laboratory associated with Part I is easy, it is of great importance. This emphasis also characterizes the films for Part I. This theme does not require frequent iteration, but the understanding and satisfaction students get from the start in seeing how physics grows will go a long way in advancing your search for understanding rather than just "learning the facts".

Chapters 1-4 deal with the range of magnitudes of intervals of time and distance and how these intervals are measured. Appropriate mathematical tools are introduced.

Chapters 5-6 present, in kinematics, a more detailed study of time and space measurement. These chapters will require slower and more detailed study than the earlier chapters.

In the PSSC course, kinematics, the study of motion apart from its causes, is separated from dynamics, which includes the forces causing motion. Dynamics is the basis of Part III. This separation was made because kinematics fits naturally into a consideration of space and time measurement, to provide time for the kinematical ideas to mature before being used again in Part III, and to break up a rather long sequence of somewhat formal subject matter if kinematics and dynamics are studied sequentially. The work on kinematics will be reviewed naturally in the introductory work on dynamics.

Chapters 7-10 complete a preliminary examination of the range and nature of physics by considering various aspects of matter. Mass is introduced and its conservation is discussed. Evidence for the existence and sizes of atoms is examined. The arrangement of atoms in various states of matter leads to the idea of a physical model in the molecular interpretation of a gas. These subjects are not explored deeply because some of the tools for a more quantitative approach are not available at this stage. Quantitative treatments are presented in the latter sections of Parts III and IV.

Part I, through a broad view of the universe, is intended to help the student see that physics is a unified subject, not a collection of seemingly unrelated fields of study. The emphasis in the text, laboratory and films on the role of experiment should help the student see something of the unfinished and continuing nature of a science and that science does not "just exist"; it is the result of the imaginative work of men and women.

SCHEDULING PART I

Part I contains a wealth of interesting material. Most classes will not be able to cover all ten chapters in detail during a one-year course. You will want to seek quantitative facility with the ideas of Chapters 5 and 6. You will be able to adjust your depth of coverage of the other chapters (especially Chapters 7-10) to suit your class and schedule requirements.

The minimum class time required for various chapters of Part I will vary considerably from one class to another, depending chiefly on the nature of students' previous work in science and mathematics, and generally cannot be judged by the number of text pages given to the various subjects.

Two possible schedules for Part I are suggested in the table below. The six and one half week schedule is a good target for a one-year course and is consistent with the earlier suggestions for a one-year schedule for the course. The ten-week schedule provides for a more detailed treatment of the later chapters. The chapter summaries indicate sections which can be deemphasized or skipped in class discussion without loss of continuity.

	Ten-week schedule for Part I			Six-week schedule for Part I		
Chapter	Class Periods	Lab Periods	Experiments	Class Periods	Lab Periods	Experiments
1	2	-	-	-	-	-
2	3	1	I-1	2	1	I-1
3	2	2	I-2, I-3	2	1	I-2
4	6	1	I-4	4	1	I-4
5	7	1	I-5	7	1	I-5
6	7	-	-	6	-	-
7	3	3	I-6, I-7, I-8	1	2	I-6, I-8
8	6	-	-	2	-	-
9	3	1	I-9	2	-	-
10	2	-	-	-	-	-

Chapter 1 - What is Physics?

Chapter 1 is an introductory chapter, intended primarily as a reading assignment. It broadly sketches the province of physics in the general scientific world and the nature of physical investigation.

It is important to recognize that this introductory chapter is intended simply as qualitative background reading. It is not meant to provide a base for extensive class development. Detailed discussion of some of the experiments and instruments mentioned here would be either hopelessly superficial or over the heads of many students. Students with strong scientific backgrounds or those preoccupied with minutiae may press for immediate discussion of many of the topics alluded to in the text and figures. Such discussions are not to the immediate point, and easily can demoralize beginning students.

CHAPTER SUMMARY

Chapter 1 introduces students to the nature and scope of physics. The chapter outlines the relationship of physics to other sciences, discusses the relationship between science and technology, considers briefly the range of instruments and machines used to extend the senses and to create the special situations one wants to study, and reminds us that physics is built by people, generally by the coordinated efforts of many people.

The chapter introduces several of the themes that are exemplified in the structure of the course: Physics is more than a coldly factual summary of our knowledge of the physical world—it is human activity in which man's ideas about the physical world are constantly refined through checking against experiment and observation. Measurement reliability and the role of instrumentation are also introduced. Such ideas do not need to be hammered at this point. As the course develops you will have many opportunities to develop them out of concrete illustrations.

SCHEDULING CHAPTER 1

For instructional purposes the entire chapter can be treated as a unit. The chapter should be treated briefly. Primarily it should be an outside reading assignment. No class time need be devoted specifically to this chapter if you are on a short schedule. If you have the time, one or two days may be used for general discussions based on the chapter (see later suggestions for class discussion).

Some of the terms and figures in this chapter can send class discussion far afield unless it is limited to introductory ideas (see suggestions under CLASS DISCUSSION and COMMENT on pages 1-2 and 1-3). Students should be assured that they are not expected to understand all the terms in the text and figures of this chapter. Avoiding a quiz on the content of this chapter will contribute to putting the chapter in proper perspective.

RELATED MATERIALS FOR CHAPTER 1

There are no problems at the end of this chapter. A film is being prepared for use as an introduction to the PSSC course but it will not be available for class use in 1960-61.

Section 1 - Physics and the Other Sciences of Nature
Section 2 - Physics and Engineering
Section 3 - The Tools of Physics
Section 4 - Who Builds Physics?

PURPOSE Introduction to the general nature of physics.

EMPHASIS Treat briefly. Can be handled entirely as a reading assignment. A single period may be devoted to a general class discussion if desired. A possible topic for discussion is described below.

CLASS DISCUSSION 1. Ask students to name devices which extend or sharpen the senses. This part is easy: "Microscope, telescope, laboratory balance, etc."

Next ask for devices which respond to phenomena which cannot be detected with the senses. Some students will not be able to think of examples. Other students will propose incorrectly devices of extreme sensitivity which still only extend existing senses, for example: "the seismograph". (Since it responds to extremely minute mechanical vibrations in the earth it is just a very good "extender".) The first successful suggestion may be an esoteric one. In one class the first was "the radio-telescope"; then hands went up all over the room to suggest "the radio". The most elegant one: "the compass". Be prepared for some surprises.

You can consider briefly the general relationship between science and technology in the development of some of these devices.

2. Ask students to outline a possible chain of events linking two observations. Start simply, perhaps with the two observations:

1. A man steps on a pedal. 2. A car stops.

As the "chains" are built, what further questions suggest themselves as requiring an answer if the "chains" are to be accepted or understood thoroughly?

You can work up to complicated situations which can provide a challenge to your students' imagination. You may want to have students suggest some pairs of events.

The purpose of such a discussion is twofold: (1) To give the students some feeling for the way a scientist uses his background and intuition in trying to hypothesize a reasonable chain of events connecting observations. (Such a hypothesis would then need to be checked experimentally.) (2) To show that hypotheses and discoveries lead to new questions.

CAUTION Some students may consider the italics used with such words as Astronomy, Geology and Meterology as a clue for memorization. Discourage this. Be careful in your exams not to stress such matters. In this chapter, the "definitions" given are easy-going introductions and are not intended as complete characterizatons.

COMMENT To many laymen, there is little distinction between what is science and what is technology. Indeed, if these terms are to be distinguished (and most scientists would agree that they represent quite different activities), the distinction probably should be on the basis of the specific aims of a particular investigation, rather than on the instrumentation used, the difficulty of the task, or the training and experience of the investigator.

Science is directed toward a new understanding of relations in the physical world, quite apart from how this understanding may affect the world. Technology, in contrast, is directed toward a useful and hopefully beneficial end result. In this sense, the person who investigates the propagation of electromagnetic waves in various media, as an exercise in the solution of boundary-value problems, is a scientist, while another person, who may perform almost identical calculations but who is primarily interested in the design of a radar set for aircarft detection, is a technologist.

The distinction between these two kinds of human endeavor is frequently fuzzy, and certainly not highly important. Each depends upon and nurtures the other. The scientist, in his quest for understanding of physical relationships, may develop an idea or device of great technological importance. The transistor, a device of great practical significance, was discovered almost by accident by scientists investigating the surface properties of semiconductors. Similarly, instruments developed by technologists for specific ends often furnish the necessary tools for major scientific advances. What starts out to be science may become technology and vice versa.

* * *

The figures in Chapter 1 illustrate some of the points in the text in a qualitative way, and are not intended as springboards for launching into discussions of complicated scientific experiments. Figure 4, for example, is simply to illustrate that scientific discovery (in this case the discovery of radioactivity) may often be made with very simple equipment. Figures 5 and 6 in contrast, show that some scientific tools are extremely complicated. Students may be intrigued by the fact that the enormous machine shown in Figure 5 is needed to observe a few of the characteristics of the behavior of submicroscopic particles.

Again, Figures 6 (a) and 6 (b) illustrate that complicated tools are sometimes used to provide information about simple interactions that cannot otherwise be seen. In Figure 6 (a), the bubble chamber (a device for detecting atomic particles, which is discussed in Chapter 7) is located inside a magnet, about halfway up on the right of the figure.

Figure 6 (b) may raise a number of questions: "Why the spiral?", "Does it spiral in or out?", "Why the little spirals?", "Is this a real photo or was the spiral drawn in?", etc. Extensive discussion of this figure is not to the point. If students ask, this is a real photo, not a fake. The particle spirals in as it loses energy and follows a curved path because of a magnetic field perpendicular to the direction of motion. The little spirals are caused by secondary low-energy particles which the primary particle knocks out of other atoms as it goes by. Only charged particles leave trails in bubble chambers. The trails are caused by tiny bubbles condensing on the ionized particles left in the wake of the moving charged particle.

* * *

Some students may be interested in some of the facts regarding physics as a profession. In this country there are about 20,000 practicing physicists. They are largely employed in industry (about 1/2), colleges and universities (about 1/3) and Government, private foundations and other non-profit organizations (about 1/6).

Like other Americans, most physicists marry, have children and belong to PTA's; some play golf and bridge and watch westerns on TV. Economically they are at about the same level as any university professor. Opportunities in the profession are essentially unlimited, without regard to race, religion or sex.

Physics is by no means restricted to people with a Ph.D. degree. There are many opportunities, particularly in industry and teaching, for persons with the broad scientific background afforded by a four-year bachelor's degree program in physics. In recent years, there has been an increasing trend for many college students who plan to enter engineering as a profession to take an undergraduate degree in physics. This broad scientific training, far from inhibiting their specialization in a single field of engineering, appears (at least in the important eyes of employers) to make physics graduates desirable in most fields of engineering.

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Chapter 2 – Time and Measurement

The opening sections of this chapter serve as an introduction not only to the measurement of time, but to one of the fundamental problems of physics (and of the teaching of physics): that of going beyond our familiar senses and experiences to develop an orderly picture of the physical world—a world whose aspects are often too big, too small, too fast, too slow, too complex, or too far away to comprehend directly through the senses.

Chapter 2 divides naturally into three parts: 1) an introductory discussion of the idea of measurement and its relation to the senses, and which identifies time as a physical variable; 2) a quantitative discussion of measuring time intervals by multiple-flash and stroboscopic techniques; and 3) a reminder that time measurement entails counting arbitrary units in an interval, and the introduction of power of ten notation and the concept of order of magnitude which leads to a discussion of the range of time intervals encountered in physics.

The qualitative material of this chapter can be covered quickly. Even the quantitative methods for measuring time intervals, can be covered quite rapidly. Many of the topics mentioned in the text, e.g., the range of time intervals, the direction of time, counting units, etc. should be treated as background information. Power of ten notation will be used throughout the text, and, if unfamiliar to students, should be stressed. Order of magnitude calculations are useful, but they are often difficult for beginners and should not be unduly emphasized at this point. Facility will come with practice on problems through the rest of the course.

Part 1 of the course is concerned with our fundamental ideas of time and space (and their combination in motion) and matter. In the organization of the course we could, perhaps, have started with space instead of time. The principal reason for beginning with time is one of simplicity: the description of space requires three dimensions, while time requires only one.

CHAPTER SUMMARY

Sections 1 through 3 These sections are introductory, and relate the measurement of time to the senses of the observer. Instruments are seen to be simply extensions of direct sensory perception. It is noted that all measurements involve detection of differences.

Sections 4 and 5 These sections deal with quantitative measurement of time intervals by multiple-flash and stroboscopic techniques for observing successive positions of physical systems.

Sections 6 through 8 We note that time intervals are measured in terms of arbitrary units. Power of ten notation is introduced as a convenient shorthand for handling very large and very small numbers of units. The idea of the order of magnitude of a physical quantity is introduced as the specification of the nearest power of ten of the units in which the quantity is measured. The chapter concludes with the observation that time has a "natural" forward direction.

SCHEDULING CHAPTER 2

The entire chapter can be given as a single reading assignment prior to class discussion. Experiment I-1 should be used prior to class discussion of Sections 4 and 5 to clarify that discussion. Class discussions can be developed around the films described below. Some problems relating to Sections 4, 5, and 7 should be discussed in class and assigned as homework.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part 1. Sections which are enclosed by brackets [] can be deemphasized or skipped in class discussion without loss of continuity. Even though these sections may be deemphasized in class, they should be assigned for home reading.

	10-week schedule for Part 1			6-week schedule for Part 1		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. [1, 2, 3]	0	0	-	0	0	-
Secs. 4, 5	2	1	I-1	1	1	I-1
Secs. [6], 7, [8]	1	0	-	1	0	-

RELATED MATERIALS FOR CHAPTER 2

Laboratory. Experiment I-1 - Short Time Intervals. This experiment employs the simple stroboscope and the paper-tape timer. Beyond their utility at this point for measuring short time intervals and introducing students to some of the problems of instrumentation, both these devices will be used extensively in later laboratory work. Hence, students should become familiar with the operation and characteristics of these devices as measuring instruments. For answers to questions in the students' Laboratory Guide and for suggestions on conducting the experiment, see the yellow pages on Experiment I-1.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 2-9; detailed comments and solutions on page 2-10 to 2-16.

Sections	Easy	Medium	Hard	Class Discussion	Home Projects
1, 2, 3	1, 2, 3			1, 2, 3	4, 5, 6
4	8, 9*	7*, 10*, 11		10	
5	14	12	13*	14	15
6		16*			
7	17	18*, 19, 20*, 21		19	

Films. "Short Time Intervals" by Campbell L. Searle of the Massachusetts Institute of Technology. This film shows techniques and applications of measuring short time intervals (a few seconds to a few microseconds) through a series of demonstrations: motion, still, and open-shutter moving-camera pictures of lightning flashes; moving pen recording distance against time; camera on turntable sweeping a neon tube enabling measurement of the frequency of the light pulses from the tube; analysis of lightning flashes; oscilloscopic analysis of motion of a gear wheel with a high-speed stroboscopic light source. The film concludes with a summary pointing out how these and other techniques have extended man's senses.

"Time and Clocks" by John G. King of the Massachusetts Institute of Technology. This film demonstrates various methods of measuring time intervals from 1 second to 10^{-21} seconds: mechanical clocks, high speed photography (movie and multiple-flash), cathode-ray oscilloscope, and atomic clock.

"Long Time Intervals" by Harrison Brown of the California Institute of Technology. This film starts with the questions "How old is the earth?" and "Has time always existed?" It traces time measurement from the ticking of clocks through the sun in its orbit as a historic clock. Ultra-historical "clocks" must be recorded: annular rings in trees, rock strata, radioactivity in rocks. The half-life concept and the processing of granite for the extraction of zircon to test for lead 206 are discussed.

Any or all of these films can be used profitably. If they are all used, the order in which they are described above is probably the most natural. If it can be arranged, it probably is wise to complete the laboratory experiment I-1 before using "Short Time Intervals" or "Time and Clocks".

Demonstrations If a good laboratory stroboscope is available it can be used in illustrating the ideas in Section 5. However, demonstrations are not an adequate substitute for laboratory work on Experiment I-1.

Section 1 - The Starting Point--The Senses
Section 2 - The Key Concepts of Physics; the Need to Extend the Senses
Section 3 - Time and Its Sweep

PURPOSE To specify what is meant by a measurement of "time", in relation to the senses of the observer and to changes in the physical world.

EMPHASIS This is introductory material which can be handled as a reading assignment. Detailed class discussion may be more confusing than helpful. Some of the ideas discussed are rather profound, and they will be reiterated later in the course. Deep understanding is not needed at this point.

COMMENT Compared to laboratory instruments, human senses are excellent for some measurements and rather crude for others. For example, we can discern small differences in color or pitch amazingly well. On the other hand, it is extremely difficult for us to measure directly any event which occurs in an elapsed time of less than a tenth of a second or to measure a period greater than perhaps a day. Instruments extend our capability of sensing time to the measurement of the wide range of time intervals with which physics is concerned.

Our senses measure differences. We have no built-in method of establishing a quantitative measure of a single physical quantity. If two bowls of water are at slightly different temperatures, it is rather easy to tell which bowl is the warmer (providing the difference is not too small) but it is difficult for us to tell the temperature of either bowl at all closely.

In these sections the student is introduced to what is meant by time in physics. When we speak of time we mean time intervals. Time intervals have meaning only in so far as we can observe changes which occur during those intervals.

Phrases such as "a very short time" and "a very long time" clearly have only relative meaning. Their significance comes only from a comparison of relative time intervals that are applicable to a particular system of physical events. A very short time in human experience, such as a millionth of a second, may be a very long time in the life of a radioactive nucleus. If we could open our eyes for only a few millionths of a second and observe the clock on the wall, we would be hard put to decide whether it was running. We would see only an instant of its cycle. However, if we could open our eyes for the same interval at the instant an atomic bomb detonates, (assuming we had on suitably dark glasses) we could observe nearly the entire life cycle of the device. To observe the "life cycle" of the universe, we would have to watch for untold billions of years.

Section 4 - Time Intervals, Long and Short; Multiple-Flash
Section 5 - The Stroboscope

PURPOSE To show how physical changes, occurring in time intervals which are longer and shorter than can be observed directly, can be measured through photographic and stroboscopic techniques. Periodic illumination, exposure, or viewing enable the measurement and analysis of motions that are too rapid or too slow to sense directly.

EMPHASIS The concept of describing motion in terms of samples of position vs. time is important, and will be used extensively in Chapter 5. However, this is a natural concept and should give little trouble. The emphasis here should be on measuring time intervals not on the analysis of motion per se, and particularly not on the mechanics of complicated flash and stroboscopic apparatus. Beginning the development of these sections with a laboratory session on Experiment I-1 will contribute both to general class discussions and to the meaning of the films you may use.

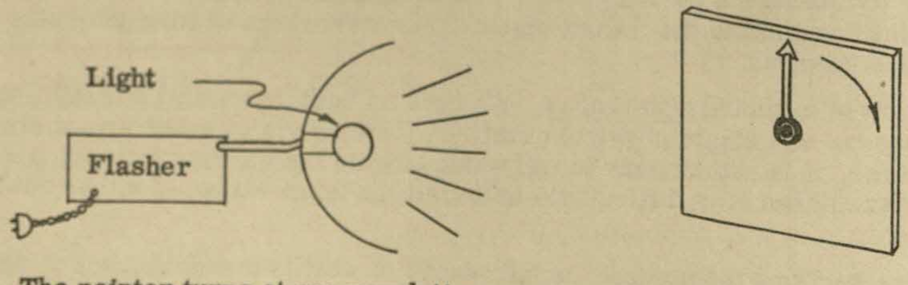
The operation and calibration of the instruments used in Experiment I-1 should be understood. The stroboscope will be used for observation of periodic waves in Part 2. The paper tape timer will be used in laboratory work on kinematics and again in Part 3.

DEVELOPMENT As indicated, after students have read the sections, Experiment I-1 is a good starting point. Students will learn much more about timing short intervals from performing this experiment than from class discussion which is not preceded by the experiment.

The films "Short Time Intervals", "Time and Clocks", and "Long Time Intervals" are appropriate at this point.

In class discussion subsequent to the laboratory, considering the following situations may help you ensure students have understood the operation of the stroboscope:

Imagine you have a pointer which can turn at various constant speeds. In addition, you have a system for flashing on the room lights at various regular intervals.



Case 1. The pointer turns at one revolution per second, and the lights flash on once per second. Since the pointer would appear to be in the same position each time the room lights flashed on, it would appear stationary.

Case 2. The pointer turns at two revolutions per second, and the lights flash once per second. Again, the pointer appears to be standing still, although it actually has turned two full revolutions between light flashes. Extend this model to the case where the pointer revolves three, four, five, etc., times per second. The students should quickly see that with this flashing rate the pointer will appear "stopped" when it turns at any whole number of revolutions per second.

Case 3. The pointer turns at 50 revolutions per second, and the lights flash 50 times per second. The situation is like case 1, and the motion appears stopped. Repeat by extending to case 2 and showing that the motion is also stopped if the pointer rotates at frequencies of 100, 150, 200, etc. times per second, that is at any whole multiple of the rate of observation. (We usually say that the observation frequency is an exact sub-multiple of the rotation frequency.)

Case 4. The pointer turns 60 times per second and the lights flash 30 times per second. Notice that this is like case 3—the pointer appears to be standing still.

Case 5. Try the reverse of case 4—the pointer turns 30 times per second and the lights flash 60 times per second. If the first flash shows the pointer at "12 o'clock", the second shows it at "6 o'clock", the third at "12 o'clock", etc. The pointer appears to be stationary at both "12 o'clock" and "6 o'clock". Thus we observe an absurd result, but one which we can easily interpret because we know it is absurd. If the flashing rate is greater than the turning rate, we will never see a sensible "stationary" pattern. This gives us a clue for how to adjust the flashing rate to be the same as the turning rate. To adjust the flashing rate to equal the turning rate, we go to the maximum flashing rate where a sensible "stopped" pattern is observed.

COMMENT In discussing the stroboscope, emphasize its use as a tool for measuring short periods. It is probably best to avoid extended discussion of the familiar "stroboscopic effect", the apparent slow forward or backward motion of a rapidly moving body when viewed at a period slightly different from the natural period. The point to make is that the motion will appear stopped when the viewing rate is any integral sub-multiple of the period of the motion, and that the period may be measured by observing the maximum viewing rate at which the motion appears stopped.

[If students persist in discussion of the stroboscopic effect, this is a good problem to have them consider at home, preferably by doing home experiments. If the viewing frequency, f_v is slightly less than the frequency of motion, f_m, the object will appear to be moving forward at a frequency $f_m - f_v$; if f_v is slightly greater than f_m, the object will appear to be moving backward at a frequency $f_v - f_m$.]

Section 6 - Comparing Times; Counting Units

CONTENT a. Counting is the basis of physical measurement.

b. The choice of a standard unit of time is arbitrary.

EMPHASIS Students should understand this section, but a moderately brief treatment should suffice. The section can be treated simply as a reading assignment.

COMMENT Students should realize that the fundamental operation in making most measurements is counting. When someone says that a room is 20 feet long, he means that a standard foot rule can be laid end to end and fitted into an edge of the room a total of 20 times.

With time, we can't get a standard second back again to measure off a time interval. We need a clock to keep on turning out standard seconds for us to count. But how do we know each one-second interval is like any other? This problem involves us in calibration procedures. Any timing device must be calibrated by comparing the standard intervals it generates with those generated by some other independent device. If the two devices agree, we may be satisfied. If they disagree, we must compare each of them with a third device. Our standard second is measured in terms of a particular device in which we have great confidence because of its agreement with more other independent devices than any other kind of clock. Quartz crystal oscillators, such as used by the National Bureau of Standards to generate time and frequency signals which are broadcast over radio station WWV (operates at 2.5, 5, 7.5, 10, 15, and 20 megacycles on the short-wave bands) are reasonably reliable standards, but considerably inferior to the modern atomic clock. At present, our primary standard of time is still based on measurement of the period of rotation of the earth.

Students should also realize that most of our standard units were arbitrarily chosen. They do not occur in nature. In many cases, they were probably chosen for convenience. There is nothing terribly special about measuring time in seconds, instead of some other unit we might invent.

An interesting class discussion which emphasizes the arbitrariness of time units (and points toward the problem of standards which is considered in the next chapter) can be developed around the invention of a new time unit. You might call it the "augenblick" (or some other name based on your school's name). Suppose one augenblick is equal to, say, 15.2 seconds. Have each student compute his age in augenblicks. (1 or 2 figure accuracy is sufficient and is good preparation for the next section.) Now you are in a position to make an important point. Probably everyone in your class would agree that it would be extremely difficult to work with the augenblick as a unit of time. But suppose there was a country in which the augenblick was a standard unit with which children were completely familiar. Suppose that in their high school physics class their teacher proposed inventing a new unit of time, "the second", which is 1/15.2 of an augenblick. Such an imaginary class would readily agree that the second would be a difficult unit to work with. Whatever units you are accustomed to, seem to be the easiest ones. In particular, a new unit seems difficult if you must always convert your measurements back to a familiar unit. A discussion such as this can give students a little background for some of the problems they often encounter in beginning to use the metric system (introduced in the next chapter).

If you use a discussion like the above you can lead toward the problems of standards and the arbitrariness of units (next chapter) by pointing out that although standard units are arbitrary, if people are to understand each other, they must agree on a definition. This is one of the reasons that a bureau of standards is required in every country.

Section 7 - Times Large and Small — Orders of Magnitude

PURPOSE To indicate the enormous range of time intervals which can be apprehended with the aid of instruments, and to introduce power of ten notation.

EMPHASIS Many students may already be familiar with power of ten notation. If so, this section may be treated briefly. Even if this notation is new to your students, intensive practice is not needed at this point since the notation will be used frequently throughout the course, and will become familiar through use. Similarly, order of magnitude calculations should not be stressed beyond what is called for in the solution of problems at the end of the chapter. Facility will be achieved more rapidly through use in laboratory and in a variety of real problems rather than drill on contrived exercises.

DEVELOPMENT As a "starter" for class discussion of power of ten notation and order of magnitude calculations, you might give the class the following quiz to be completed in five minutes:

> Estimate to the nearest power of ten (that is, find the order of magnitude of) the number of ping-pong balls that it would take to fill completely this classroom. Do not make any measurements or leave your seat until you have completed your estimates.

Students are likely to complain, "We don't know the dimensions of the room!" "How big is a ping-pong ball?" Most students will work in the English system and carry out elaborate computations. In classes in which this has been tried, less than a third of the class arrived at a satisfactory answer, and most of these did too much computation. But they were ready to learn something about estimation.

After the quiz, show them how to rough out such an estimate. Ask a student how long the room is. A likely reply is, "I don't know". In answer to further questions he may even claim that he has no idea how long the room is. This is a notion that must be dispelled. Ask him if the room is more than an inch long! The fact that this is a joke indicates that he knows a great deal about the length of the room. Is it a mile long (another joke)? Thus we know that the length of the room is more than an inch and less than a mile. We know a lot! Can we do better than that? For most classrooms, students will be able to agree that the length is more than 10 and less than 100 feet long. A typical classroom might be about 30 feet long, 20 feet wide, and 12 feet high. It is not important, in making an order of magnitude estimate, whether the room is 20 feet or 25 feet in width. The order of magnitude of the volume of most classrooms will be between 10^3 and 10^4 cubic feet.

Similarly, while students may not know the precise dimensions of a ping-pong ball, they will probably concede that it would take nearer 10 balls in a row to make up a foot than 1 or 100. Thus we can estimate that a cubic foot would contain about 10^3 ping-pong balls. The entire room would then hold between 10^6 and 10^7 ping-pong balls.

As a check that the class is getting the idea, you might repeat the quiz on the next day, this time asking how many basketballs would be required to fill the school building.

COMMENTS Table 1 is intended to stimulate reflection, not memorization. It is meant only to give students a feeling for the range of time intervals we are able to sense with the help of physical instruments.

A question which frequently arises in discussions of order of magnitude calculations is where to draw a dividing line between two adjacent powers of ten. While this is not a matter of great importance, the dividing line is ordinarily taken to be the square root of 10. Thus numbers less than 3.16 but greater than 0.316 would be of order of magnitude 1, numbers between 3.16 and 31.6 would be of order of magnitude 10, etc. If the distinction between numbers like 300 (order of magnitude 10^2) and 400 (order of magnitude 10^3) is important in determining an answer, it is best simply to refine the calculation to carry one significant figure in addition to the order of magnitude (i.e., 3×10^2). With order of magnitude estimations we are simply trying to get "in the right ballpark" and we are not troubled by knowing that we may be off by a factor of 10 either way.

Section 8 - The Direction of Time

PURPOSE From the observation of our senses, it seems to us that time has a "natural" forward direction.

EMPHASIS Treat briefly, as a reading assignment.

COMMENTS We always like to talk about the "relentless march of time", and it is fun to look at moving pictures which are running backwards or running speeded up. From such familiar experiences we generally tend to think of time as having a "natural" forward direction and a "natural" rate. Any natural direction or rate of time is meaningful only in terms of extremely complicated situations which occur during such periods.

Whether or not time has a "natural" direction in a fundamental sense is a matter of considerable scientific interest, but beyond the scope of the present course. In order to avoid probably extended and most certainly fruitless class discussion at this level, treat this present section quite briefly. For some of the best students, there have been interesting articles about "time reversal" in the Scientific American.[1]

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[1] John M. Blatt, "Time Reversal", Scientific American (New York: Scientific American, Inc.), Vol. 195, No. 2 (August, 1956), pp. 107-114.

"Letters" (number 3) Scientific American (New York: Scientific American, Inc.), Vol. 196, No. 2 (February, 1957), p. 10.

Chapter 2 - Time and Measurement

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 2-10 to 2-16 .

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1-3	1, 2, 3			1, 2, 3	4, 5, 6
4	8, 9*	7*, 10*, 11		10	
5	14	12	13*	14	15
6		16*			
7	17	18*, 19, 20*, 21		19	

SHORT ANSWERS

1. See discussion on page 2-10.
2. See discussion on page 2-10.
3. See discussion on page 2-11.
4. A home project.
5. A home project. See discussion on page 2-11.
6. A home project. See discussion on page 2-12.
7. 3600 flashes/sec.
8. 0.003 sec.
9. Friend is correct.
10. 5 min.
11. (a) 140 ft.
    (b) 2.4×10^{-6} sec.
12. 80 times.
13. (a) 8, 16, 24, 32, 40, 48, ... etc. revolutions/sec.
    (b) 32, 64, 96, ... etc. revolutions/sec.
14. 0.42 sec.
15. A home project.
16. (a) 10^4 life spans.
    (b) 10^{19} times.
17. (a) $1.76 \times 10^8 \approx 10^8$

(b) $\$7.1 \times 10^{10} \approx \$10^{11}$

(c) $\$4 \times 10^{2} \approx \$10^{3}$

18. (a) 10^{12} miles in 1 year.

    (b) 10^{3} sec.

19. (a) 1.66×10^{-1}

    (b) 5×10^{5}

    (c) 3.52×10^{-8}

20. (a) 10^{6} ticks/day.

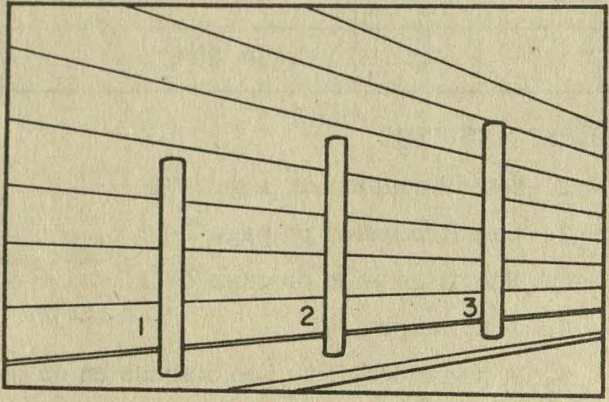
    (b) 10^{8} ticks/yr.

21. 1.6×10^{8}

SOLUTIONS AND COMMENTS

PROBLEM 1

Compare the apparent heights of columns 1, 2, and 3 in Fig. 2-9. What is the actual relationship of heights? Did you use a device to check up on your senses?



To most people the columns to the right seem progressively taller because the background of converging lines suggests they are further away. In a perspective drawing, the columns to the right should be progressively "smaller" if they are to appear to be the "same" size.

If you discuss this problem it may be well to point out that there are several different ways to check up on the actual heights. Some students will think only of a ruler. But units are not important here, only a comparison. One could mark the height of one column on the edge of a scrap of paper and compare this length with the other heights. From the tip of the thumb to a handy wrinkle in the knuckle might provide another quick measure. The straight edges of sheets of paper could be laid along the tops and bottoms of the columns, then estimate whether the paper edges are parallel. For comparative measures that need to be more precise, a pair of dividers is handy.

PROBLEM 2

Check your time sense by trying to estimate when an interval of $\frac{1}{2}$, 1, or 3 minutes has elapsed.

(a) Record your error in each case.

(b) Are the errors appreciably different fractions of the time intervals involved?

This problem can be handled nicely by students working in pairs. Expressing the errors as per cents of the various intervals will facilitate comparison of the errors for intervals of different lengths.

PROBLEM 3

Since it is difficult to see a watch in a photographic darkroom, some photographers have learned to measure time in seconds by counting at their normal conversational rate but interspersing a long word between numbers, as "Mississippi one, Mississippi two, Mississippi three . . ." Try this method and compare its accuracy with that of Question 2.

This method is almost sure to increase the reliability (reproducibility) of a time measurement. The same person repeating a measurement with the same method is likely to come out with a similar result. Increased accuracy is also likely but a little practice may be required to get a rhythm that closely approximates seconds.

PROBLEM 4

Make a pendulum from a piece of string and a small weight. Adjust the length until it takes one second to make a complete swing to and fro. How great is the error of your pendulum over a one-minute period? What fraction of the total time is the error?

Students who are on their toes may immediately see the advantage of checking a time interval for a number of swings instead of only one. Even with relatively crude equipment the student should get the feeling that it is possible to reduce the error to an extremely low point. Pendulums are used in clocks for a purpose!

PROBLEM 5

A project for a homemade timepiece. Galileo performed some of the first important experiments with the motion of falling bodies before the days of accurate clocks. To measure the short time intervals involved in his experiments, he used a simple clock that you can make for yourself. In the bottom of a tin can, punch a small hole with a nail. Keep the can nearly full of water and measure the amount of water that falls out in 10, 20, and 30 seconds. In this way you can learn to read the clock in seconds.

What are the major sources of error in this device? Can you reduce the errors? Remember Galileo used a clock like this in discovering some of the major principles of physics. (See "Two New Sciences" by Galileo.)

In this project, the amount of water can be measured either by weight or by volume. For example, the stream of water can be caught in a container during the interval to be measured, then poured into a graduated cylinder for measurement of its volume.

The major sources of error in such a device would include possible awkwardness in turning the device on and off at the correct instants, and variation in rate of flow with pressure differences if the level of water in the "clock" is allowed to vary. Students should be able to think of ways to improve both factors. An overflow device which would permit a continuous input and a constant level of water is one possible improvement.

PROBLEM 6

Galileo, in observing the swinging of a cathedral lamp, "found that the same number of his pulse beats marked each swing regardless of the distance the lamp traveled during each swing." Check on the correctness of this statement by constructing and timing a simple pendulum. If you find difficulty in counting your pulse beats and pendulum swings at the same time, try working with a partner or using a watch (which Galileo did not have available).

This project will clearly show that the period of a pendulum is independent of the amplitude of its swing. (It is independent at relatively small amplitudes. At large amplitudes, the period is greater than for small amplitudes.)

PROBLEM 7

How many times per second should a light flash to show images 10 inches apart of a projectile traveling 1000 yards per second?

Since 1000 yards = 36,000 inches, the speed is 36,000 inches/sec. We want the light to flash every 10 inches of the projectile's trip. In 1 sec, the number of times the projectile will travel 10 in. (the number of flashes/sec required) is

$$\frac{36,000\ \text{in/sec}}{10\ \text{in/flash}} = 3600\ \text{flashes/sec.}$$

A light flashing 3600 times/second would show images 10 inches apart.

PROBLEM 8

A study of the entire series of pictures of which the strip in Fig. 2-3 is only a part would show that the balloon was completely deflated 12 frames after the bullet entered it. What period of time does this represent?

The interval between frames is 1/4000th second. If the balloon is completely deflated 12 frames after the frame which shows the bullet entering the balloon, the time required for deflation is 12 × 1/4000 = 0.003 sec.

PROBLEM 9

A student has a movie projector which shows pictures at the rate of 16 frames per second. His movie camera can take pictures at various speeds. He wishes to take pictures of a football game so that the action will be slowed down to ½ normal speed when projected. He wants to set the speed of the movie camera at 8 frames per second but his friend insists he should set it at 32 frames per second. Who is correct? Why?

The friend is correct. The camera must run faster than the projector. When the camera takes pictures 1/32 second apart in time, the changes in position of the players are half as great as in 1/16 sec. Thus, when these same pictures are shown 1/16 sec apart, the speed of motion is cut in half.

Some students may understand this situation better through an alternative way of looking at it. At 16 frames per second, three seconds of action on the football field will result in 3 × 16 = 48 frames, which projected at 16 frames per second will show this action taking place in three seconds. If taken at 32 frames per second, the three seconds of action will be recorded on 3 × 32 = 96 frames. If these 96 frames are projected at the normal 16 frames/sec rate, the original three seconds of action will be displayed in 96/16 = 6 seconds.

PROBLEM 10

In time-lapse photography, plants and their blossoms may be seen to develop at a rate comparable to that of the motion of animals. If the total growth of a plant which would require 50 days is to be shown on a 10-minute film, what interval should elapse between the successive pictures? (Assume that the film is to be shown at 24 frames/sec.)

Dividing the total growth time by the number of frames in a 10 min film will give the required interval between successive pictures.

$$\frac{\text{growth time in min}}{\text{frames in 10 min film}} = \frac{50 \text{ days} \times 24 \text{ hrs/day} \times 60 \text{ min/hr}}{24 \text{ frames/sec} \times 60 \text{ sec/min} \times 10 \text{ min}} = 5 \text{ min/frame}$$

A picture should be taken every 5 min.

PROBLEM 11

The shutter speed of a camera is $\frac{1}{25}$ of a second. How far would a rifle bullet travel while a student attempts to take its picture with a $\frac{1}{25}$-second exposure? (The bullet's speed is 3500 ft per sec.) What shutter speed would be necessary to limit the bullet's movement to 0.1 inch during the exposure?

a) In $\frac{1}{25}$ sec, the bullet travels 3500 ft/sec × $\frac{1}{25}$ sec = 140 ft.

b) 3500 ft/sec = 12 × 3500 inches/sec = 42,000 in/sec.

Thus the bullet travels .1 inch in $\frac{0.1 \text{ in}}{42,000 \text{ in/sec}} = 2.4 \times 10^{-6}$ sec.

PROBLEM 12

A 10-slit stroboscope is rotated at a constant rate for 20 seconds to determine that a motor shaft takes $\frac{1}{40}$ sec to make one rotation. How many rotations does the stroboscope make?

The stroboscope moves 1/10 of one revolution (from 1 slit to the next) during the 1/40 sec in which the motor shaft rotates once. Then the speed of rotation of the stroboscope is $\frac{1}{10}$ rotation ÷ $\frac{1}{40}$ sec = 4 rotations/sec. In 20 seconds, it rotates 20 × 4 = 80 times.

PROBLEM 13

A stroboscope disc with 4 openings must be rotated 80 times in 10 seconds to make a fan with four identical blades appear to stand still. What are the possible rates of motion of the fan? How would your answer be different if one of the blades was a different color from the others?

This problem is a bit complicated and should not be assigned before a simpler development (such as described on pages 2-4 and 2-5 of the guide) has been done in class. The frequency of observation is 4 × 80/10 = 32 per second. For students who are overwhelmed with this problem, you can convert it into a somewhat easier problem by proposing a "one-bladed fan" rather than a four-bladed fan. This is really the same problem as is given in the second part where one blade has a different color but may be a somewhat easier way for students to understand it. For such a fan to appear "stopped" it must make either one complete turn between observations or two complete turns or three complete turns, etc. Thus it would appear stopped at revolution frequencies of 32, 64, 96, etc. Assuming that the four blades of the fan are identical and are arranged symmetrically on the shaft, the motion will appear "stopped" if the fan turns any integral number of 90° angles between observations. The lowest possible rate of rotation of the fan is therefore 32/4 = 8 revolutions per second. Other possible rates are 16, 24, 32, 40, 48, ... etc.

There is a quibble here which some of your good students may bring up. When we say that a fan or other rotating object appears "stopped", we mean that we see an apparently stationary image---what we see looks the way the fan would look if it were stopped (excepting dimness of the image and such details). There are observation frequencies that will cause a four-bladed fan to look like a stationary eight-bladed fan. We are not including such cases when we say that the fan appears "stopped" Of course, you can generate many problems by taking up such cases, but it is unlikely that students would learn enough extra to be worth the complexity.

PROBLEM 14

A disc stroboscope contains 4 slits and is turned 6 times in 10 seconds. What time interval can be measured by it?

The time interval between the passage of two consecutive slits is

$$\frac{10 \text{ sec}}{6 \times 4} = \frac{10}{24} = \underline{0.42 \text{ sec.}}$$

This is the time interval that can be measured.

PROBLEM 15

Use a stroboscope (see Laboratory Guide) to look at a phonograph turntable, a dripping faucet, a fluorescent lamp, a fan, a TV screen, a small weight tied to a string and whirled about someone's head, and a falling weight. For which of these motions can you measure the time interval?

In this home project students should find that it is easy to measure stroboscopically the time interval for almost any repetitive phenomena. The falling weight is not in that category, but ingenious students may come up with ways to do it.

PROBLEM 16

(a) How many human life spans have elapsed since earliest man?

(b) Approximately how many times would an air molecule spin around its axis while the earth revolves once around the sun? See Table 1.

a) From Table 1, a human life span is about 10^9 seconds, and 10^{13} seconds have elapsed since earliest man. The number of life spans elapsed since that time = $10^{13}/10^9 = \underline{10^4 \text{ human life spans}}$.

b) An air molecule spins once in 10^{-12} seconds. The earth revolves around the sun in 10^7 seconds. In a year, the number of times an air molecule would spin = $10^7/10^{-12} = \underline{10^{19} \text{ times}}$.

PROBLEM 17

(a) In 1959 the population of the United States was about 176,000,000. Express this number in powers-of-ten notation. What is the order of this magnitude?

(b) Express as a power of ten the budget of the United States for a year when it is 71 billion dollars.

(c) Use the powers-of-ten notation to determine the average amount of the budget that must be paid by an individual citizen.

a) $176,000,000 = 1.76 \times 10^8$. The order of magnitude is $\underline{10^8}$.

b) $\$71,000,000,000 = \$7.1 \times 10^{10}$. The order of magnitude is $\underline{\$10^{11}}$.

c) If the population is 1.76×10^{8} when the budget is $\$7.1 \times 10^{10}$, then the average amount that must be paid by an individual is $\$7.1 \times 10^{10}/1.76 \times 10^{8}$. The average payment per citizen is $\$4 \times 10^{2}$. The order of magnitude is $\underline{\$10^{3}}$.

PROBLEM 18 Express only the order of magnitude of your answer to the following:

(a) Light travels on the order of 10^5 miles each second. How far will light travel in a year?

(b) Calculate the length of time it takes the light from the sun to travel the 93 million miles to the earth.

a) Since there are 10^7 seconds in a year, light travels

$$10^5 \text{ mi/sec} \times 10^7 \text{ sec/yr} = \underline{10^{12} \text{ mi/yr}}.$$

b) The order of magnitude of 93,000,000 miles is 10^8 miles. Traveling at the rate of 10^5 miles/sec, light requires 10^8 miles $\div 10^5$ miles/sec $= \underline{10^3 \text{ seconds}}$. Some students may remember the answer to be 8 minutes. Since 8 is of order 10, and a minute is of order 10^2 seconds, 10^3 seconds is the right order of magnitude.

PROBLEM 19 Using powers-of-ten notation, find the following:

(a) 0.00418×39.7

(b) $\frac{6000}{.012}$

(c) $\frac{.703 \times .014}{280{,}000}$

a) $0.00418 \times 39.7 = 4.18 \times 10^{-3} \times 3.97 \times 10^{1} = 4.18 \times 3.97 \times 10^{-2} = 16.6 \times 10^{-2}$ $= \underline{1.66 \times 10^{-1}}$

b) $$\frac{6000}{.012} = \frac{6 \times 10^3}{1.2 \times 10^{-2}} = \underline{5 \times 10^5}$$

c) $$\frac{.703 \times .014}{280{,}000} = \frac{7.03 \times 10^{-1} \times 1.4 \times 10^{-2}}{2.8 \times 10^5} = \frac{7.03 \times 10^{-3}}{2 \times 10^5} = \underline{3.52 \times 10^{-8}}$$

PROBLEM 20 A watch ticks 5 times each second. Expressing only the order of magnitude of your answer, find how many times it ticks:

(a) during a day.

(b) during a year.

If we use only orders of magnitude, we would have

$$\text{Ticks/day} = 10 \, \frac{\text{ticks}}{\text{sec}} \times 10^2 \, \frac{\text{sec}}{\text{min}} \times 10^2 \, \frac{\text{min}}{\text{hr}} \times 10 \, \frac{\text{hrs}}{\text{day}} = \underline{10^6}$$

$$\text{Ticks/yr} = 10 \, \frac{\text{ticks}}{\text{sec}} \times 10^2 \, \frac{\text{sec}}{\text{min}} \times 10^2 \, \frac{\text{min}}{\text{hr}} \times 10 \, \frac{\text{hrs}}{\text{day}} \times 10^2 \, \frac{\text{days}}{\text{yr}} = \underline{10^8}$$

RELATED MATERIALS FOR CHAPTER 3

Laboratory. Experiment I-2 - Large Distances. In this experiment students become familiar with the use of triangulation methods through the use of two devices, the range finder and the parallax viewer.

You will note that in the experiment, the theory of the range finder is not worked out. It is simply calibrated on known or measurable distances. On the other hand, the theory of a parallax measurement is worked out, so that students can see how to go beyond the range within which calibration is possible. This is a minor illustration of how science goes ahead – a combination of checks. See the yellow pages on Experiment I-2 for suggestions.

Experiment I-3 - Small Distances. In this experiment students assemble and calibrate an optical micrometer which is capable of measuring distances of the order of 10^{-4} cm. See the yellow pages on Experiment I-3 for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 3-11 ; detailed comments and solutions on pages 3-12 to 3-23 .

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1			1	2
2		4, 9*		10	3*, 5, 6*, 7, 8
3	11			11	
4	12, 13, 14	15			
5		16*, 17		16*	
6		18		18	
7	23, 24	19, 20*, 21 22		21, 24	

Films. "Measuring Large Distances", by Dr. Fletcher Watson of Harvard University. Measuring distances on earth by triangulation is explained, and, by use of models, extended to measurement of the distances to the moon and the nearest stars. Orders of magnitude of astronomical distances are strikingly illustrated, and the apparent brightness method of estimating distances to stars is explained. Running time: 30 minutes.

"Measuring Small Distances", by Professor Dorothy Montgomery of Hollins College. Starting with a centimeter scale, methods of measurement of successively smaller distances are discussed, including the optical, electron, and field ion microscopes. How the instruments are calibrated is discussed. Running time: 20 minutes.

Section 1 - The Unit of Distance

PURPOSE To show how systems of units are developed and, secondarily, to introduce the metric units of distance.

EMPHASIS Treat briefly, mostly as a reading assignment. Most students already will have a little familiarity with metric units and will quickly pick them up as they work through the text and problems. Metric units do not require emphasis at this point. Suggestions for a possible brief class discussion are given below.

DEVELOPMENT Give examples of measuring lengths by counting the number of standard units in an interval. Example: laying a ruler end to end across a desk. Mention other methods of measuring, such as using a tape measure (which is essentially a set of rulers already laid end to end for convenience), pacing, and spanning with the arms extended.

To lightly push the idea that in measuring distance any convenient unit can be used as a standard, you might propose a new unit of distance on the spot. Taking as a model the light-year used in astronomical measurements, you might introduce the "sound-second" as the distance sould travels in a second. Do not emphasize the conversion of the sound-second into other units but try (in so far as possible) to work with it directly. For example, "We often hear the thunder from lightning when the bolts get as close to us as 15 sound-seconds away. When they get as close as 1 sound-second, they are very loud!" Problem 1 and 2 (Problem 2 is a home project) are appropriate at this point.

In discussing the metric system, stress the ease of working with decimal notation rather than any absolute "correctness" of the metric system. You might also point out that people who work in English units extensively often use a decimal system by picking the foot or the mile as a unit and then speaking of decimal fractions of a mile. "It is three and seven tenths miles from our house to the library." No one would think of converting this measurement to miles and feet.

One of the reasons that students avoid the metric system, especially in their daily conversation, is that they have no intuitive feeling for its units. It is probably best to avoid direct conversion between British and metric units. Instead, encourage students to try to develop direct mental pictures of the meter, centimeter and millimeter by practice in estimating the metric dimensions of familiar objects. They then should check their estimates by measuring. Ask questions such as "Are most men more or less than two meters tall?" "How many city blocks in a kilometer?" Laboratory work should play an important role in developing a real feeling for the metric system.

Emphasize particularly the consistency of the prefixes in the metric system (kilo, centi, and milli). The meter is our chosen standard unit of length, and other metric units of length are simply convenient submultiples or multiples of the meter.

COMMENT Someone may ask how the distance from the equator to the poles was measured in the 18th century, in terms of "surveyed base lines". The technique is graphically illustrated in the film "Measuring Large Distances", and involves measuring a portion of a meridian between known latitudes, as discussed in the Teacher's Guide for Section 2. The length of the meridian for one degree difference in latitude is 1/90 of the length of a quadrant (the length of a degree increases somewhat with latitude on account of the non-spherical shape of the earth). The Greeks measured the size of the earth quite well by this method.

Section 2 - Measuring Large Distances — Triangulation

PURPOSE To show that enormous extensions in the range of distance measurements can be accomplished through the use of simple geometric techniques.

CONTENT a. The intersection of the lines of sight to a distant object from the two ends of a "base line" determines the position of an object.

b. By solving the triangle formed by the two lines of sight and the base line we can determine the distance to the object. This can be done graphically by constructing a similar triangle, to a reduced scale, or by trigonometry.

c. Triangulation methods have been used to measure distances to planets and even to some of the nearest stars.

d. The orders of magnitude of distances determined through various sense extending techniques is discussed briefly.

EMPHASIS The basic idea of triangulation is ingenious, but quite simple. While this is an important and interesting topic, it can be treated quite briefly. The laboratory can carry the principal burden of getting understanding of triangulation methods. Experiment I-2 is appropriate at this point. HDL 3, 5, and 9 will be helpful.

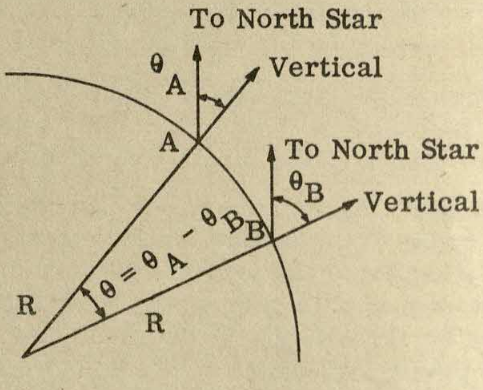
DEVELOPMENT The following simple class experiment as a pre-laboratory experiment may help get the idea of triangulation across: Ask your class to measure a distance of a few meters using a convenient base line such as a meter stick or the edge of a table. The two lines of sight can be marked directly on a normal-sized sheet of paper, moving the paper to each sighting point and making sure both times that one edge of the sheet is parallel to the base line. This yields directly a triangle similar to the triangle formed by the base line and the two lines of sight, but smaller. Using the proportionality between corresponding sides of the similar triangles the distance is easily worked out. The distance should be checked by direct measurement with a meter stick or a tape. Pins may be used to improve the accuracy with which lines of sight are laid out on the paper.

COMMENTS There are many applications of the basic idea of triangulation; several are mentioned in the text and others will occur to your class. The following comments are not intended for class presentation but to help answer questions that may come up.

The method of determining the size of the earth mentioned in the Teacher's Guide for Section 1, is essentially a triangulation method. Here the measured base line is an arc of a circle along a meridian and the distance to be measured is the radius of the earth. The angle subtended at the circle of the earth by the arc is established by noting the change in latitude. A and B are two points on the surface of the earth on the same meridian, i.e., A is due north of B. $90^\circ - \theta_A$ is the latitude of A. We can work out the radius of the earth by making a scale drawing or by noting that the length of an arc of a circle is proportional to the angle subtended:

$$\frac{AB}{2\pi R} = \frac{\theta}{360^\circ}$$

From which $R = \frac{360^\circ}{\theta} \times \frac{AB}{2\pi}$.



* * *

A good student may try to check the distance to Barnard's star, mentioned in the caption of Figure 3-5, from the observed apparent motion. This is not possible since the required length (focal length) of the telescope is not given; the 24-inch dimension mentioned in the caption refers to the diameter of the telescope mirror. To get the stated distance, the focal length should be 6 meters. Using similar triangles (see Figure 3-4) we have:

$\ell = 6$ meters

$s = 0.03 \text{ mm} = 3 \times 10^{-5}$ meters

$b =$ twice the earth-sun distance $= 3 \times 10^{11}$ meters

$$d = \frac{\ell}{s} b = \frac{6 \text{ meters}}{3 \times 10^{-5} \text{ meters}} \times 3 \times 10^{11} \text{ meters} = 6 \times 10^{16} \text{ meters}.$$

Some students may think about the possibility that the star may not stand still relative to the sun, and that this may interfere with the distance measurement. This is not a real problem, since it is easy to separate the effect due to the motion of the earth (relative to the sun) and the effect due to the motion of the star (relative to the sun). The first effect results in a reciprocating motion with a period of one year, while the part that comes from the motion of the star is essentially a uniform straight line motion. Presumably both effects are seen in Figure 3-5.

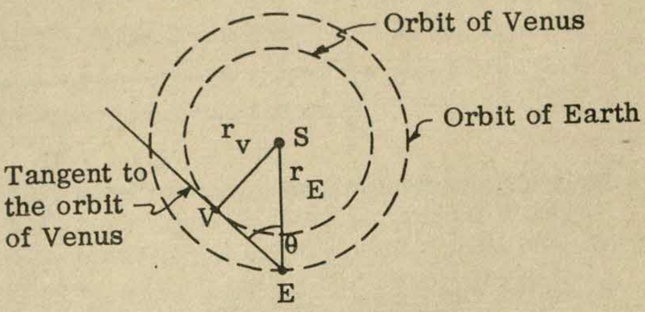
* * *

The question raised at the end of the caption of Figure 3-7 is simply a general invitation to the student to see if he can make sense of it. Definite answers should not be expected. Astronomers use a variety of methods to estimate distances too large to be obtained by triangulation. Most methods are based on the amount of light received from a star, and a rough estimate of the total amount of light emitted by the star (intrinsic luminosity). The intrinsic luminosity can be estimated with the help of an empirical relationship between the intrinsic luminosity and some other observable quantity, e.g., the color of the emitted light, or the period of a variable star. Such relationships have been established by observations on near stars whose distance is known by more direct methods. Some of the methods are discussed in the film "Measuring Large Distances".

SUPPLEMENT Some students may raise questions regarding how the astronomical distances listed in Table 1 are measured. While this is an interesting topic, you will want to avoid discussing it in class because it would take too much time. Partial answers to some of the more obvious questions are given below.

(1) While most interplanetary distances can be measured quite well by triangulation and the earth's diameter as a base line, other techniques are sometimes employed to give greater accuracy. Observations of the motion of the planets relative to the sun and to the stars make it possible to draw a scale map of the solar system, establishing the ratio of various distances within the solar system to each other. As an example, the maximum angle between Venus and the Sun essentially fixes the ratio of the radius of the orbit of Venus to the radius of the orbit of Earth. To keep things simple, suppose that both orbits are circular, with the Sun at the center, and that the two orbits are in the same plane. At a time when the angle between Venus and the Sun is a maximum, Sun, Venus, and Earth are at the vertices of a right triangle. The angle θ is about 46°; from a scale drawing, or by trigonometry, we find that the ratio

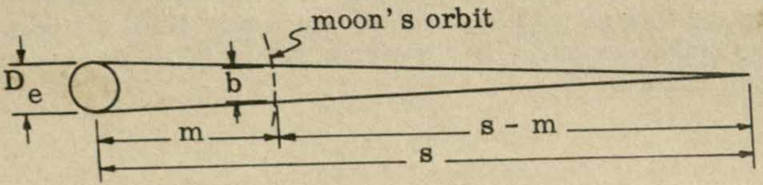
$$\frac{r_v}{r_e} = 0.72.$$



In a similar but more complex manner, ratios can be determined for planets which have orbital radii greater than that of the earth. To find the actual distances another equation is needed that relates the radius of the earth's orbit to that of one of the other planets. The actual difference between these two radii, say the distance between Mars and the earth at closest approach, can be computed from parallax measurements (triangulation) using two widely separated points on the earth's surface as a base line. As an alternative, noting the position of the planet in early evening and again in early morning provides a base line of approximately the earth's diameter because of the rotation of the earth on its axis. In practice the best measurements have been those of the distance to a minor planet (e.g., Eros) which passes much closer to the earth than Venus or Mars. Recently the distance to Venus has been measured by noting the time for the round trip of radio waves bounced off the surface of venus. If one of your students is interested in pursuing this subject in more detail a good source of information can be found in William Lee Kennon's "Astronomy", Ginn and Company, 1948 (pp. 199-206 and 247-257).

(2) The length of the earth's shadow could be determined from the diameter of the

shadow at the position of the moon and the distance of the moon from the earth.

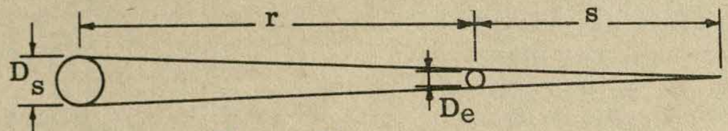


By similar triangles: $\frac{D_e}{s} = \frac{b}{s - m}$ which can be solved to give $s = \frac{D_e \times m}{D_e - b}$. The diameter, b, of the shadow of the earth at the moon's position can be determined by observing the time required for the moon to traverse it during a lunar eclipse. For instance, one may observe that the moon moves by a distance equal to its own diameter in about 3500 seconds, but it takes about 9200 seconds to traverse the earth's shadow. This means that b is equal to $\frac{9.2 \times 10^3}{3.5 \times 10^3}$ times the diameter of the moon. The diameter of the moon is about 3.48×10^6 meters (see HDL 6). This gives $b = \frac{9.2}{3.5} \times 3.48 \times 10^6$ meters $=$ 9.2×10^6 meters. With $D_e = 12.7 \times 10^6$ meters and $m = 3.84 \times 10^8$ meters we get:

$$s = \frac{12.7 \times 10^6 \text{ meters}}{(12.7 - 9.2) \times 10^6 \text{ meters}} \times 3.84 \times 10^8 \text{ meters}$$

$$= 14 \times 10^8 \text{ meters} = 1.4 \times 10^9 \text{ meters.}$$

Alternatively, we can compute the length of the shadow from the diameter of the Sun, the diameter of the Earth and the distance from the Sun to the Earth.



By similar triangles we have: $\frac{D_s}{r + s} = \frac{D_e}{s}$

we solve and find $s = \frac{rD_e}{D_s - D_e} = \frac{r}{D_s} \times \frac{D_e}{1 - \frac{D_e}{D_s}}$.

We know the ratio $\frac{D_s}{r}$ by measuring the angle subtended by the disk of the sun at the earth (see HDL 6). We have $\frac{D_s}{r} = 9.3 \times 10^{-3}$. The diameter of the earth is $D_e = 12.7 \times 10^6$ meters. Therefore we calculate:

$$s = \frac{1}{9.3 \times 10^{-3}} \times \frac{12.7 \times 10^6}{1 - \frac{D_e}{D_s}} = \frac{1.4 \times 10^9 \text{ meters}}{1 - \frac{D_e}{D_s}}$$

Comparing the two results we see that the diameter of the Earth must be quite small compared to the diameter of the Sun. Very accurate measurements would be required to give even a rough estimate of the actual ratio (the diameter of the sun turns out to be about 110 times that of the earth).

(3) As indicated in Table 1, the maximum distance which can be measured by triangulation using the diameter of the earth's orbit as a base line is about 10^{18} meters. This maximum is set by the limit on the ability of a telescope to distinguish between small angles (angular resolution). The diameter of the earth's orbit is about 3×10^{11} meters. This implies an angular resolution of about $\frac{3 \times 10^{11}}{10^{18}} = 3 \times 10^{-7}$. This is close to the resolving power of the largest telescope. The maximum angular resolution cannot exceed the ratio of the wavelength of light to the aperture of the telescope. For visible light the wavelength is about 5×10^{-7} meters, and the diameter of the largest telescope is about 5 meters. This gives a resolution in angle which cannot exceed about 10^{-7}. Other limitations, e.g., irregular refraction through the atmosphere, slightly reduce the resolving power. Note that this limitation allows us to measure distances only to stars which are relatively near neighbors. 10^{18} meters is about 10^2 light years (1 light year $\approx 10^{16}$); the diameter of our galaxy is about 10^5 light years.

Section 3 - Small Distances

CONTENT Several methods or devices can be used to enable us to infer the size of a small distance from the measurement of a larger one.

EMPHASIS The principal emphasis here should be to give students a feeling for how and how far measurement can be extended toward the very small. At this point, in the laboratory the student can get to 10^{-5} meters with the optical micrometer. In the monolayer experiment (I-8) in connection with Chapter 7, he will get to 10^{-9} meters. Before the course is over he will be familiar with techniques for getting down to subatomic dimensions.

This section may be treated quite briefly since most of the ideas involved are similar to those used in the preceding section. If you have time, the construction and calibration of the optical micrometer (Experiment I-3) will be appropriate at this point. If you do the experiment, you should not introduce the micrometer caliper until later, since it may detract from the students' interest in building and calibrating their own device.

COMMENTS It would take you too far afield to discuss now how the distances listed in Table 3 are measured. You can point out that this table is meant to show students where they are going, not how to get there.

In all the examples you choose, try to emphasize that we construct a device or procedure to extend the results of direct measurement. As the text points out, we can take a hundred sheets of paper instead of only one to bring the necessary measurement into the range where we can carry it out directly.

Students may notice that in Figure 3-11 (b) on page 31 a human hair is about 8×10^{-5} meters in diameter whereas on page 35 near the top of the first column a hair is said to be 3×10^{-5} meters in diameter. Let students settle this in laboratory with the optical micrometer. (The thickness varies considerably.)

Section 4 - The Dimensions of Space

PURPOSE To make more explicit the intuitive physical connections between lengths, areas, and volumes, that is, between one, two, and three dimensions.

EMPHASIS Treat very briefly, mostly as a reading assignment.

COMMENTS That it requires three numbers to specify the location of a point in space will be apparent to most students. It will certainly be apparent that you can do it with three numbers; some of them may not have considered the possibility of trying to do it with less than three. Three coordinates are needed any way you try it.

When you move a line in order to "sweep out" a surface, you must move it perpendicular to itself (or at least partially so; you cannot slide it along its length). Similarly, you move the surface at least partially perpendicular to itself in order to generate a volume. This is as far as you can go. As the text indicates, this is because you only get another volume when you move the original solid. However, students with mathematical sophistication may point out that the difficulty is that we cannot move a solid perpendicular to itself. We are forced into a case similar to that of sliding a line along itself or a surface along itself where another line or surface, respectively, is obtained.

Try to point out that we deal in physics with quantities of no dimensions, like π, as well as with quantities of one or more dimensions. A dimensionless quantity is measured by a pure number. A one-dimensional quantity, like a distance or time interval, is measured by a number with units. A two-dimensional quantity, such as an area, can be described in terms of two one-dimensional quantities, such as two distances (e.g., the length and width of a rectangle). A three-dimensional quantity, such as a volume, can in turn be described by three one-dimensional quantities (three sides of a cube) or by a one-dimensional quantity and a two-dimensional quantity (the base area and height of a cylinder). Students may point out that even in our everyday world, it takes four one-dimensional quantities, three distances and a time measurement, to specify the state of a particular system which is changing in time. (This concept is fundamental in the theory of relativity, and aroused wide and wild speculation about "the fourth dimension" in the public mind at the time Einstein published this theory.)

Section 5 - Measuring Surfaces and Volumes

PURPOSE To show that the measurement of areas and volumes is simply an extension of the process of counting units.

EMPHASIS Treat briefly.

COMMENT We define what we mean by the area of a square and volume of a cube using the standard equations, $A = s^2$ and $V = s^3$, respectively. To find the area and volume of irregular bodies, we count (or compute) the number of standard squares or cubes that will fit into the given figure. While this material hints at some rather important advanced mathematics such as integral calculus, it is enough to leave it as a hint here.

Some of Problems 12 through 17 at the end of the chapter may be appropriately used for homework or class discussion here. These problems will give students a little practice with area and volume measure. This is a good opportunity to point out an advantage of the metric system over the English system. In the metric system we choose as the unit of volume, the volume of a cube having an edge of a meter: the unit of volume is constructed in the simplest possible way from the unit of length. This is not the case for most common English units. You will really drive this home by asking your class to calculate the length of the edge of a cube which is to have a volume of a bushel. They will get the point, if not the answer.

Section 6 - On the Limitations of Measuring
Section 7 - Significant Figures

PURPOSE To point out some of the limitations on measurement and to introduce the use of significant figures in reporting the results of measurement.

CONTENT a. Physical measurements are not exact.

b. Indirect measurements depend upon assumptions. The recognition and testing of these assumptions is an important part of measurement.

c. The accuracy of a measurement can be expressed by the number of significant figures used in reporting the result.

d. Arithmetical processes with measured quantities must take account of the uncertainties in the original data.

EMPHASIS The several examples in the text should be sufficient at this point as an introduction to measurement limitations.

While significant figures should be treated as important, they should not become a fetish since significant figures are a long way from being a completely satisfactory answer to the problem of errors and their representation. To get further in this regard we would have to get into fractional errors. This is hardly necessary in a first course.

Many students will require months of reminding to get into the habit of paying attention to significant figures. Practice at this point will not be as helpful as pointing out appropriate use when problems are discussed in class, and especially in the laboratory when students are working with their own measurements.

COMMENTS The relevant ideas on significant figures are presented in the text through examples, avoiding any hard and fast rules. It will be well to stick to this method. The student should recognize: (1) Any number obtained by measurement is uncertain to some degree. (2) When quantities are computed from such numbers the numerical results are also uncertain to some degree. In doing arithmetic with the results of measurement we estimate the uncertainty in the answer by asking: If we change the measured result by the possible error, how much is the result of the calculation changed? If students once get into the habit of asking this question, there is little difficulty in getting a proper answer.

It will be well to avoid hard and fast rules and carefully refrain from going beyond a simple, common sense approach in answering questions which may arise in connection with significant figures. In most of the cases where two "common sense" answers to the same question appear to exist, the difference between the two answers will be unimportant. Such an example is given in Problem 19. In this problem both answers are correct. The difference between the two answers (12.76 and 12.77 meters) is not significant, since the last digit is uncertain in both cases.

Problems 19 through 24 deal with significant figures. Class discussion of perhaps two of these (e.g., 21 and 24) should be enough to present most of the ideas involved.

* * *

It may be necessary to point out explicitly that the errors in measurement result from natural and instrumental limitations quite unrelated to the decimal number system. Thus writing a number to a given number of significant figures does not imply that the last figure given is known to an accuracy of ± 1; e.g., if we state that a certain distance is 3.523 meters, this does not necessarily mean 3.523 ± .001 meters. The statement simply means that our best estimate of the last digit is 3, and that we know nothing at all about the following digit. If the uncertainty in a particular measurement is known, for example by many independent measurements of the same quantity, we conventionally state the uncertainty together with the results, e.g., 3.523 ± .007 meters. Students cannot be expected to adopt such notation at this point, since they do not know how to estimate errors.

* * *

In physics, we deal with exact as well as with measured quantities. Exact quantities are those defined by law or by mathematics. For example, there are exactly 2.54 centimeters in an inch, by law. Therefore, it is perfectly all right to write this quantity as 2.5400000000... Similarly, a number like π can be written with any number of digits since it has an exact definition. We could, of course, "measure" π by measuring the diameter and circumference of a circle and determining π "experimentally" from the ratio of the circumference to the diameter. Then, the number of significant figures we could use in reporting the value of our "experimental" π would be determined by the precision of the measurements of a circumference and diameter. However, if we want to know the circumference of a circle, it is much better to take the mathematical definition of π and the measured value of the diameter. Then, of course, the reported "measured" value of the circumference can have no more significant figures than the measured value of the diameter. In computing by longhand, of course, we can save work by using only an appropriate number of digits of π. Note that when we multiply or divide a measured quantity, like a radius, by an exact quantity like π, we do not change the number of significant figures.

Work carefully through the arithmetic of dealing with physical quantities. Show that the number of significant digits can never increase as a result of a mathematical operation like addition, multiplication, or division, but can, in fact, be reduced by subtraction. Point out that the number of significant digits in a measurement is independent of the units chosen to express it. For example, if the diameter of a circle measured with a meter stick is reported as 2.55 centimeters, the digits are all significant, but the last one on the right is somewhat in doubt because it comes from an estimate of tenths between the millimeter divisions on the scale. The number of significant digits is not changed by converting units and writing 0.0255 meters or 25,500 microns. The zeros at the left in the first case, and at the right in the second case, serve to locate the decimal point, and are not counted as significant digits. The true number of significant digits can always be indicated by using powers of ten to put the decimal point to the right of the first digit and writing 2.55×10^{-2} meters or 2.55×10^{4} microns. This sort of notation is sometimes called "scientific" notation and is a good thing to encourage at this point.

Chapter 3 - Space and its Measures

For Home, Desk and Lab — Answers to Problems

GENERAL In these problems and in much of the work of the following chapters, a fair amount of routine arithmetic is frequently encountered. Some teachers may wish to take some time at about this point to train students in the use of a slide rule to simplify routine calculations. While a knowledge of slide rule technique is generally useful, and a valuable aid in computation, the progress of the course will not be impeded if students are not specifically trained in this technique.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 3-12 to 3-23.

Section	Easy	Medium	Hard	Class Discussion	Home Projects
1	1			1	2
2		4, 9*		10	3*, 5, 6*, 7, 8
3	11			11	
4	12, 13, 14	15			
5		16*, 17		16*	
6		18		18	
7	23, 24	19, 20*, 21 22		21, 24	

SHORT ANSWERS

1. See discussion on page 3-12.
2. A home project. See discussion on page 3-13.
3. A home project. See discussion on page 3-13.
4. 220 meters.
5. A home project. See discussion on page 3-15.
6. 4×10^3 km.
7. A home project. See discussion on page 3-16.
8. A home project.
9. (a) 25 meters
   (b) Dif. = 1 meter. Error 4.0%
   (c) Dif. = 20 meters
10. A class project. See discussion on page 3-18.
11. See discussion on page 3-19.
12. 78 cm^2

13. 200 cm^2

14. 780 cm^3

15. 900 cm^3

16. Pattern and measured square of pattern material are of uniform thickness and composition.

17. 5×10^{-3} inches per groove.

18. See discussion on page 3-21.

19. (a) 12.77 meters and 12.76 meters.
    (b) Both are correct. Either way the digit representing hundredths of a meter is uncertain.

20. (a) 5.4×10^{-3}
    (b) 2.3×10^{-1}
    (c) 36.7
    (d) 0.458

21. (a) 64 cm^3
    (b) 64 ± 1 cm^3
    (c) 1/64 (or 1.6%)

22. 8.2 cm

23. 44.0 cm^3

24. (a) 9.1 hrs.
    (b) 0.60
    (c) 3.5 hrs.

COMMENTS AND SOLUTIONS

PROBLEM 1

Choose an arbitrary unit such as the length of the first joint of the thumb or the length of a fingernail. Mark off a scale, using this unit, along the edge of a piece of paper. Use this scale to measure the length and the width of this page. Determine the ratio of the length to the width. What are the units of this ratio? Compare the ratio that you obtained with the ratios that others obtained for this same object using different units. What is the effect of different units in measuring ratios?

To demonstrate that the choice of unit in a measurement is arbitrary. Typical results:

width of paper = 5 1/2 thumb-joints; length = 7 thumb-joints

ratio of length to width = 7/5.5 = 1.3 (no units)

(with a ruler) width = 19 cm; length = 25.5 cm

ratio of length to width = 25.5/19 = 1.3 (no units)

The ratio of length to width is the same for any set of units, as long as the same units are used for measuring both distances.

Some students will object to making crude measurements such as using a thumb joint when they know they could do much better with rulers. If the ratios computed by the thumb method and the ruler method disagree, they readily will discard the thumb measurement and keep the ruler measurement. If your students' experience with science thus far is such that they do not take to this exercise, you can use it in at least two modified ways and still put across the important ideas.

1. You can ask the class to perform a thought experiment by asking them what would happen if they did compute this ratio using different units of measurement. In principle, the ratio should stay the same. However, students may point out that in practice there would be slight variations in the ratio due to errors in measurement.

2. You can engage the class in competition. The intention of the exercise as it is stated is for each student to play the game by himself to see what happens and perhaps to see how accurately he can make such a measurement with his thumb. You can assign the exercise in class before students have previously read it or measured a page with a ruler. Have each student measure the two distances as accurately as he can with his thumb. In doing this he will face in a conspicuous way most of the problems which one faces in trying to make a precision measurement with good equipment. Have students estimate tenths of thumb joint lengths as accurately as they can. Record the results obtained by each student. Have students use a ruler to decide the winner.

It also might be interesting to have a student record all of the readings and average them to see if the average turns out (as it will if you are lucky) surprisingly close to the results obtained with a ruler.

Whether you do this exercise as it stands or introduce variations in it, it is important that students realize here that the ratio of two linear measurements is a pure, dimensionless number. Also by the time they leave this chapter students should be thoroughly convinced that all measurements are approximations.

PROBLEM 2

Stand 75 to 150 yards from a building with a fairly flat front and several stories high (an alley between buildings with one dead end is also suitable). Clap your hands sharply and listen to the echo, which is, of course, much fainter than the original clap.

Now clap in rhythm and adjust your rate of clapping until you cannot hear the echo because it comes back at the same time the louder sound is being created. Have someone count the number of times you clap in ten seconds. Measure the distance and compute the speed of sound (not forgetting to double the distance to get the path traversed).

An answer between 900 and 1500 ft/sec is fine.

Unless a student has done this with great care, one significant digit is all that he is justified in keeping. Also, by assuming the speed of sound, this method becomes a way of measuring the distance to the building.

At the distance suggested, unless the arrangements are ideal, (no wind, no background noise, etc.) it may be difficult to hear an echo from a hand clap. As an alternative to clapping, the students might try pounding with a hammer on a vertical plank.

PROBLEM 3

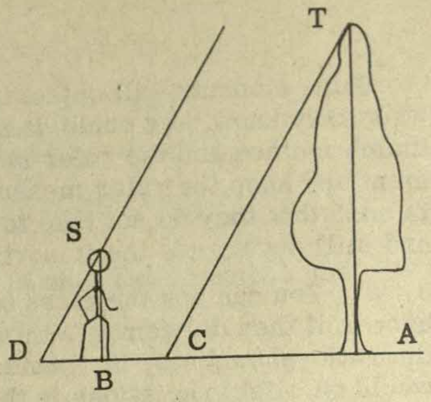
Find the height of a tree or a building on a sunny day. The information you need is the length of both your shadow and that of the tree and your own height. From these data the tree's height may be found by using similar triangles. Does the sun's position affect your results?

An exercise in triangulation, in which the student uses his own body as a gauge length.

Numerical answers, of course, depend on the tree or building selected. We must assume only that the sun's rays are parallel. Then, triangles TAC and SBD are similar, and

$$\frac{\text{tree height (TA)}}{\text{student height (SB)}} = \frac{\text{tree shadow (AC)}}{\text{student shadow (BD)}}. \text{ Then}$$

$$\text{TA} = \frac{\text{(SB) (AC)}}{\text{(BD)}}.$$



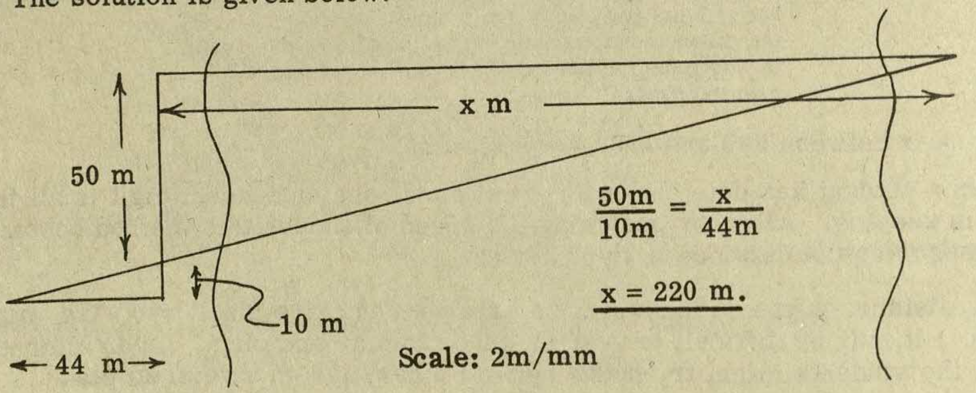
For example, if SB = 5.0 ft, AC = 27 ft, and BD = 3.0 ft; then TA = 45 ft. Any position of the sun will, of course, work as well as any other position, since the light rays striking both the tree and student are parallel.

However, students may point out that the accuracy of this method necessarily goes down as the sun nears a "directly overhead" position. When the sun is in such a position, it will not work at all. The method would be most inconvenient at sunset due to the great length of the shadows.

For classes that do not do this problem experimentally, the question of the effect of the sun's position can be used as the basis of a short class discussion.

PROBLEM 4 In using the triangulation method of measuring a distance, an observer sights upon an object directly opposite him on the far bank of a river. He places a stake where he is standing and then moves 50 meters upstream parallel to the river. Here he places a second stake. Then he moves 10 meters farther upstream and places a third stake. From this stake he walks directly away from the river until he is on a line with the second stake and the object on the far side of the river. He finds that he is 44 meters from the third stake. What is the width of the river? See Fig. 3-1 of the text for a diagram of this method.

A straightforward construction problem in triangulation. Even if students know trigonometry, they will learn a great deal by solving problems with accurate scale drawings. The solution is given below.



$$\frac{50\text{m}}{10\text{m}} = \frac{x}{44\text{m}}$$

$$\underline{x = 220 \text{ m.}}$$

Scale: 2m/mm

PROBLEM 5

It is possible to determine your distance from a landmark by measuring two angles and a base line. Choose for a base line the side of a building or a football-field side line of known length. By sighting to an object from each end of the base line, the angle which each line of sight makes with the base line may be determined by using two sticks and a protractor. If these results are scaled so as to fit on a piece of paper, the distance may be roughly measured. The distance may also be determined trigonometrically, since two angles and a side are known.

This triangulation problem can be solved either trigonometrically or by constructing a scale drawing. For example, in Figure (a): $AB = 20$ meters; $\angle CAB = 71.6°$; $\angle CBA = 84.3°$.

We can construct the line DC perpendicular to AB, forming two right triangles. Then, solving trigonometrically,

$$DC = AD \tan 71.6° = DB \tan 84.3°.$$

From tables, $\tan 71.6° = 3.01$, and $\tan 84.3° = 10$. So,

$$DC = 3.01\ AD \qquad DC = 10\ DB$$

$$= 10\ (AB - AD)$$

$$= 10\ (20 - AD)$$

$$10\ (20 - AD) = 3.01\ AD$$

$$200 = 3.01\ AD + 10\ AD$$

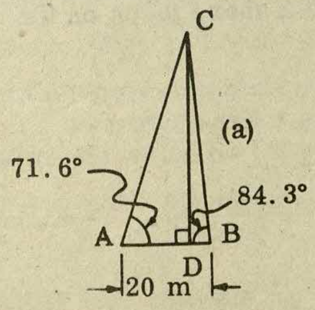
$$AD = 15.4\ \text{m}$$

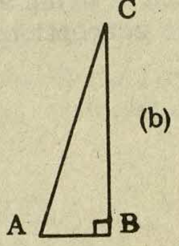
To determine AC, $\cos 71.6° = \dfrac{AD}{AC}$ or $AC = \dfrac{15.4\ \text{m}}{.316} = 48$ meters. Or, if we want CD, $CD = AD \times \tan 71.6° = 15.4 \times 3.01 = 46$ meters.

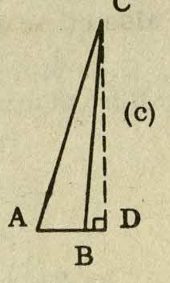
If this problem is solved by making a scale drawing, the protractor can be eliminated. As the angles are sighted they can be recorded directly on a sheet of paper. Point out here, especially for students who are facile with trigonometry, that a careful scale drawing is just as good as a trigonometric computation. Students can fool themselves by using trigonometry to bring in many extra "significant" digits but a scale drawing will preserve all the accuracy they have in their basic measurements when they are measuring angles by sighting. In fact, the computations given above in the solution of the problem are to show a "limiting case" of accuracy. It is unlikely that by sighting over a protractor a student can justify a reading such as 71.6°. Two digit accuracy is all that should normally be expected or accepted.

If it is practical the student should select a point on the base line so that a line connecting it to the object will form a right angle with the base, (BC in Figure (b)). This will greatly simplify the calculations since AC will equal $AB/\cos \angle BAC$, and $CB = AB \times \tan \angle BAC$.

Figure (c) illustrates a third possible relationship of base line and the distance, (AC), to be measured. Constructing DC and DB again produces two right triangles and the solution of the problem is similar to that for (a).

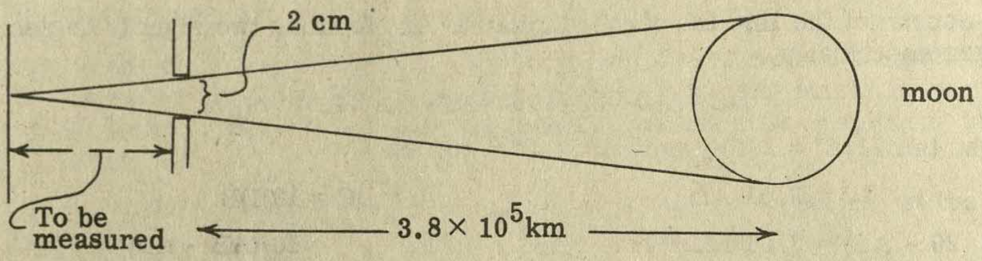






PROBLEM 6

What is the diameter of the moon? Assuming that the moon is 3.8×10^5 km from the earth, you can determine its diameter by the following method. Place two strips of opaque tape 2 cm apart on a windowpane. Make a pinhole in a card and then observe the moon through the pinhole and between the two strips. Back away from the window until the moon just fills the space between the two strips. Measure the distance from the card to the window. Using the geometry of similar triangles, calculate the diameter of the moon. Do **not** attempt to use this method to determine the diameter of the sun. The bright sun will be harmful to the eye. Could this method be used to determine the size of a star?



$$\text{Using similar triangles: } \frac{2\text{ cm}}{\text{distance from card to window}} = \frac{d\text{ (moon's diameter)}}{3.8 \times 10^5\text{ km}}$$

The student should find that the distance from the card to the window is of the order of 10^2 cm. If the student measures carefully, he will obtain a value of approximately 2 meters and from this we have:

$$\frac{D\text{ km}}{3.8 \times 10^5\text{ km}} = \frac{2\text{ cm}}{2 \times 10^2\text{ cm}}, \text{ and } D = \underline{4 \times 10^3\text{ km}}.$$

The distance from a star to the window is so great in proportion to the star's diameter that rays from all parts of the star are too nearly parallel to permit the use of triangulation for finding the diameter.

PROBLEM 7

Cover one eye with your hand and try to judge distances with only one eye. Observe that most of the hunting animals and birds have a unique placement of the eyes which permits them to estimate the striking distance to their prey. Consider the difficulty of almost any of the many tasks we perform if we were denied this depth perception.

While people with normal vision generally use parallax to judge distances, people with sight in only one eye do have some depth perception. This can be realized through sensing the muscular activity required to bring an image to a sharp focus on the retina. There are also other cues to distance perception.

PROBLEM 8 A camera with one lens cannot show relative distance unless the picture it takes shows the depth of space which separates objects. Show how this could be used to make a camera "lie." Take a picture of objects on a table, from slightly below the edge. Notice that similar objects appear to be of different size depending upon their position on the table. Effects such as this are used to show the illusion of a person seemingly inside a bottle or larger than a tree.

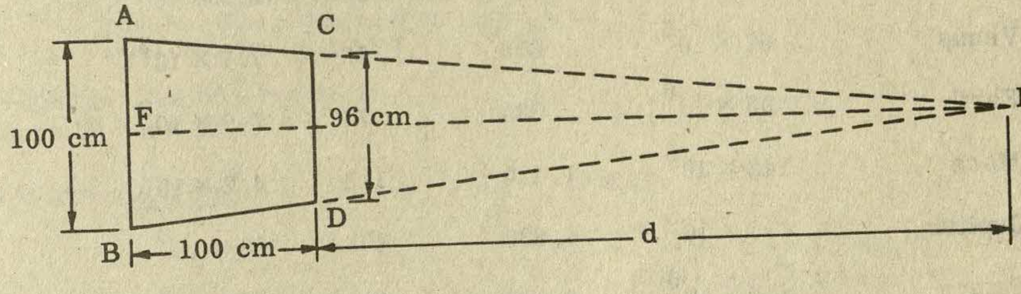
A home project for photo hobbyists.

PROBLEM 9 (a) A simple range finder may be improvised by using two meter sticks to establish sight lines to the object. The amount these deviate from parallel positions is a measure of the nearness of the object sighted. If the meter sticks form equal angles with their base line, and are separated by one meter at the near end and 96 centimeters at the end away from the observer, what is the distance to the sighted object?

(b) If the meter sticks are not sufficiently straight to give an accurate reading on the separations, an error in range estimation will result. What difference in the estimated range would result if the front measurement had been 96.2 instead of 96.0 cm when the backs of the sticks remained at 1 meter? Express this error as a per cent.

(c) As the sticks are made more nearly parallel, failure to sight the sticks exactly produces a much larger error of range estimation. Compare the ranges when the fronts of the sticks are 99.0 and 99.2 cm apart while the backs remain at one meter.

A good exercise in triangulation--could well be done as a class activity.



a) From similar triangles in the above diagram:

$$\frac{d}{96\text{ cm}} = \frac{100\text{ cm} + d}{100\text{ cm}};\quad 100\,d = 9600 + 96\,d$$

$$d = 9600/4 = 2400\text{ cm} = 24\text{ meters}$$

The range, EF, is 24 m + 1 m = 25 m.

b) From the above calculations we see that if the front measurement is 96.2 cm then $d = 9620/3.8 = 25$ meters and the range, EF, would be 26 meters. Thus the difference in estimated range would be 1 meter.

$$\%\ \text{error} = \frac{(26 - 25)\text{ meters}}{25\text{ meters}} \times 100 = \underline{4.0\%}$$

c) When CD = 99 cm: $d = \frac{9900}{1} = 9900$ cm or 99 m and the range is 100 m.

When CD = 99.2 cm: $d = \frac{9920}{0.8} = 12,400$ cm and the range is 120 m.

In this case, a difference of .2 cm in the front measurement produces a difference in the estimated range of 20 meters. It is obvious that as the range increases the error resulting from measurement of $\overline{CD}$ is magnified. Note that in subtracting 99.2 from 100, while our answer (0.8) has only one significant figure, the possible error is only about 10% so we are justified in keeping two significant figures in our range.

PROBLEM 10

Constructing a scale model of the solar system: A volleyball representing the sun and placed near one end of a football field will give a scale on which an inch will equal approximately 10^5 miles (0.1 megamile). To such a scale, the earth will be only a little more than $\frac{1}{16}$ inch in diameter and 930 inches (77 feet) away from the sun. Distant Neptune would be almost half a mile away and only a little over $\frac{1}{4}$ inch in diameter. What sizes will our other planets and their satellites have on our model? By what factor must all dimensions be reduced if this scale model is to fit easily on your football field? What object could you use to represent the earth on this new scale?

A possible class project. Students are surprised to see that the "scale-models" of the solar system in museums are often far from true scale!

The following table contains the necessary data for constructing the scale model. Distances and sizes of the planets are also given.

Planet	Distance in miles from the sun	Distance to scale 1 inch = 10^5 mile		Actual diameter in miles	Scale size diameter in inches
		inches	feet		
Mercury	36×10^6	360	30	3.0×10^3	0.03
Venus	67×10^6	670	56	7.7×10^3	0.08
Earth	93×10^6	930	77	7.9×10^3	0.08
Mars	142×10^6	1,420	118	4.2×10^3	0.04
Jupiter	483×10^6	4,830	403	88×10^3	0.88
Saturn	886×10^6	8,860	738	75×10^3	0.75
Uranus	$1,780 \times 10^6$	17,800	1,483	31×10^3	0.31
Neptune	$2,790 \times 10^6$	27,900	2,325	33×10^3	0.33
Pluto	$3,670 \times 10^6$	36,700	3,058	--------	----

Using a scale of 1 inch = 10^5 miles, Pluto would be about 3000 feet from the sun. Since a football field is 360 feet, goalpost to goalpost, this scale would have to be reduced by a factor of 10 f the model were to fit on the field. On this new scale, earth's diameter would be about 0.008 inches—approximately that of a fine human hair.

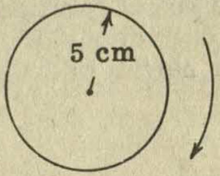
PROBLEM 11 Determine the thickness of the leaves in one of your books by measuring the thickness of 200 pages (100 sheets). Why is it suggested that you use 100 sheets? What assumptions are made in this measurement?

To show how it is possible to measure small distances with fairly crude instruments. A typical book might give a measurement of 0.9 cm for the thickness of 100 sheets, giving an average thickness for each sheet of 0.009 cm. Assuming the error in measuring the thickness (due to inaccuracies in reading the ruler) to be perhaps 0.05 cm, the error in the thickness of each sheet is 0.05/100 cm, or 0.0005 cm, about 6% of the actual thickness. If we had taken only ten sheets, the error in measurement would have been 0.05/10 cm, or 0.005 cm—an error more than half as large as the distance we were measuring. The major assumption made in this measurement is that the sheets in the book have uniform thickness. In any case, the answer obtained is an average.

Be sure that this example of a method for measuring the thickness of a single sheet of paper comes to the attention of students either by assigning this exercise, by treating it in class (previous references to this method occur in the chapter), or in laboratory work.

PROBLEM 12 A straight line 5.0 cm long is rotated in a plane about one end. What is the area swept out by this motion?

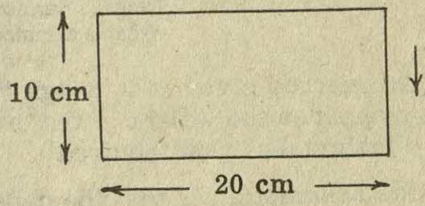
The line sweeps the area of a circle with a 5.0 cm radius. Using $A = \pi r^2$, $A = 3.14 \times 25\ \text{cm}^2 = \underline{78\ \text{cm}^2}$.



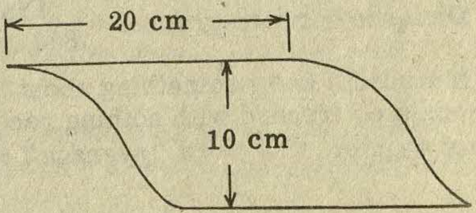
PROBLEM 13 A straight line 20 cm in length is moved to a new position parallel to and 10 cm from its original position. What is the area of the surface that is swept out in moving the line?

The line sweeps the area of a rectangle with the dimensions 20 cm × 10 cm.

$\text{Area} = 20\ \text{cm} \times 10\ \text{cm} = \underline{200\ \text{cm}^2}$.



Some students may quibble with the wording of this problem. It does not say that we must generate a rectangle as assumed above. The new position could be parallel to the old position with twisting and turning in between. One rewording is: "A straight line twenty centimeters in length is moved parallel to itself to a new position. . ." This still does not require that the line sweep out a rectangle. As an additional attempt to pin it down, you might require that throughout its travel the line segment stay in a single plane. In this case you still are allowing an area to be swept out such as shown at the right.



Probably most of your students will not be familiar with the fact that in such cases the area *is* $200\ \text{cm}^2$. The requirement is that the line stay in a plane and move parallel to itself and that the two distances are measured as indicated rather than "along a curve".

PROBLEM 14

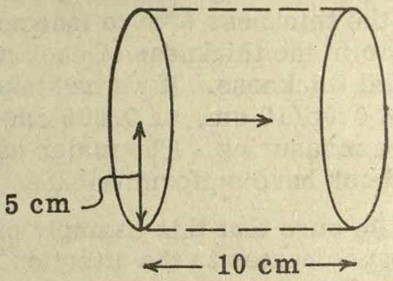
A circle whose radius is 5.0 cm is moved along its axis to a new position parallel to the original plane. If it has been moved 10 cm, what volume has been generated by this motion?

The circle generates a cylinder with a radius of 5 cm and a height of 10 cm.

Volume of a cylinder = $\pi r^2 h$

$= 3.14 \times (5.0 \text{ cm})^2 \times 10 \text{ cm}$

$= 785 = \underline{780 \text{ cm}^3}$.



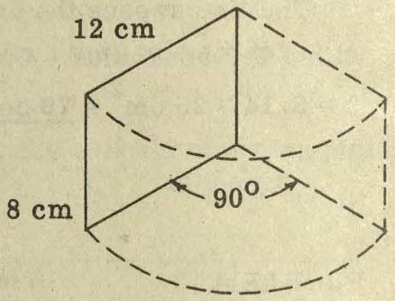
PROBLEM 15

A piece of cardboard whose dimensions are 12 cm by 8.0 cm is tilted about the 8.0 cm edge through an angle of 90 deg. What volume has been swept out by this motion?

The piece of cardboard sweeps out one-fourth of the volume of a cylinder with a length (or height) of 8.0 cm and a radius of 12 cm.

$$\text{Vol.} = \frac{\pi r^2 h}{4}$$

$$\frac{3.14 \times (12 \text{ cm})^2 \times 8.0 \text{ cm}}{4} = \underline{900 \text{ cm}^3}$$



PROBLEM 16

An indirect method for determining the area of an irregular surface was mentioned in this chapter. What is assumed in this method?

The method given is to weigh " a paper pattern cut to fit the surface neatly. One then compares the weight of the cut pattern with the weight of a measured square of the same material to find the area".

One assumption is that the paper pattern and the measured square of the pattern material are of uniform thickness and composition.

PROBLEM 17

A 12-inch LP record which plays for 25 minutes at 33⅓ RPM has 4 inches of grooved surface along the radius. What is the average distance between grooves?

Total number = 25 min × 33 1/3 grooves/min = 833 grooves.

Distance between grooves $= \frac{4 \text{ inches}}{833 \text{ grooves}} = \underline{5 \times 10^{-3} \text{ inches per groove}}$.

If students know something about the grooves of an LP record, we are thinking of an unmodulated (record with nothing recorded on it, just blank grooves) where the grooves do not wiggle. The word "average" in the exercise takes care of this, but you may relieve the students' worries if you tell them that the grooves are smooth. You should also call the attention of the class to the fact that in carrying out this computation we are assuming that the long spiral can be replaced by an equivalent group of concentric circles.

PROBLEM 18

A person reports seeing a "flying saucer" 200 feet long, at an altitude of 6000 feet and flying at an estimated speed of 2000 miles per hour. Discuss the accuracy of this observation.

This problem requires the student to exercise his perception of the interrelationship between "measurements". Consider what the observer could infer if he knew one or another of his estimates.

If he knew the altitude, he might be able to estimate actual size from "apparent" size. From the altitude, and the angular field of view covered in a given time, he might also estimate speed.

If he knew the actual size, he might estimate altitude from "apparent" size, and, again, estimate speed using altitude and angular field covered in a given time.

If he knew only the speed, he could not infer anything specific about either altitude or size.

Since our observer probably has not seen flying saucers "close-up" so as to know their real size, or speed capability, his estimates of size and speed depend upon the accuracy of his altitude estimate. Of course, if he had seen a jet fighter (whose size he might know) engage the saucer in a dog-fight, and the saucer appeared to be about three times the length of the fighter, he might conclude that the saucer and the fighter were about the same altitude, and that the saucer was about 200 ft long. But this is going a bit beyond the report.

How did he determine the altitude? Not knowing the size, he would, at the least, need clouds of a known altitude as a benchmark. A long baseline rangefinder or a height-finding radar would be preferable. One must conclude that the accuracy of the report is highly questionable.

As a general check on the credibility of the observer, we might ask him how long he kept the object in view. Flying a straight course at 6,000 ft and at 2,000 mi/hr, an object would be in sight from horizon to horizon for, at the most, 5-6 minutes.

PROBLEM 19

Two students made the following length measurements in the lab and wished to find their sum — 3.52 m, 4.213 m, and 5.034 m. One student insisted that they should round off to the nearest hundredth of a meter first and then add, while the other student argued that they should be able to add the measurements directly and then round off their sum.

(a) Try both of these methods and compare the results.

(b) Which student was correct?

a)

$$\begin{array}{l} 3.52\ \text{m} \\ 4.213\ \text{m} \\ \underline{5.034\ \text{m}} \\ 12.767 = 12.77\ \text{m} \end{array} \qquad\qquad \begin{array}{r} 3.52\ \text{m} \\ 4.21\ \text{m} \\ \underline{5.03\ \text{m}} \\ 12.76\ \text{m} \end{array}$$

b) There is no reason for preferring one method over the other. In either case, the digit representing hundredths of a meter is uncertain.

There are some situations, however, in which the student who added first, then rounded off, would have the better result. Suppose that one hundred and one measurements were made of successive segments of a long straight line. One hundred of these made with an accurate rule, give 2.714 m each. The 101st measurement made with a coarse rule, gives 2.71 m. Clearly it would be wrong to throw out the 40 centimeters represented by the round-off carried out one hundred times, i.e. 271.400 + 2.71 = 274.11 m. If the numbers were rounded before adding, the result would be: 271.00 + 2.71 = 273.71 m. The difference of .4 meters between the two results is much greater than the uncertainty (± .005 m) in the single coarse measurement.

PROBLEM 20

Solve the following problems with due consideration to significant figures.

(a) $\frac{1.4 \times 10^3}{2.6 \times 10^5} = ?$

(b) $3.7(6.27 \times 10^{-2}) = ?$

(c) $46.7 - 10.04 = ?$

(d) $\frac{8.34 \times 0.659}{12.03} = ?$

a) $\frac{1.4 \times 10^3}{2.6 \times 10^5} = 0.54 \times 10^{-2} = \underline{5.4 \times 10^{-3}}$

b) $3.7\,(6.27 \times 10^{-2}) = 23 \times 10^{-2} = \underline{2.3 \times 10^{-1}}$

c) $46.7 - 10.04 = \underline{36.7}$

d) $8.34 \times 0.659 = \underline{5.50}$. $\frac{5.50}{12.0} = \underline{0.458}$

PROBLEM 21

A student measures a block of wood and records the following results: length = 6.3 cm, width = 12.1 cm, and height = .84 cm.

(a) What is the volume of this block?

(b) Assume the length and width measurements to be correct; however, you can see that the height measurement may be off by 0.01 cm either way. How would this change your answer for the volume?

(c) What fraction is this of the total volume?

a) $V = 6.3 \times 12.1 \times .84 \text{ cm}^3$, $6.3 \times .84 = 5.3 \text{ cm}^2$, $5.3 \times 12 = \underline{64 \text{ cm}^3}$

b) $V' = 6.3 \times 12.1 \times .83 = 63 \text{ cm}^3$

$V'' = 6.3 \times 12.1 \times .85 = 65 \text{ cm}^3$

Thus the increased uncertainty in height introduces an increase in the uncertainty of the volume. The answer would still be given as 64 cm^3, but whereas in a) the answer implied $V = 64 \pm 0.5 \text{ cm}^3$, the new answer would be written as $\underline{64 \pm 1 \text{ cm}^3}$.

c) The uncertainty is $\underline{1/64, \text{or } 1.6\%}$, of the total volume.

PROBLEM 22

If a circular piece of tin has a measured radius of 2.6 cm, what is its circumference?

$C = \pi D = 3.14 \times 2.6 = 8.164 = \underline{8.2 \text{ cm.}}$

PROBLEM 23

A bag contains 25 identical marbles. A student makes measurements on one of them and determines its volume to be 1.76 cm^3. What should his estimate for the total volume of all the marbles be?

$V = 1.76 \times 25 = 44.0\ \text{cm}^3$. Here, the number 25 is exact. There is no error in the count. Therefore, the answer is properly expressed to three significant figures.

PROBLEM 24

A bus driver clocked the following times for portions of his route:

Station A to Station B	1.63 hr
Station B to Station C	4.7 hr
Station C to Station D	0.755 hr
Station D to Station E	2.00 hr

(a) How long did it take him to drive from Station A to Station E?

(b) What part of the whole trip does the time between Stations B and D represent?

(c) The time to go from Station A to Station C is how much more than the time to go from Station C to Station E?

a) 1.63
4.7
0.755
2.00
9.085 = 9.1 hrs.

b) 4.7
0.755
5.455 = 5.5 hrs.

$\frac{5.5}{9.1} = 0.60 = $ 6/10 of the trip

c) 1.63
4.7
6.33 = 6.3 hrs.

0.755
2.00
2.755 = 2.76 hrs.

6.3 - 2.8 = 3.5 hrs. more

Chapter 4 - Functions and Scaling

Throughout this course we shall have to express many mathematical relations. Some of the techniques for doing this job are collected for convenience in this chapter. A few important applications are also included both to introduce relations that will be used later and to add interest. Nevertheless, the mathematical formalities will take some time to sink in, particularly if this is the first exposure for students. Practice in the course of later work will be required for students to become proficient in the use of these ideas.

CHAPTER SUMMARY

Sections 1 through 3 One simple relationship between two quantities is expressed by direct proportion. Alternative ways of conveying the same information are by an equation involving only first powers, or a straight-line graph. The variation of the weight of a given substance with its volume is given as an example of a direct proportion. Second and third power laws, and the alternative ways of expressing them, are considered, with areas and volumes as the particular examples.

The hazards of inferring results for cases not covered by measurements, either by interpolation or extrapolation, are discussed.

As an application of these ideas, and as a further example, the inverse-square law for light intensity is derived, and applied to the problem of measuring the distances to stars.

Section 4 By considering the effect of a change of scale on various characteristics of physical and biological structures, the power laws are combined to obtain interesting new information about limitations on the size of things.

SCHEDULING CHAPTER 4

The relationships between physical quantities, and the various ways these relationships can be formulated and expressed, are the main subjects of the chapter. Much of the material may be familiar to students through their mathematics classes, in which case the main task is to link up what they know with physical observations and measurements. Only the simplest kinds of relationships are discussed and illustrated here. It is not the purpose of the chapter to learn about complicated graphs and their interpretation, or to delve deeply into measurement of stellar distances or the strength of materials. Treat all of the examples as introductions to physical relationships. The rest of the course is devoted essentially to amplifying just this theme.

Even students who have used graphical techniques in their mathematics courses often do not have much insight into the use of graphs to represent quantities or phenomena in the real world. We shall lean heavily on graphical techniques in Chapters 5 and 6, and in later work. Now is a good time to begin to show the power of graphical tools.

Sections 1 through 3 should be assigned for home reading prior to class discussion. If your class has a strong mathematical background, you may be able to cover the material in these sections quite rapidly. In this case, an extended discussion of Section 4 may prove novel and stimulating to your students. However, if your students need help in developing their mathematics for this course, spend most of the class time on the first three sections. Section 4, if necessary, can be handled largely as a reading assignment. The film, "Change of Scale", succinctly summarizes the important material in Section 4. Doing Experiment I-4 in class will give you a chance to help individual students when needed.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part 1.

	10-week schedule for Part 1			6-week schedule for Part 1		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2, 3	4	1	I-4	3	1	I-4
Sec. 4	2	0	-	1	0	-

RELATED MATERIALS FOR THE CHAPTER

Laboratory. Experiment I-4 - Analysis of an Experiment. This experiment presents data for students to graph and analyze. It is set up as an experiment rather than a homework problem to provide an opportunity for students to analyze the mathematical relationships involved under your guidance. There are several optional extensions to accommodate students with different mathematical backgrounds. The best time for this experiment is toward the end of your classwork on Chapter 4. See the yellow pages on Experiment I-4 for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 4-9; detailed comments and solutions on page 4-10 to 4-24.

Section	Easy	Medium	Hard	Class Discussion
1	1, 2*, 3	4*	5*, 6	2*, 5*, 6
2	8	9*, 10*	7*	7*, 9*, 10*
3	11*	14	12, 13	13, 14
4	15*	16, 17, 18* 21, 22*	19, 20*	15*, 16, 18* 20*, 22

Film. "Change of Scale", by Professor R. W. Williams of Massachusetts Institute of Technology. This film highlights the material of Section 4 with effective demonstrations which show that changing the size of objects necessitates a change of structure. Specially constructed props are used to illustrate the mathematical rules of scaling. The movie concludes with practical applications of scaling in the design of ships and harbors, and in the creation of special effects in motion pictures. Running time: 23 minutes.

Section 1 - Mathematical Relations

PURPOSE To introduce and illustrate the ways in which physical laws are formulated: proportions, equations (functions), tables, and graphs.

CONTENT a. Direct proportion is a particularly simple relation between measured physical quantities. It can be represented by symbolic notation such as $W \propto V$ (W is proportional to V); by an equation $W = kV$, which gives the actual numerical relation; by a table which gives the value of W for certain chosen or measured values of V; and by a straight line graph.

b. Areas and volumes are used as examples of second and third power laws, respectively. The equations, tables and graphs of these power laws are discussed.

c. The term "function" is introduced to include any dependence of one measured quantity on another.

EMPHASIS This material will be used in most of the quantitative work that follows in the course. It is very important that students grasp thoroughly the various ways in which relations are expressed – proportionalities, equations, tables and graphs – and appreciate that these are simply alternative ways of describing the same information. You may find that a considerable part of this material is familiar to your students through their algebra and geometry classes; they may be well aware that a relation between two unknowns x and y, $y = f(x)$, can also be represented by an equation, a table or a graph. The important job remaining is to link up algebraic unknowns, or scales on a graph, with physically measured quantities.

The purpose of this section is not to make students completely familiar with complicated graphs and their interpretation, but merely to introduce them to the very simple first, second and third power laws. More complicated cases will arise naturally in the later chapters, and can be discussed when they occur.

DEVELOPMENT – FIRST POWER RELATIONS These ideas are best introduced by considering a simple physical problem. Problems 1 and 2 are suitable examples, or you could use a problem such as determining the relation between the height of liquid in a container and the volume of the liquid. Standard measures of liquid can be used to fill a variety of containers, and the height of the liquid measured with a meter stick. Make up tables of data, listing volume of liquid in standard measures, height of liquid, and ratio of height to volume. Make graphs of height as the ordinate (vertical coordinate) versus volume as the abscissa (horizontal coordinate). Go through this procedure for containers of various shapes.

Students will see that a straight-line graph results whenever the third column of the table (ratio of height to volume) shows essentially constant values, and that this occurs for any vessel of constant cross sectional area. In fact, they should be able to deduce that the slope of the line (ratio of Y-coordinate to X-coordinate) is identical to the third column of the table, and is simply the reciprocal of the actual cross sectional area of the container (if zero height corresponds to zero volume).

For each case, formulate an equation representing the dependent variable (height) as a function of the independent variable (volume). Show that the linearity still holds even though the zero of height is shifted upwards or downwards (i.e., the equation $y = kx + b$ is a linear relation that becomes a direct proportion between x and y for the special case of $b = 0$).

You can include experiments on containers of variable cross sectional area, so that students can see that the results do not fall on a straight line for such cases, and that they cannot be expressed as direct proportionalities.

SECOND AND THIRD POWER RELATIONS The main points to stress here are the ways in which area and volume scale, and the appearance and behavior of second and third power equations and graphs. Students should note that the graph of a second power relation frequently can be represented by a straight line by plotting the square of one of the variables on one axis vs. the first power of the other variable.

The scaling of areas and volumes of regular figures will probably be familiar to students from their previous work. A little time will be required to convince them that the scaling is the same for a figure of any shape, provided that all linear dimensions are scaled in the same way. The most convincing approach is probably the one presented in the text, and illustrated in Figure 4-2, of subdividing an irregular figure into many regular figures (plus the small pieces at the boundary), each of which scales in the familiar way. For volumes, you could consider the analogous method of constructing distinctive shapes from child's building blocks, and doubling all linear dimensions, but this may be unnecessary.

For practice with graphs, the suggestions in the caption to Figure 4-4 are apt, and can be applied also to the cube-law graph, Figure 4-5. Adjusting the vertical scales of these graphs to make them appropriate to the cases of a circle and a sphere, respectively, will emphasize the conclusions on scaling, and is good practice for scale-choosing.

Problem 6 will bring home the point of scaling all dimensions of a body. Problems 7 and 10 provide good general illustrations of the material.

Comment on functions: In mathematics, if y = f (x) (y is a function of x), then for any value of x, there is a mathematical prescription for y. Power relations, $y = x^n$, are particularly simple functions. Any measurement of two physically correlated quantities, such that when we measure one of them we get a unique answer for the other, defines a functional relation between them. The procedure of analyzing and plotting the data is simply a way of finding out whether this functional relation is a simple power law, or something more complicated. It is very kind of nature that some laws of physics are simple power relations, but this is by no means always the case. Problem 9 of HDL is a good example of a simple experiment which involves a more complicated function.

Section 2 - Interpolation and Extrapolation

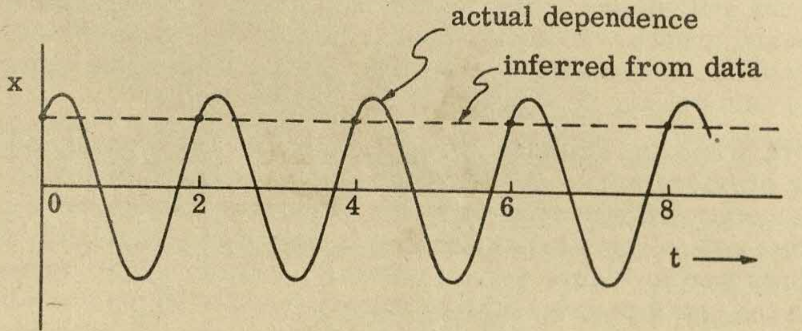
PURPOSE To point out possible difficulties in interpolating or extrapolating the results of an experiment.

CONTENT a. Interpolation is meaningful only if the dependence described by the measured data is known on physical grounds to be smooth.

b. Extrapolation assumes that the dependence described by the measured data persists outside the ranges examined. This may or may not be correct (and therefore useful). It depends on the particular experiment.

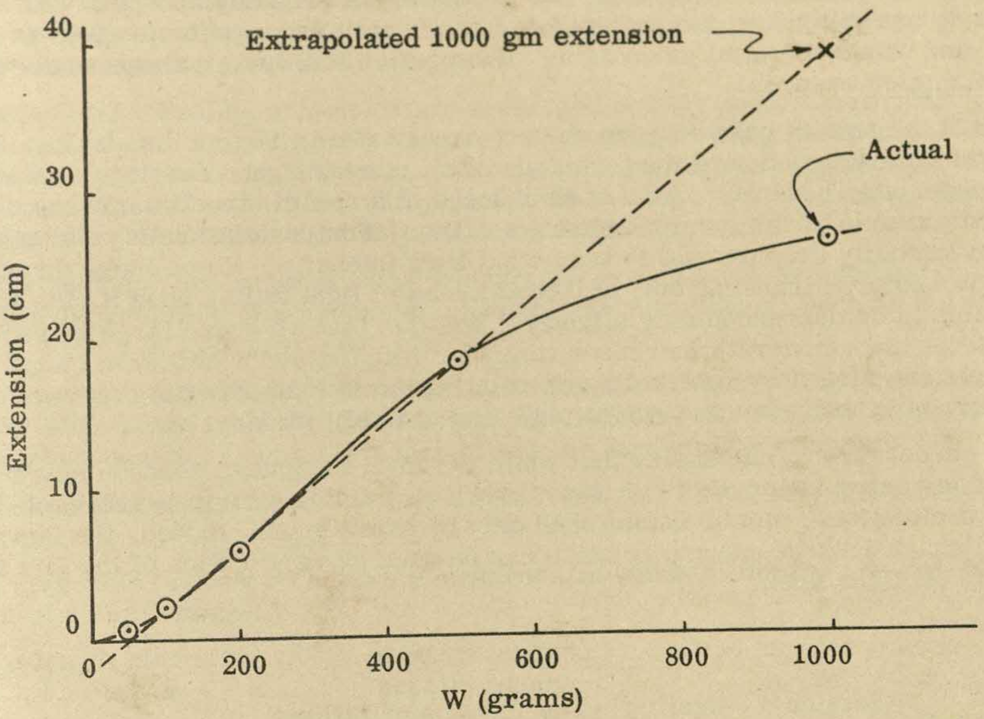
EMPHASIS This material presents an important qualification of the previous section, but a reasonable and easily acceptable one. Not much time should be needed.

DEVELOPMENT The examples chosen to illustrate the material of Section 1 all involve essentially geometrical relationships. It is clear that the dependences obtained are smooth, and that the relationship between the measured quantities lies on the smooth curve for all intermediate values. The only way students will appreciate the warning being given to them is to consider an example where interpolation will not work. A possible one is as follows: consider the pendulum of a grandfather clock which makes one complete oscillation every two seconds. Suppose you observe its position, x, at two-second intervals. x will be the same for every measurement. Given this set of points, and not knowing the experiment, one would be tempted to draw a horizontal straight line through the points, and say that for all intermediate values of t, x is constant. In this case, it would be obviously



incorrect. The actual function is the curve shown, as one would find out if one made observations at intermediate times. The students should get the point without a detailed discussion of the actual shape of the curve, and not much time need be spent over it. To summarize, one can say that whether interpolation is warranted depends on the particular experiment considered. If the physics of the situation is too involved for a prediction about the smoothness of the dependence, the only way to find out is to repeat the experiment at intermediate values.

The text contains examples of cases where extrapolation from known data does and does not work. Students will probably understand well enough the case of aircraft breaking the sound barrier, but a detailed discussion of this and the other examples is clearly not called for. As a homely example, you could experimentally examine the stretch of a rubber band. Plot its extension as a function of the weight suspended from it, keeping the weights small enough so that the band behaves elastically. You will get a straight line. You can then attempt to predict the stretch which you would expect for, say, twice the maximum weight used previously. You do this by extrapolating the approximate straight line dependence obtained already. Now actually apply twice the maximum weight already used.



If you have chosen suitable values, you should find an extension considerably less than was predicted. (See the figure.) The moral is that extrapolation was unwarranted in this case. Be sure to use a new, unstrained rubber band. You will not get an exact straight line except for small extensions. Problem 8 provides an alternative example of the perils of extrapolation, but the effect may not be as marked as with a rubber band.

Section 3 - The Inverse-Square Relation

PURPOSE To illustrate the methods and ideas of Section 1 by considering the emission of light from a point source, which introduces the inverse-square relation.

CONTENT a. By geometry, assuming that light travels in straight lines, $I \propto \frac{1}{d^2}$ (I the light intensity, d the distance from the point source).

b. By using this relation, the relative distances to two identical lamps can be determined by measuring the relative intensity.

c. This method is used to measure distances to stars.

d. The inverse-square relation will only hold if the light travels in straight lines, and if there is no obscuring material in its path.

EMPHASIS The inverse-square relation is an interesting physical law that everyone should know about, but it is not essential for the development of the rest of Part 1. (It is treated more fully in Part 2, Chapter 15, and used extensively in Parts 3 and 4.) It is worth spending a short time on here, both for its intrinsic interest and the astronomical application, provided the physics of light propagation is not allowed to become a stumbling block. A brief treatment should suffice.

COMMENT As with the area and volume relations of Section 1, the "proof" of the inverse-square relation for light intensity is geometric. Any physically measurable quantity emitted from a point source, and traveling in straight lines, will obey the inverse-square relation. The essential physical assumption is thus that light does travel in a straight line. This is stated in the text, and students will probably accept it without question. Do not stress this point; the evidence for rectilinear propagation is given in Part 2, Chapter 12, and would be out of place here. We mention it only to indicate where the physical assumption crept in.

As noted in the text on page 45, you are not expected to get into a discussion of the nature of light or a discussion of the technique of measuring light intensity. One warning might be appropriate, however. As a student looks at a row of street lamps, he does not "see" a good example of the inverse-square relation. Fortunately for us, our eyes do not "see" an intensity proportional to the actual light intensity. If one looks through a filter which is known to transmit only half the amount of light falling upon it, the light intensity seems to be decreased only slightly. The fact that we sense large increases in light intensity as only moderate increases (together with the eye's ability to change the amount of light entering it by contracting or dilating the iris diaphragm), allows us to see reasonably well, both on a moonlight night and at bright midday.

It may be necessary to emphasize that while the inverse-square relation may be "proved" by geometry, the validity of this "law", as it applies to propagation of light and other physical situations, can be established only by experiment. Indeed, the fact that light does follow an inverse-square relation can be used as verification of the fact that it must travel in straight lines.

Section 4 - Scaling — The Physics of Lilliput

PURPOSE To show applications of first, second and third power relations in an introduction to scaling, and to show the role of scaling in scientific investigation.

CONTENT Structures whose dimensions are proportional but quite different in magnitude have very different physical behavior:

a. The strength of structural members (bones, and tendons, girders, etc.) depends on their cross sectional area, i.e., on L^2, where L characterizes the linear dimension. The weight that must be carried by the structure depends on its volume, i.e., on L^3. Together, these two relations (strength $\propto L^2$ and weight $\propto L^3$) place, for instance, an upper limit on the size of living creatures. Animals of different sizes (different scales) must be differently proportioned to preserve an adequate ratio between their strength and weight.

b. Surface area is proportional to L^2. Hence, biological and physical characteristics that are affected by interrelationships between surface area (L^2) and volume (L^3) are altered markedly by changes in scale. Examples: The weight of the film of water clinging to a body surface is negligible compared to body weight (volume) for a human, but not for a fly. A small animal's food intake requirements are a much larger proportion of the animal's volume than for a large animal.

c. As any system is scaled up, the load will eventually be greater than the strength of the structure. Thus, conventional buildings cannot be scaled up without limit, mountains are solid or nearly so, and planets tend to be spherical.

d. In physical investigations, it is often helpful to examine how a system's behavior changes as its dimensions are scaled up or down. A study of systems built on unusual scales often discloses unsuspected physical relationships.

EMPHASIS Although scaling is fascinating to most people and proper regard for scaling factors is an important part of many physical investigations, it will be wise to limit your classwork pretty much to the level presented in the text. At this point we simply want to implant the ideas. While there are many other interesting examples of scaling in biology and physics, they can take you too far afield. As the course proceeds, you will be able to point out many scaling considerations and assumptions in the laboratory experiments and in the experiments described in the text.

DEVELOPMENT Scaling principles are presented clearly and dramatically in the movie "Change of Scale". The film includes many interesting scaling illustrations which would be difficult to present in a classroom. After students have read the section, a preliminary discussion, followed by the film, will probably be the best way of handling this section. The preliminary discussion need only make the following points:

(a) The behavior and appearance of a structure or system is determined to a large extent by the interrelation of a number of physical characteristics, such as the weight, strength, energy intake, heat loss, etc.

(b) When the linear dimensions of the structure are scaled up or down, these physical characteristics scale according to different powers of the dimensions. The changed relationship between these characteristics alters the structure's behavior.

(c) The effects of scaling result in a limit on the size (maximum and minimum) of any structure.

You may wish to draw up a table of squares and cubes to impress on students the marked effect of scale changes:

If the length of the edge of a cube in meters is L	then its surface area in square meters is $6 \times L^2$,	and its volume in cubic meters is L^3.
1	6	1
2	24	8
3	54	27
4	96	64
5	150	125
6	216	216
10	600	1,000
50	15,000	125,000

Notice how rapidly the numbers for volume catch and pass the number for surface area, and that the numbers for surface area increase much more rapidly than the linear dimensions.

The movie can then follow immediately. Physical laws and effects not known to the students are shown in the film, but they are presented plausibly (surface tension, simpleharmonic motion, for example). Some of these will be treated later in the course. Try to avoid detailed discussion of small points – the over-all picture is all that is important here.

You may want to recommend to interested students the brilliant six-page essay on this topic, "On Being the Right Size", by J. B. S. Haldane. It is available among other places in Volume II of the World of Mathematics.[1] You can also suggest that they view with a critical eye (from the physicists's viewpoint) the horrific movies which, in their portrayal of creatures from remote lagoons and outer space, violate most of the natural laws discussed in this section. Problem 19 of HDL is a start in this direction.

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[1] James R. Newman, editor, "On Being the Right Size", World of Mathematics, Vol. II, Simon and Schuster, New York, (November, 1956).

Chapter 4 - Functions and Scaling

For Home, Desk and Lab — Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Many of the early problems introduce very simple scaling factors as an introduction to the work on scaling in Section 4. Problems which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 4-10 to 4-24.

Section	Easy	Medium	Hard	Class Discussion
1	1, 2*, 3	4*	5*, 6	2*, 5*, 6
2	8	9*, 10*	7*	7*, 9*, 10*
3	11*	14	12, 13	13, 14
4	15*	16, 17, 18* 21, 22*	19, 20*	15*, 16, 18* 20*, 22

SHORT ANSWERS

1. 200 sheets: 16 mm; 300 sheets: 24 mm; 600 sheets: 48 mm.
2. (a) See graph on page 4-10.
   (b) The points cannot be joined by a smooth line.
3. A increased by a factor 4.
4. See discussion on page 4-11.
5. (a) 1.0 m.
   (b) 1.41 m.
6. $3^{1/3} = 1.44$.
7. (a) See table in detailed solution.
   (b) See graph on page 4-14.
   (c) h = 40 m; D = 22.5 km;
   h = 70 m; D $\approx$ 30 km.
   (d) Plot D versus $\sqrt{h}$ or D^2 versus h.
8. (a) Yes.
   (b) See discussion on page 4-15.
9. (a) See graph on page 4-16.
   (b) See graph and discussion on page 4-16.
   (c) .69 atmospheres.
10. (a) See graph on page 4-18.
    (b) Yes, for large values of h.
    (c) .32 atmospheres.
    (d) 3×10^{-6} atmospheres. Not on this evidence.
11. 20 cm from one bulb, and 40 cm from the other bulb.
12. 18.
13. (a) 2.56.
    (b) $(2.56)^2 = 6.55$.
14. 22 meters.
15. (a) Increased 8 times.
    (b) Decreased by factor 2.
16. (a) 1.26 } or (a) 1.41
    (b) 2 } (b) 2.83
    See discussion on page 4-21.
17. 1.59 meters on an edge.
18. See discussion on page 4-22.
19. 3×10^2. This compares favorably with the stated food intake ratio.
20. See discussion on page 4-23.
21. (a) .224 cm.
    (b) 1.12 cm.
22. Pillars 630 cm in diameter. The tank will hold 1000 times as much water.

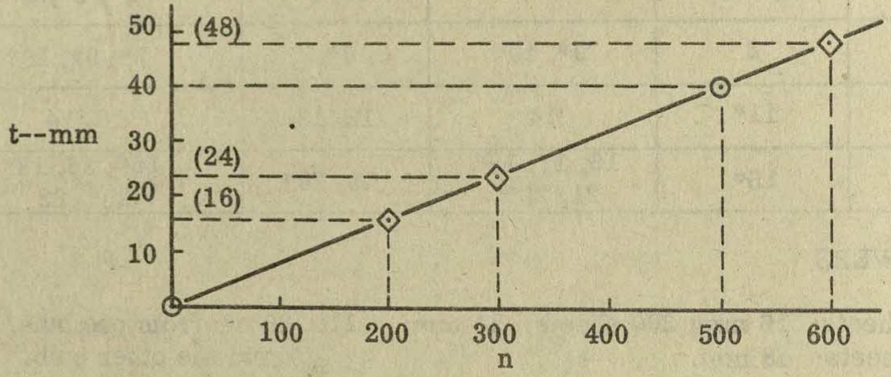
COMMENTS AND SOLUTIONS

PROBLEM 1

The total thickness of 500 sheets of a particular sample of paper is 40 mm. Assuming that they are of uniform thickness, plot a graph of the total thickness against the number of sheets. From your graph determine the thickness of 200 sheets. Of 300 sheets. Of 600 sheets.

This problem is best used as a simple exercise in graphing. However, it is probably too much to expect that students will not verify their answers by mental arithmetic.

Most students will see intuitively that the thickness, t, will be proportional to the number of sheets, n,($t \propto n$). The graph of t versus n is thus a straight line passing through the origin (zero sheets have zero thickness). We are given one other point: $n = 500$ corresponds to $t = 40$ mm. The straight line is determined by these two points.



The thicknesses for 200, 300 and 600 sheets may then be read off.

PROBLEM 2

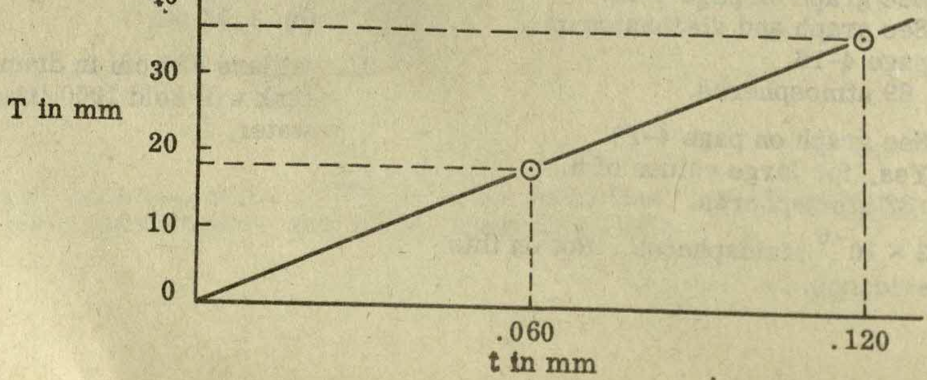
A book is to have a total of 600 pages.

(a) Plot the thickness of the book against the thickness of each sheet of paper (1 sheet is 2 pages), assuming that the paper is available in *any* thickness from 0.060 mm to 0.120 mm.

(b) Actually, paper is available only in certain specified thicknesses. If we assume that the thicknesses are multiples of 0.005 mm between these two limits, can we connect the points of our graph with a smooth line? Why?

Part (a) is to be treated as a problem in graphing. Part (b) is an illustration of a case where interpolation is not warranted.

a) We know that the thickness of 300 identical sheets of paper will be proportional to the thickness of one sheet, so that the graph of total thickness, T, versus thickness of one sheet, t, will be a straight line. In fact, we can see immediately that $T = 300\ t$. The graph can be drawn by connecting the two points $t = .060$ mm, $T = 18$ mm; and $t = .120$ mm, $T = 36$ mm. The thickness T for any intermediate value of t can then be read off the graph.



For students who have not had much graphing experience, scale choice may present some problems. T and t both have units of mm, and some students will feel that the vertical and horizontal scales should be the same. Point out to them that T and t are different quantities: one is total thickness,. the other the thickness of one sheet. Although they happen to have the same units, the scales for each can still be chosen arbitrarily, and independently.

b) With the relationship $T = 300\ t$, one usually assumes that t can be any value, and the corresponding value of T is determined by the equation or the graph. In this problem the physical situation restricts t to the values .060 mm, .065 mm, .070 mm, etc. The intermediate values of t on a line running through these points correspond to physical situations (thickness) which do not exist. In this case it is incorrect to join the points by a line.

PROBLEM 3 The total surface area of a rectangular solid is the sum of the areas of the six faces. If each dimension of a given rectangular solid is doubled, what effect does this have on the total surface area?

This problem provides an easy preliminary example on how scaling of linear dimensions affects areas.

It is unnecessary to make a detailed calculation. Each face of the solid is a rectangle with each side double the original length. The area of each face is thus increased by a a factor 4 ($=2^2$). Therefore, the total area also is increased by a factor 4.

PROBLEM 4 Carefully plot the area of a square against length of side for lengths from 1 to 10. (This may be used as a square-root table in solving other problems.)

This problem gives practice in graph plotting, enables visualizing the second-power law graph, and provides an opportunity to discuss power-of-ten problems when taking square roots.

The students should use a full page of graph paper, choose their scales to utilize the whole sheet, and draw a careful graph.

The graph can be used to find the square of any number, simply by expressing the number as the product of a number between 1 and 10 times the appropriate power of 10, noting that $(10^n)^2 = 10^{2n}$, for any value of n, positive or negative.

In finding the square root of a number, it is most convenient to express the number in power-of-ten notation using an even power of ten, since $\sqrt{10^n} = 10^{1/2\,n}$, which is simple if n is even. Thus, to find $\sqrt{7500}$, we could write

$$\sqrt{7500} = \sqrt{75 \times 10^2} = 10\sqrt{75} = 87,$$

or

$$\sqrt{7500} = \sqrt{0.75 \times 10^4} = 100\sqrt{0.75} = 87.$$

Similarly, $\sqrt{750} = \sqrt{7.5 \times 10^2} = 10\sqrt{7.5} = 27.$

It will be helpful to give students practice with negative powers-of-ten. For example, ask them to find the square root of .75 and .075.

After students have practiced using a graph of squares an interesting class activity is to have them put away the graphs and use their knowledge of powers-of-ten and memory of the graphs to make rough estimates of squares and square roots.

PROBLEM 5

The volume of one cylinder is eight times the volume of another similar cylinder.

(a) If the circumference of the base of the smaller cylinder is 0.5 m, what is the circumference of the base of the larger cylinder?

(b) If another cylinder, also having eight times the volume, has the same height as the smaller cylinder, what is the circumference of its base?

The volume of a cylinder is equal to the base area, A, times the height, h.

$$V = Ah$$

a) The cylinders (1) and (2) (cylinder 1 is the larger of the two) are similar, i.e., all corresponding linear dimensions are in the same proportion. Thus, if r_1 and r_2 are the radii, and h_1 and h_2 the heights, then

$$h_1 = a\,h_2$$
$$r_1 = a\,r_2$$

(a is the constant of linear proportionality)

Since $A = \pi r^2$, $A_1 = \pi r_1^2 = \pi a^2 r_2^2 = a^2 A_2$

so that $V_1 = A_1 h_1 = a^2 A_2 \times a\,h_2 = a^3 V_2$

Since we are given that $V_1 = 8\,V_2$, then $a^3 = 8$, and $a = 2$.

Circumference of the base C is equal to $2\pi r$, so $C_1 = 2\pi r_1 = 2\pi a r_2 = aC_2$.

Hence, $C_1 = 2C_2$. We need not have worked this out explicitly: since circumference is a linear dimension, it must also scale by a factor $a = 2$. The circumference of the larger cylinder is thus $.5 \times 2 = \underline{1.0\text{ m}}$.

b) Now the heights are the same. Suppose the radii scale by a factor b.

$$h_1 = h_2$$
$$A_1 = b^2 A_2$$
$$V_1 = A_1 h_1 = b^2 A_2 \times h_2 = b^2 V_2.$$

Since we are given that $V_1 = 8\,V_2$,

$$b^2 = 8$$
$$b = 2\sqrt{2} \approx 2.83.$$

The circumference of the larger cylinder is $.5 \times 2.83 = \underline{1.41\text{ m}}$.

PROBLEM 6

A hollow metal sphere has a wall thickness of 2 cm. If you increase both the diameter and thickness of this sphere so that the overall volume is three times the original overall volume, how thick will the shell of the new sphere be?

To handle this problem, we must assume that the outside diameter, the thickness, and all other linear properties of the sphere are scaled up by the same factor. Suppose this factor is a.

The new diameter d′ is a times the old one, d. The new volume V′ thus depends on the original volume as follows:

$$V' = \frac{4}{3}\pi\left(\frac{d'}{2}\right)^3 = a^3\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = a^3 V.$$

Since we are given that V′ = 3V, then $a^3 = 3$, and $a = 3^{1/3} = 1.44$. The new thickness will be 1.44 times the old one.

Some students may tend to think that since the thickness increases, the inside diameter does not. You may have to draw a cross section on the blackboard to convince them that all linear dimensions scale by the same factor.

PROBLEM 7

The range of a television station is a function of the height of the antenna. The range is given by the formula $D = 3.56\sqrt{h}$, where D is the range in kilometers and h is the height of the antenna in meters.

(a) Calculate the range for antenna heights of 10 meters, 20 m, 30 m, and 50 m.

(b) Plot the range against the antenna height.

(c) Can you determine from the graph the range for an antenna height of 40 m? Of 70 m?

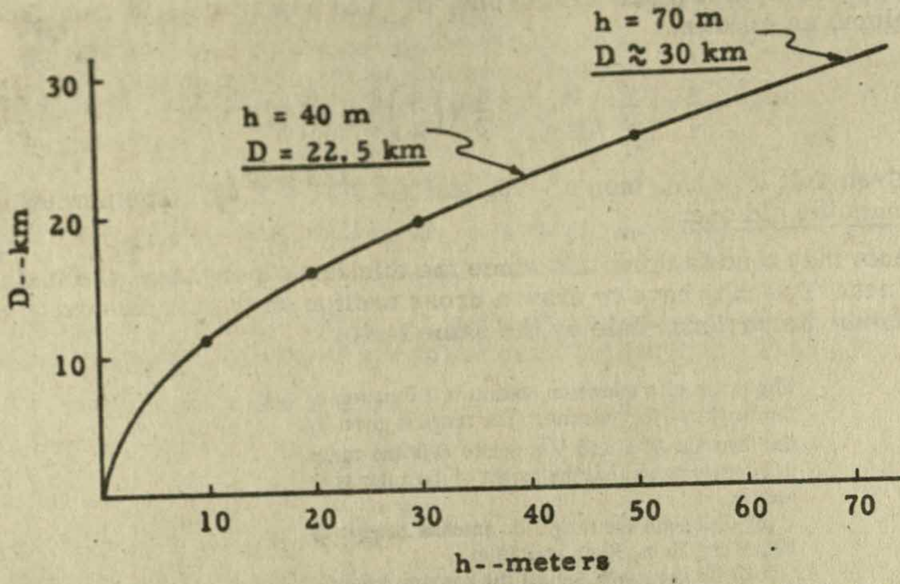
(d) How would you plot the data to get a straight-line graph?

This interesting practical problem involves obtaining tables from a given equation, and graphing. Students who are overcautious about significant figures may get in a wrangle. The question of significant figures should be regarded from a practical point of view: in part a) heights are given as 10, 20,----meters, but in setting up an antenna you can probably adjust its height accurately to less than 10 cm. So assume that these are 10.0, 20.0,----, and give the values of $\sqrt{h}$ and D accordingly. (The accuracy of the formula depends sensitively on the assumption that the earth's surface is that of a perfect sphere. Local variations in terrain can make quite a difference, as you may have found when erecting your own television antenna for fringe reception.)

a) The values required are given in the following table:

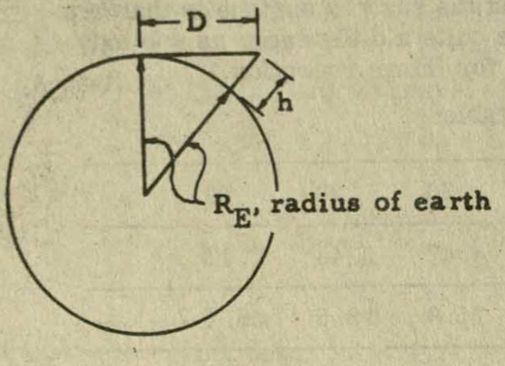
Height in m	10	20	30	50
$\sqrt{h}$	3.16	4.47	5.48	7.07
Range in km	11.3	15.9	19.5	25.2

b) and c) The graph is smooth. To maintain interest, ask the students to draw it using only the above points, using their judgment as to how it goes in between 30 and 50 m, and above 50 m. Their interpolated value at 40 km, and their extrapolated value at 70 km, can then be checked by using the equation.



d) Some students may have observed that the curve they have drawn looks like the second-power law curve if it is tipped on its side. This is exactly right: since $D = 3.56\sqrt{h}$, $D^2 = (3.56)^2 h$, so if we plotted h vertically and D horizontally, we should get the second-power law curve (a parabola). This gives us a clue as to what to plot in order to get a straight line. We can either plot D against $\sqrt{h}$ or D^2 against h. In both cases, the second-power law graph of Problem 4 is useful either to obtain $\sqrt{h}$ from h or D from D^2. We can then determine D for any h from our two graphs.

Note: the relation $D = 3.56\sqrt{h}$ can be deduced by geometry, by making the assumption that h is much less than the radius of the earth. By Pythagoras' theorem,



$$D^2 + R_E^2 = (h + R_E)^2$$

$$= h^2 + 2h\,R_E + R_E^2,$$

and if we neglect the term h^2 compared with $2h\,R_E$, we get

$$D^2 \approx 2R_E\,h.$$

Inserting R_E = 6400 km, we get the relation given in the text. Some of your more inquisitive students might be interested in seeing this approximate derivation.

PROBLEM 8

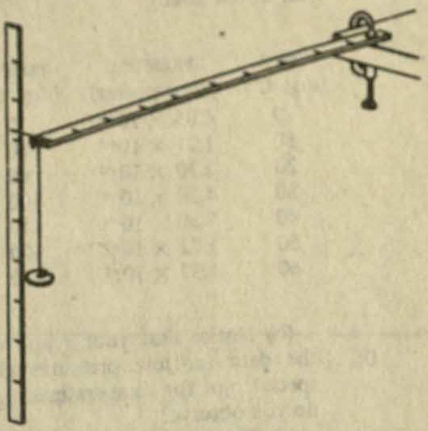
Solid materials resist the application of forces tending to distort them. Clamp a meter stick to a table and hang various known weights from the end. Measure the amount of bending in each case and plot these values against the corresponding weight.

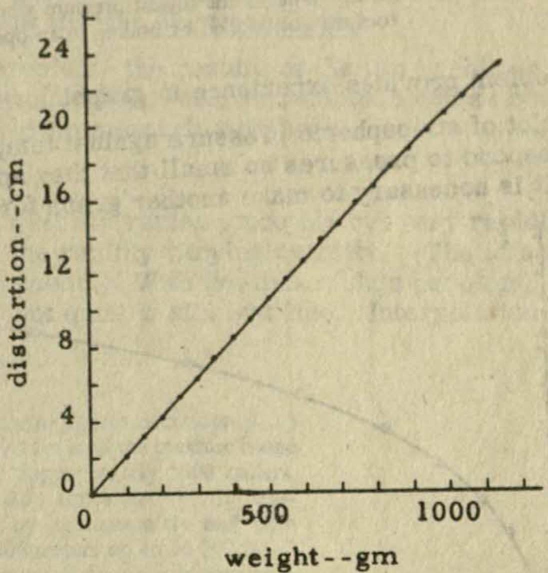
(a) Does your graph suggest that the amount of distortion is directly proportional to the weights applied?

(b) Does this relation hold for extremely small weights? Large weights?

A wooden meter stick was clamped to a desk as shown below with 95 cm beyond the edge of the table. As weights were hung on a wire attached to the end of the meter stick, the distances from the floor to the end of the meter stick were measured. The table and graph below show the results of these measurements. Caution.—Our meter stick cracked under a 1200 gm load without much indication of being close to the yield point.

Weight (gm)	Vertical Position (cm)	Distortion (cm)
0	76.6	0
50	75.5	1.1
100	74.5	2.1
150	73.4	3.2
200	72.4	4.2
250	71.3	5.3
300	70.3	6.3
350	69.2	7.4
400	68.1	8.5
450	67.1	9.5
500	66.0	10.6
550	65.0	11.6
600	64.0	12.6
650	62.9	13.7
700	61.0	14.6
750	60.9	15.7
800	60.0	16.6
850	59.0	17.6
900	58.0	18.6
950	57.0	19.6
1000	56.1	20.5
1050	55.2	21.4
1100	54.3	22.3
1150	53.4	23.2





a) The graph of distortion vs. weight is nearly a straight line so it appears that the distortion is directly proportional to the weights applied.

b) Since the graph shows a straight line relationship at the lower end it may be presumed that one could interpolate for distortions of very small weights. However, for weights of 1, 2, or even 5 grams a method of measuring more precisely than that shown would be needed as these weights produced very small distortions. From about 600 gm to a 1000 gm the displacement increased at a lower rate than for smaller weights. In the last few readings from 1000 gm to 1150 gm a slightly lower distortion rate was noted, and at 1200 gm the stick cracked.

It appears that if one could accurately determine the position of the end of the meter stick, the proportional relation would hold for small weights but not for large weights.

PROBLEM 9

(a) The temperature at which a liquid boils depends upon the air pressure acting on its surface. Plot the pressures and the corresponding boiling temperatures of water from the data in the following table.

(One atmosphere is the pressure exerted by the air at sea level.)

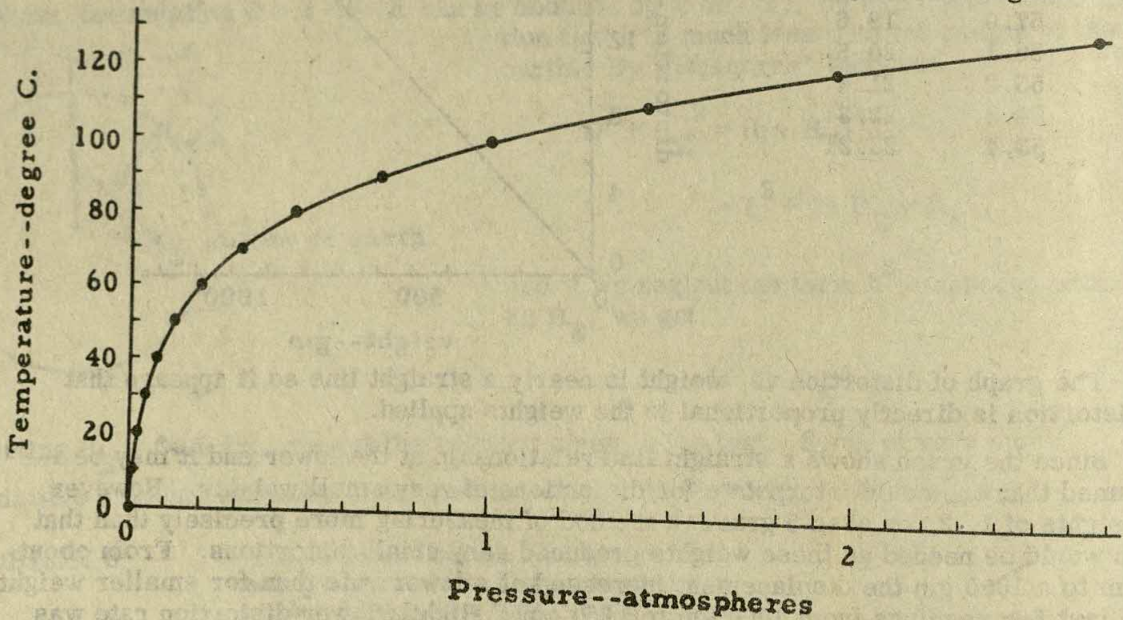
TEMP. (*deg. C.*)	PRESSURE (*atmospheres*)	TEMP. (*deg. C.*)	PRESSURE (*atmospheres*)
0	6.05×10^{-3}	70	3.12×10^{-1}
10	1.21×10^{-2}	80	4.67×10^{-1}
20	2.30×10^{-2}	90	6.92×10^{-1}
30	4.20×10^{-2}	100	1.00
40	7.30×10^{-2}	110	1.43
50	1.22×10^{-1}	120	1.96
60	1.97×10^{-1}	130	2.67

(b) Notice that your graph does not present the data for low pressures clearly. Make a special plot for temperatures up to 60°. What do you observe?

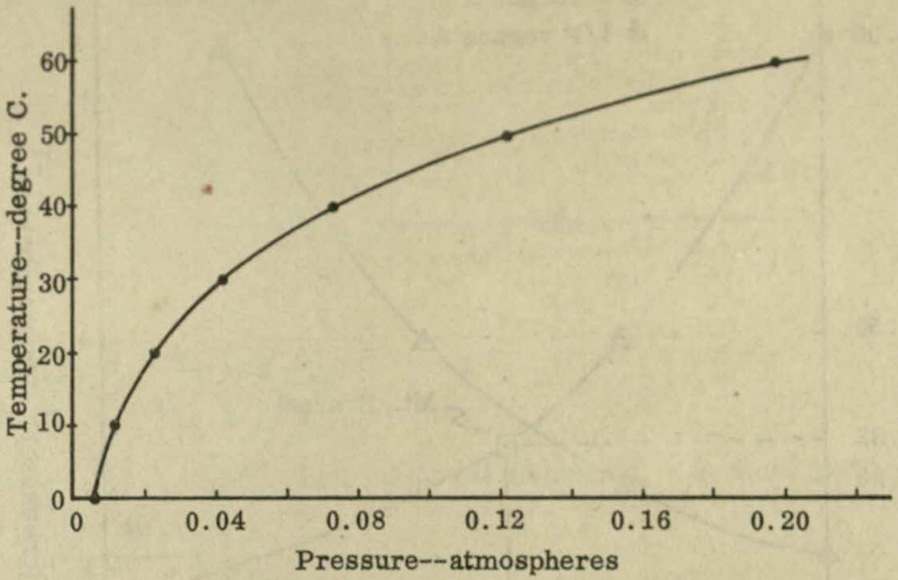
(c) The starch in potatoes will not convert to the edible form at temperatures of less than about 90°C. What is the lowest pressure where potatoes may be cooked by boiling in an open pan?

This problem provides experience in graphing with a new, unfamiliar law.

a) In a plot of atmospheric pressure against temperature, the low-temperature points correspond to pressures so small that they appear as zero on the graph. Hence, in part b), it is necessary to make another graph for the low-temperature region.



b) The interesting feature of the two graphs is that they are very similar: the rough shape of the curve is still the same, although we are now plotting only the lower half of the temperature scale.



c) The answer can be given by looking at the table, without reference to the graph. Water boils at 90°C (i. e., you can't heat it to a higher temperature than 90° in an open pan) when the atmospheric pressure is about 0.69 atmospheres.

If Problem 10 is done with this problem, the results of the two problems can be combined to deduce the temperatures of boiling water at various heights above sea-level, important information for would-be mountain climbers.

Note on a better way to plot the data:

With data in which the dependent quantity varies smoothly but very rapidly, it is often helpful to plot the logarithm of the rapidly varying quantity. (The effect can also be achieved by using semi-log graph paper.) With the data of this problem, a graph of $\log_{10} P$ versus T is very smooth, not quite a straight line. Interpolation over the whole range of T is now convenient.

PROBLEM 10

The pressure of the atmosphere decreases as the altitude increases. At sea level the pressure is one atmosphere and at approximately 5500 meters, the pressure is 0.5 atmospheres. It continues to decrease by approximately half with each increase of 5500 meters up to 16,000 m.

(a) Plot the pressure *P* in atmospheres as a function of altitude *A* in meters.

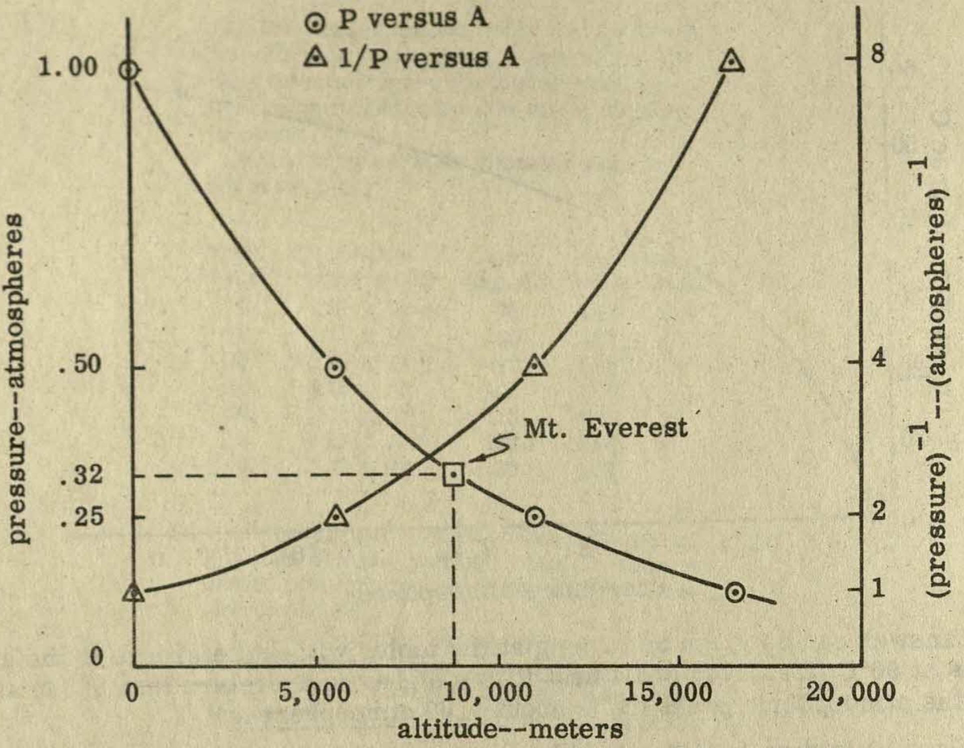
(b) Is the representation improved by plotting $1/P$?

(c) What is the pressure at the top of Mt. Everest (8800 meters)?

(d) What is the pressure in atmospheres at an altitude of 100 kilometers? Do you believe your answer?

This problem will help students see that translating physical information into graphs, helps in visualizing the physics of the situation. The problem can be combined with Problem 9, to produce interesting new information.

a)



b) We have used the same horizontal scale for 1/P versus A. The two curves are very similar, and complement each other in that at small altitudes, where the P versus A curve is easy to read, the 1/P versus A curve is compressed, and hard to read. At large altitudes the reverse is true. So there is some use in plotting 1/P versus A, to display the behavior in the large-A region.

c) From the P graph, the pressure at the top of Mount Everest is 0.32 atmospheres. (The same information can be read from the 1/P graph.)

d) We have not carried on the horizontal scale to include 100 km = 100,000 m. If the stated dependence were assumed to persist up to this altitude, the pressure would be $(\frac{1}{2})^{\frac{100,000}{5,500}} \approx \underline{3 \times 10^{-6}}$ atmospheres. It would be impossible to read this from a graph such as we have drawn. In fact, the dependence is given as reliable only up to 16 km in altitude, and we have gone far beyond this point. This result is therefore not to be believed.

Connection with Problem 9:

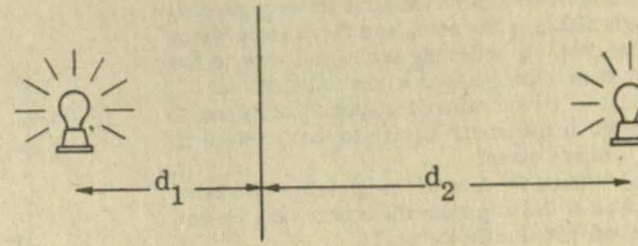
In Problem 9 we found the temperature of boiling water as a function of atmospheric pressure. Here we have the atmospheric pressure as a function of altitude. By using both graphs, we can give the temperature of boiling water as a function of the altitude. In fact, a graph of T versus A can be constructed. It turns out to be much more slowly varying than either of the curves T versus P, or P versus A. This is because P is the rapidly varying quantity in each case, and we have eliminated it.

Note that, as in Problem 9, a plot of log P versus A will be much smoother than the curves we have drawn. In fact, it is a straight line in this case.

PROBLEM 11

If two identical lamps are 60 cm apart, where should a screen be placed between them so that the intensity on one side of the screen is four times the intensity on the other side?

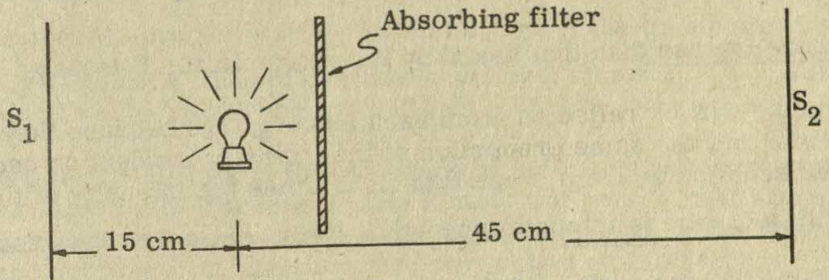
The lamps are identical, so $I = \frac{k}{d^2}$ for each lamp, with the same constant of proportionality. We are given



that $I_1 = 4\,I_2$, i.e., $\frac{k}{d_1^{\,2}} = 4\frac{k}{d_2^{\,2}}$

Thus $d_1^{\,2} = \frac{1}{4}d_2^{\,2}$, $d_1 = \frac{1}{2}d_2$. Since the total distance $d_1 + d_2$ must be 60 cm, we have $\underline{d_1 = 20 \text{ cm}}$, $\underline{d_2 = 40 \text{ cm}}$.

PROBLEM 12 A small source of light is placed between two parallel, white screens (*S*₁ and *S*₂). It is 15 cm from *S*₁ and 45 cm from *S*₂. An absorbing filter that allows one half the light that falls upon it to pass through is placed between the light source and *S*₂. What is the ratio of the intensities of the light that falls on *S*₁ and *S*₂?



Suppose that the intensity law for the lamp, without filter, is

$$I = \frac{k}{d^2}.$$

This is what applies to the illumination of S_1. For S_2, only half of the intensity gets through the filter. So for S_2, the intensity law is

$$I_2 = \frac{1}{2}\,\frac{k}{d^2}.$$

Thus, $I_1 = \frac{k}{(15)^2}$, $I_2 = \frac{1}{2}\,\frac{k}{(45)^2}$,

and $\frac{I_1}{I_2} = \frac{2 \times (45)^2}{(15)^2} = \underline{18}.$

This is simply a formal way of saying: One the grounds of distance alone, the ratio of distances being 3:1, the intensity ratio would be 3^2:1 = 9:1. But because the intensity on the more distance screen is reduced by the filter to 1/2, the ratio is 18:1.

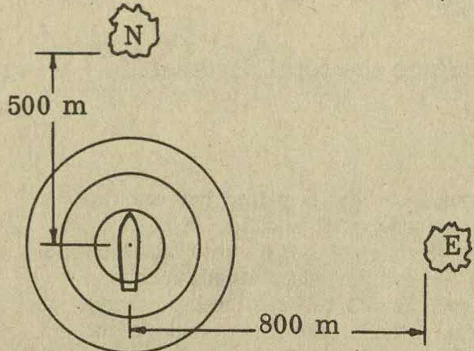
PROBLEM 13

A ship sends out a sonar signal (a short pulse of sound that radiates uniformly in all directions). There are two nearly identical icebergs nearby. One is 500 m to the north and the other is 800 m to the east. In reflecting this signal back to the ship, each iceberg acts as a new sound source.

(a) What is the ratio of the intensity of the sound received at the nearer iceberg to that received at the further iceberg?

(b) What is the ratio of the intensity of the echo received at the ship from the near iceberg to that received from the far iceberg?

This is a rather hard problem in the application of the inverse square law. (We are given that the sonar pulse radiates in all directions, and since it will presumably travel in straight lines, we are to apply the inverse square law.)



a) Since $I \propto \frac{1}{d^2}$, the sound intensity that a polar bear on the N iceberg hears is $\left(\frac{800}{500}\right)^2$ = 2.56 more intense than that heard by his cousin on the E iceberg.

b) The sonar pulse will be reflected from each iceberg, and because they are nearly identical, we assume that the same proportion of the intensity incident on each berg is reflected. The reflected pulse from N has thus 2.56 times the intensity of the pulse from E. Each pulse will decrease in intensity according to the $\frac{1}{d^2}$ law as it radiates from each berg. If I_N and I_E are the intensities of these reflected pulses, then at distances d_N and d_E from each berg,

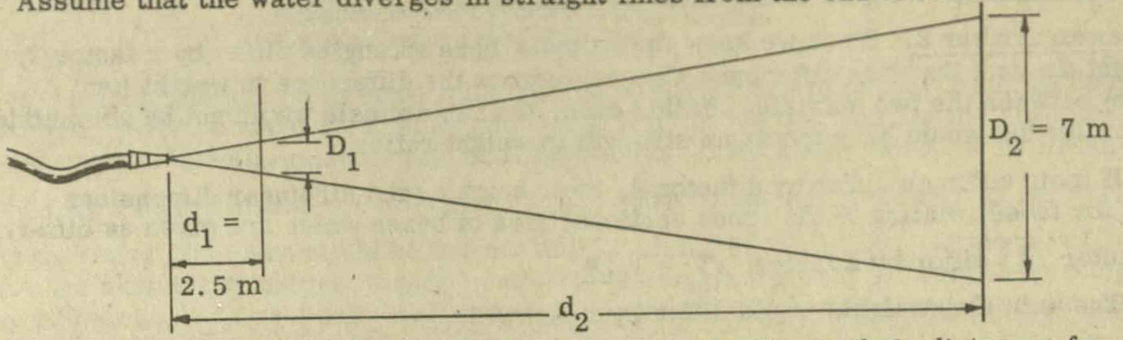
$$\frac{I_N}{I_E} = 2.56 \, \frac{d_E^{\,2}}{d_N^{\,2}} = (2.56)^2 = 6.55.$$

The echo from the nearer iceberg will thus be 6.55 times as intense as that from the further one. For bergs of equal reflecting ability, the intensity of the reflected pulse thus varies as $1/d^4$, an inverse fourth-power law. This rapid dependence on distance is a very useful property of devices of this kind.

PROBLEM 14

A fireman holds a high-pressure hose 2.5 meters from the midpoint of the wall of a building and finds that he can wet a circular area of 0.5 square meter on the wall. If the building is 7 meters wide, how far must he stand from the building so as to wet its entire width?

Assume that the water diverges in straight lines from the end of the hose.



The diameters of the areas covered will be in proportion to their distances from the nozzle: $D_1/D_2 = d_1/d_2$. We are asked to find d_2. We are given D_2 and d_1 directly, but instead of D_1, we are given the area of the circle for which D_1 is the diameter.

To derive D_1 from the area, $A = \pi (D_1/2)^2$, or $D_1 = \sqrt{4A/\pi} = \sqrt{2/3.14} = 0.8$ meters.

Then in $D_1/D_2 = d_1/d_2$, $0.8/7 = 2.5/d_2$, and $d_2 = 22$ m. The fireman must stand 22 meters from the building.

PROBLEM 15

If your height and all your other dimensions were doubled, by what factor would this change
(a) your weight?
(b) the ability of your leg bones to support your weight?

a) Weight increases proportionally to volume, and if the linear dimensions of a body are scaled up by a factor 2, the volume increases by a factor $2^3 = 8$. A student eleven or twelve feet high would weight about 1200 lbs!

b) The cross sectional area of the leg bones would increase by a factor $2^2 = 4$. The strength of your legs would increase by this factor, but your weight would increase by a factor 8. Thus the strength to weight ratio would be halved. This would be inconvenient but not impossible (a man can carry his own weight on his back).

PROBLEM 16

The leg bones of one animal are twice as strong as those of another closely related animal of similar shape.
(a) what would you expect to be the ratio of these animals' heights?
(b) what would you expect to be the ratio of their weights?

The answer to this problem will depend upon how students interpret "of similar shape". If this phrase is interpreted to mean that the two animals are geometrically exactly similar (all linear dimensions in the same proportion), answer number 1 follows. If the phrase is interpreted, as it often is in common usage, to mean "pretty much alike", answer 2 is logical.

Answer number 1. In this answer we assume that "of similar shape" means that all linear dimensions differ by the same factor. While the two animals would be geometrically similar, the smaller animal would have a greater strength to weight ratio.

a) Since the animals' bone strengths differ by a factor 2, the cross sectional area of the bones must differ by a factor 2. Then all linear dimensions would differ by a factor $\sqrt{2} = 1.41$. Thus the animals' heights would differ by a factor 1.41.

b) The animals' weights would differ by a factor $(\sqrt{2})^3$ = 2.83.

Answer number 2. Since we know the animals' bone strengths differ by a factor 2, we might assume that this difference also represents the difference in weight (or volume) between the two animals. In this case, the two animals would not be geometrically similar but would have the same strength to weight ratios.

a) If their volumes differ by a factor 2, their heights (and all linear dimensions except for those relating to the cross sectional area of bones which are given as differing by a factor $\sqrt{2}$) differ by a factor $\sqrt[3]{2}$ = 1.26.

b) The animals' weights would differ by a factor 2.

PROBLEM 17 A square column 15 cm by 15 cm can barely support a cube 1.0 m on an edge. What size cube can be supported by a column 30 cm by 30 cm made of the same material?

If the column's linear dimensions are doubled, its cross sectional area, and therefore its strength, are increased by a factor 4. The weight of the supported cube can thus be increased by a factor 4. This means that the linear dimension of the load (a cube) can be increased by a factor $4^{1/3} = 1.59$. Thus a cube of side 1.59 m can be supported by the 30 cm × 30 cm column.

PROBLEM 18 A hummingbird must eat very frequently and even then must have a highly concentrated form of food such as sugar. What does the concept of scaling tell you about the size of a hummingbird?

Food enables a body to do work, like moving itself around, and for warm-blooded animals also provides heat to maintain body-temperature at a fixed level. The food-for-movement requirements will be roughly proportional to the weight of the animal, the food-for-heat requirements will be proportional to the animal's surface area. If we are given that an animal needs a large amount of food in proportion to its weight, we deduce that the ratio of its surface area to its volume is large, i.e., that it is a very small animal. Hummingbirds are small animals.

PROBLEM 19 According to the zoo, an elephant of mass 4.0×10^3 kg consumes 3.4×10^2 times as much food as a guinea pig of mass 0.70 kg. They are both warm-blooded, plant-eating, similarly shaped animals. Find the ratio of their surface areas, which is approximately the ratio of their heat losses, and compare it with the known ratio of food consumed.

If students don't note it, it may be worthwhile to point out the assumption that must be made here: That the density of an elephant and a guinea pig is about the same. Since density has not yet been discussed, you can point out that we must assume that the weight per unit volume is about the same for elephants and guinea pigs (a cubic inch of elephant weighs the same as a cubic inch of guinea pig). Assuming this, their volumes would be in the same proportion as their weights:

$$4 \times 10^3 \text{ kg}/0.7 \text{ kg} \approx 5.7 \times 10^3.$$

The ratio of their linear dimensions would be $\sqrt[3]{5.7 \times 10^3} \approx 18$. The ratio of their surface areas would be $18^2 \approx 3 \times 10^2$. This compares favorably with the stated ratio of their food intakes: 3.4×10^2.

PROBLEM 20 Horror movies sometimes present a creature such as a spider large enough to devour a human being. Is such a creature a possibility? Why?

We might assume a man-size spider would be big enough to devour a human being. Taking a spider as about 2 cm in length, and a man as 180 cm, the horror creature would require an enlargement ratio of nearly 100:1. This would increase the creature's weight by a factor of $100^3 = 10^6$ (weight $\propto L^3$). The cross sectional area of the skeletal or supporting structure would be increased by a factor of $100^2 = 10^4$ (strength $\propto L^2$). Thus the skeletal structure would be suited to the support of only 1/100th of the weight. Any adjustment in cross sectional area to increase the supporting strength would lead to a very differently proportioned spider.

One might argue that the size could be achieved by using air spaces and thin tubular struts for extra support. Such a creature would be much too fragile to devour a man!

Students may worry about the fact that we are not dealing with solid bones, but an exoskeleton—the tubular shell of an insect. But if all of the animals' dimensions are scaled up by a factor 100, both the thickness of the shell and its circumference go up by a factor 100. Since the cross sectional area of the material forming a tube is equal to the thickness × circumference, increasing both these dimensions by 100 increases the cross sectional area (and the strength) by the square of the scaling factor.

It may be fun to play with this example a little further. For our 2 cm spider we might assume that at a given point on its leg, the diameter is 0.5 mm, and the thickness of the "shell" (chitin) is 0.05 mm. For a tubular structure the cross sectional area is approximated by circumference × thickness. Thus the cross sectional area of the spider leg at this particular point would be $A \approx \pi dT = \pi \times 0.5 \times 0.05 = 0.025\,\pi\ \text{mm}^2$. If our spider is scaled up by a factor 100, the volume and weight are increased by a factor $100^3 = 10^6$. Hence, for our scaled-up spider to have the same strength to weight ratio as our "normal" spider, the cross sectional area at a comparable point on the leg would have to be $10^6 \times .025\ \text{mm}^2 = 2.5 \times 10^4\ \text{mm}^2$. But, to keep our big spider to scale, the leg diameter at this point will be only $100 \times 0.5\ \text{mm} = 50\ \text{mm}$. Even if the leg at this point were solid, so as to provide maximum strength, but leaving no room for muscle and other tissue, its cross sectional area would be

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times (25\ \text{mm})^2 \\ &\approx 2 \times 10^3\ \text{mm}^2. \end{aligned}$$

This falls far short of the $2.5 \times 10^4\ \text{mm}^2$ required to provide the needed strength for our scaled up spider. We must conclude that a spider that was scaled up by a factor 100 would have to look more like a baby elephant than a spider.

PROBLEM 21 A sphere of iron is suspended from a wire, 0.1 cm in diameter, which is just strong enough to support it. What must be the diameter of the wire needed to support a similar iron sphere having

(a) five times the original volume?
(b) five times the original diameter?

a) If the volume is increased five times, the weight is increased by this same factor. The strength of the wire is proportional to its area, i.e., to its $(\text{diameter})^2$. Hence, the diameter must be increased by a factor $5^{1/2} = 2.24$. It must now be .224 cm.

b) If the diameter of the sphere is increased by a factor 5, its weight is increased by a factor 5^3. The diameter of the wire must therefore be increased by a factor $(5^3)^{1/2} = 11.2$. It must now be 1.12 cm in diameter.

PROBLEM 22 A rectangular water tank is supported above the ground by four pillars 5 meters long whose diameters are 20 cm. If the tank were made ten times longer, wider, and deeper, what diameter pillars would be needed? How much more water would the tank hold?

The weight of the tank will be increased by the cube of the factor by which the linear dimensions are increased, i.e., by 10^3. The cross sectional area of the pillars must be increased by a factor 10^3 to support this increased weight. Their diameter must be increased by a factor $10^{3/2} = 31.6$. They must now be 630 cm in diameter.

The tank will now hold 10^3 times as much water.

Note that the height of the pillars is immaterial. Their strength depends only on their cross sectional area.

Chapter 5 - Motion Along a Path

The ideas of time and distance measurement developed in Chapters 2 and 3 are applied in Chapters 5 and 6 to the description of motion. In Chapter 5 the study of kinematics is limited to motion along a path; and the relevant concepts such as speed, acceleration, etc., are defined for simple one-dimensional motion. In Chapter 6 these ideas are extended to motion in three-dimensional space through the introduction of vectors.

In the design of the course, kinematics is introduced at this point because it is natural, to provide a "soaking in" period before studying dynamics in Part 3, and to break up a rather long sequence of somewhat formal subject matter if kinematics and dynamics are taken up sequentially. Some of the concepts introduced here will be used in Part 2 and much of the material will be reviewed at the beginning of Part 3.

In developing the fundamental concepts of kinematics in Chapters 5 and 6, trigonometry and calculus are not assumed to be part of the student's background; instead, the development relies heavily on graphical constructions. In addition to providing a means for calculating various aspects of motion, graphical presentation has a major advantage over analytical methods in furnishing a "picture" of what is actually going on in kinematic problems. Students at this level can handle problems graphically which ordinarily would be difficult for beginning students in calculus.

Graphical analysis should be made a quantitative approach. Students should be urged to draw accurate graphs and make precise measurements of the slopes of lines and of areas under curves. Their ability to relate algebraic and graphical approaches should grow as they proceed through these chapters, but not all students can be expected to become familiar with purely algebraic methods the first time through.

CHAPTER SUMMARY

The two principal topics of this chapter are speed and acceleration, both defined for one-dimensional motion. We begin with the concept of speed, assuming that it is a natural notion for the student. The distance traveled is given by the area under a graph of speed vs. time, and speed is given by the slope of a graph of distance vs. time. Acceleration is defined in terms of changes of speed, and is shown to be the slope of a speed-time graph. For instructional purposes, the nine sections of this chapter can be grouped conveniently as follows:

Sections 1 and 2 The first sections of the chapter review the basic notion that distance equals the product of speed and time, and introduce the idea of a graph of speed vs. time as a useful record for a trip. The distance traveled during the trip is given by the area under the speed-time graph both for constant and for varying speed.

Section 3 through 5 Graphical constructions are extended to plots of distance vs. time, and the instantaneous speed is shown to be given by the slope of the distance-time graph. Speed is a signed (positive or negative) quantity, where the sign indicates the direction along the path. Both constant and varying speeds are discussed.

Sections 6 and 7 Acceleration is introduced as the change of speed which occurs in a given time. Acceleration is noted as given by the slope of a graph of speed vs. time. The graphical construction techniques provide a base for the derivation of a number of useful algebraic relations for handling motion with constant acceleration.

Sections 8 and 9 Section 8 describes the principle of measuring distance by timing the round trip of a signal which has a known speed. Section 9 summarizes the chapter.

SCHEDULING CHAPTER 5

Chapters 5 and 6 include a great deal of new and important material which will require an extensive amount of class time for a full, quantitative development. Considerable time should be given to the solution of problems, both in and out of class. For this reason, the Guide contains a few supplementary problems, which may be used for quizzes or for class discussion. Reading assignments should generally precede class discussion.

Sections 1 and 2 make a good single reading assignment. Sections 3 through 5 can be handled as a second reading assignment, perhaps after some of the ideas of these sections have been developed in class during discussion of the earlier sections. Study of Section 6 should precede class discussion of acceleration, but you may wish to have a class discussion of the formulas given in Section 7 before assigning this section for reading. At the conclusion of the chapter, about two periods spent in review and problem solving will increase students' facility with both graphical and algebraic techniques for handling kinematic problems. Problem 22 in HDL is particularly appropriate, since it utilizes both algebraic and graphical methods for solution. Section 8, if discussed in class, can be taken up during such a review. The film "Straight Line Kinematics", is useful in summarizing the chapter (the first part of the film can be used as an introduction).

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part 1. Sections which are enclosed by brackets [] can be deemphasized or skipped in class discussion without loss of continuity.

6-week or 10-week schedule for Part 1

Subject	Class Periods	Lab Periods	Experiment
Secs. 1, 2	2	-	-
Secs. 3, 4, 5	2	1	I-5 (any time after Section 3)
Secs. 6, 7	2	-	-
Secs. [8], 9,review	2	-	-

RELATED MATERIALS FOR CHAPTER 5

Laboratory. Experiment I-5 - Motion: Speed and Acceleration. This experiment uses the paper tape timer to provide data for the construction of distance, speed, and acceleration vs. time graphs. The experiment will contribute markedly to giving concrete meaning to the ideas of Chapter 5. It may be performed any time after completion of Section 3. See the yellow pages on Experiment I-5 for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Problems 16 and 17 comprise a pair which tests the key concepts of the chapter. Answers to problems are given in the green pages: short answers on page 5-17 ; detailed comments and solutions on page 5-18 to 5-39.

Sections	Easy	Medium	Hard	Class Discussion
1	1, 2*, 5*	3*, 4	6	5
2	7a*, b	8*	9*, 17*	7, 9
3	7c, 10, 11			
4, 5	14	12, 13*		12
6	15*		16*, 26, 29*	[16*, 17*], 26, 29
7		18*, 19, 20, 21 22*, 23*, 24*, 25	27	18, 22, 23
9			28	

Film. "Straight Line Kinematics", by Professor Everett Hafner, University of Rochester. This film develops the relations between distance, speed, and acceleration by measurement in a car traveling along a highway. A wheel trailing behind the car is geared to three recording meters which indicate the values of distance, speed and acceleration as functions of time while the car is speeding up, traveling at a steady rate, and slowing down. After calibrating the instruments and gathering data, the instructor-driver returns to the classroom for analysis of the graphs. Running time: 33 minutes.

The film can be used in at least two ways. A summary and review of the chapter can be built around the film. Alternatively, the first part of the film (up to the return to the classroom) can be used as an excellent introduction to the chapter, making real to students the production of graphs of distance, speed and acceleration versus time. The second part of the film goes a bit fast if used before the students have had a chance to discuss simpler types of motion in class.

Section 1 - Speed and Distance
Section 2 - Varying Speeds

PURPOSE To show that the area under a graph of speed vs. time for a trip gives the distance traveled.

CONTENT a. $d = vt$.

b. Speed vs. time graphs.

c. Total distance traveled can be found from the area under a speed-time graph.

d. Units of "area" in a speed-time graph.

e. Extension (by a limiting process) of cases of constant speed to cases of continuously varying speed.

EMPHASIS This is extremely important material and should be developed fully through class discussion, problem solving in class, and problem assignments. About two class periods will be needed initially, and the material should be reviewed at the end of the chapter. Section 8 discusses practical applications (radar and sonar) of some of the ideas in these sections and can be taken up at this point. However, you probably will prefer to incorporate Section 8 in your review and summary of the chapter.

These two sections should be treated as a single unit, since if graphical methods are applied only to problems involving constant speed they may seem unnecessarily cumbersome. Students should realize that the simple introduction to graphical techniques is laying the groundwork for more complicated problems which they cannot solve by simple formulas.

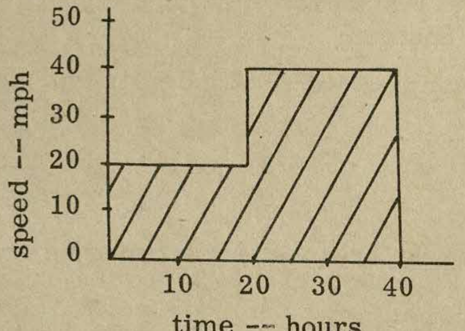
COMMENTS In these sections it is assumed that the concept of speed is familiar to students. They should already be able to compute how far a car goes in one half hour if it is traveling at 30 miles per hour. The equation $d = vt$ is introduced as a shorthand notation for this familiar relationship.

The student who could answer the previous problem easily might stumble if asked how far the car went if it traveled at 20 meters per second for 1800 seconds. Since the text at this point relies on everyday experience, the more familiar British units have been extensively used, rather than the metric units favored generally throughout the text.

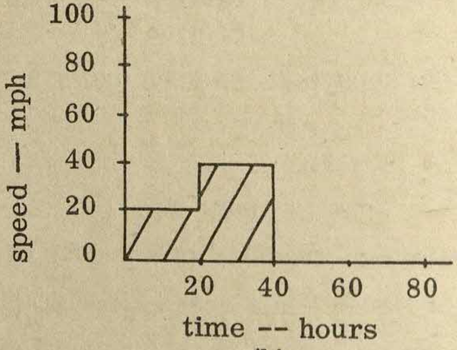
The plotting of a graph can become so mechanical that a student may not realize what the graph represents, particularly if he is plotting a quantity like speed against time. Some effort must be spent in making sure that the student realizes that the horizontal line in Figure 5-1 means that the car is going at a constant speed. The more complicated trip of Table I and Figure 5-2 should also be thoroughly understood, particularly that speed is being plotted against time, not, for instance, distance against time.

Some students may ask how, in the trip graphed in Figure 5-2, a car can change instantaneously from one constant speed to another. A real car, under full acceleration, can speed up from 30 mph to 50 mph in a few seconds, and while the process is certainly not discontinuous, a graph of the motion might indeed be quite like that shown in Figure 5-2. This is shown strikingly in the film "Straight Line Kinematics". Students should realize that the "area" under the steeply rising portion of the graph, while not zero, is negligible in comparision with that under the rest of the graph, so that the idealization is a reasonable simplifying assumption. When continuously varying speeds are considered, this restriction can be removed, and any value of speed considered quantitatively.

The idea that the distance, d, the product of speed and time, vt, can be thought of as an "area" on a speed-time graph will probably be new to most students. For the case of constant speed, the result is almost trivial, but is, nevertheless, worth considerable attention, since the idea that an "area" can have the units of miles may be puzzling. Students also may need help to appreciate that the "areas" in such graphs as Figures (a) and (b) below are really the same; the difference in the scale of the drawings does not affect the distance traveled in the trip.

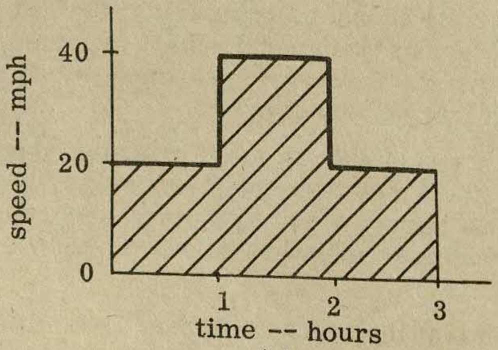


(a)

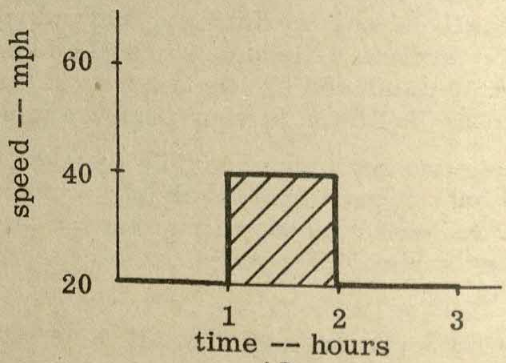


(b)

One other point may give trouble. The "area" which represents the distance traveled is the area between the speed curve and the axis of zero speed. See Figures (c) and (d) below.



(c)



(d)

Both Figures (c) and (d) are correct graphs of the speed of the car vs. time. By reading either graph one can ascertain the speed of the car at any time. However, for the purpose of finding the distance traveled by determining the "area", only Figure (c) is appropriate. It is clear that the two shaded areas differ considerably.

Introduce the idea of a speed which varies steadily with time as a natural extension of the previous work rather than as something radically new. The process of showing that the area under any speed-time graph gives the distance traversed by "squeezing" the final answer between two limits, is fundamental mathematics, and might well be emphasized here. If the notion of the limit is made clear now, later material will come more easily.

The text employs a two-stage development of the idea that distance traveled is represented by the "area" under a speed vs. time curve. The first stage is the qualitative idea that any curve can be approximated by a stairstep graph which can be handled by the methods of Section 1. The second stage is the more formal and rigorous "squeezing" procedure which really proves the assertions of Section 2.

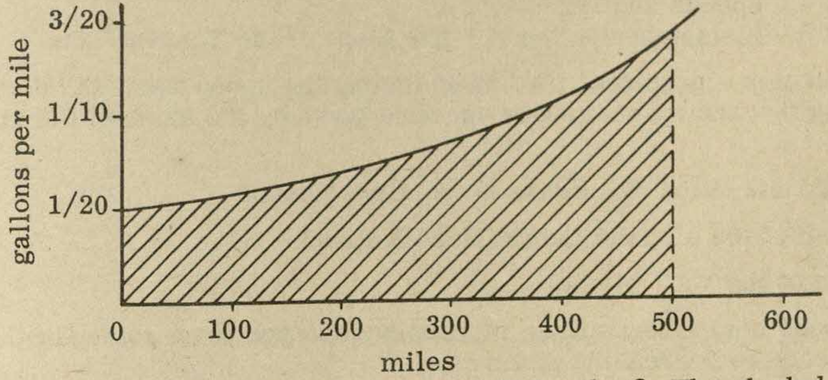
Frequent blackboard illustration is probably the best way to put across the ideas in these sections. With each new example, the students should be asked to describe what is happening to the moving object, before proceeding with a more detailed analysis. Do not let them forget that these graphs really tell how fast the car is moving at any particular time.

Figure 5-3 shows a car moving at a constantly increasing speed during the first part of the time interval. The case of uniform acceleration is particularly important since it describes bodies falling near the earth's surface, and approximately describes many other motions. The "area" involved is a trapezoid and can be computed directly.

Some more complicated cases should be given which must be computed by square counting. If the curves are drawn carefully on fine-grid coordinate paper, so that a few hundred squares are included in the area between the curve and the axis, this method is capable of great precision. You probably can arouse a great deal of interest by asking for answers correct to within 3/10 of one percent.

In counting squares or computing to get an "area" which represents distance, care always should be taken to emphasize the units involved; miles per hour must be plotted against hours if the area is to represent distance in miles, etc. You might drive home this idea by giving students a graph of speed in miles per minute against time in hours. See how many students fail to convert to consistent time units before finding the "area".

The technique of determining the distance traveled by taking the area under a curve is a simple way of obtaining results which could otherwise be obtained only by use of the calculus. The method of measuring areas is of much more general use than just for calculation of distance from a speed vs. time graph. As a simple illustration, you might ask the following question of the class: A car starts a trip using 1/20 gallon of gasoline per mile. As the sparkplugs foul up, the car uses more and more gasoline per mile as shown in the accompanying graph:

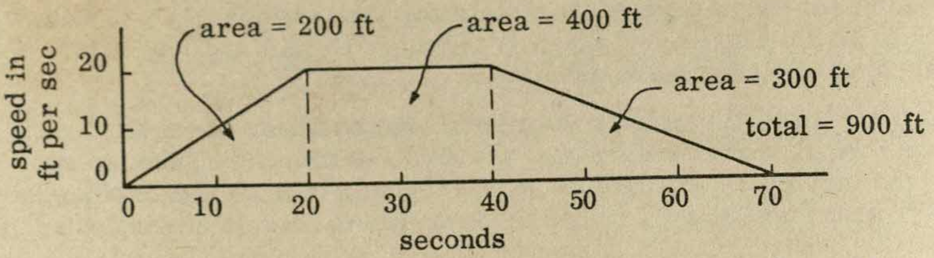


How many gallons of gasoline were used to travel 500 miles? The shaded area gives the answer (gallons per mile) × (miles) = (gallons).

QUIZ QUESTIONS If you decide you need extra problems for class discussion or quizzes, the following may be helpful.

1) A cyclist rides along a straight road for two minutes at a speed of 1000 feet per minute, then continues for three more minutes at 1500 feet per minute. How far did he go? (6500 feet)

2) A cyclist starting from rest, speeds up uniformly to a speed of 20 feet per second in 20 seconds, continues at this speed for 20 seconds, and then slows down uniformly to a stop in 30 seconds. Plot his speed vs. time and determine how far he went.



3) You can draw almost any kind of speed versus time graph and have the students find the distance traveled. An interesting variation is to find the distance traveled between two particular times rather than from the start of the trip.

4) A much more difficult problem which might serve as an introduction to uniform acceleration (Section 7) is the following: A car speeds up uniformly from rest so that it is going 80 feet per second after 20 seconds. How far does it travel during the first 10 seconds? How long does it take to travel 600 feet?

The first question can be answered by the standard methods of the chapter. After 10 seconds the car is going 40 feet per second. The speed-time graph is a triangle, so the distance traveled during the first ten seconds is 1/2 × 10 sec × 40 ft/sec = 200 ft. The second question is more difficult. It can be approached in several ways. One way: plot the speed vs. time and count squares until an area of 600 feet is reached. Another: simply try different times. After 14 seconds the car's speed is 56 ft/sec, and it has traveled 1/2 × 56 ft/sec × 14 sec = 392 ft; after 18 seconds, it has traveled 648 ft. Most elegant way; every second the car increases its speed by 4 feet per second, so after t seconds its speed is 4t ft/sec. The distance traveled in t seconds is then $1/2 \times t \times 4t = 2t^2$ ft. Setting $2\,t^2$ ft = 600 ft; $t = \sqrt{300} = 17.3$ seconds.

Section 3 - Graphs of Distance Versus Time
Section 4 - Speeds and Directions
Section 5 - Instantaneous Speed — The Slope of the Tangent Line

PURPOSE To develop a graphical method of finding the speed from a distance vs. time graph, by showing that the instantaneous speed is given by the slope of the graph at any point.

CONTENT a. Construction of a distance vs. time graph.

b. Identification of the slope of the graph with speed.

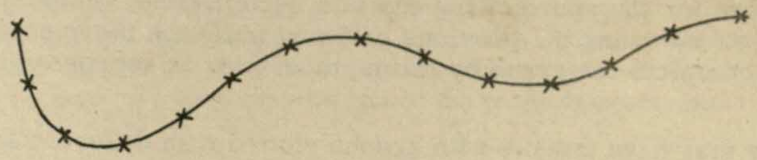
c. Introduction of the "Δ" symbol.

d. Positive speed corresponds to an increasing distance from some fixed reference point,. negative speed to decreasing distance.

e. Instantaneous speed is defined as the limit of the average speed over a very short time interval, as the interval becomes arbitrarily small.

EMPHASIS This is very important material, and will need at least two days class discussion. The identification of the speed with the slope of the curve is much more important than the calculus-like notation. Care taken here will help to insure that students do not confuse speed with acceleration in later work.

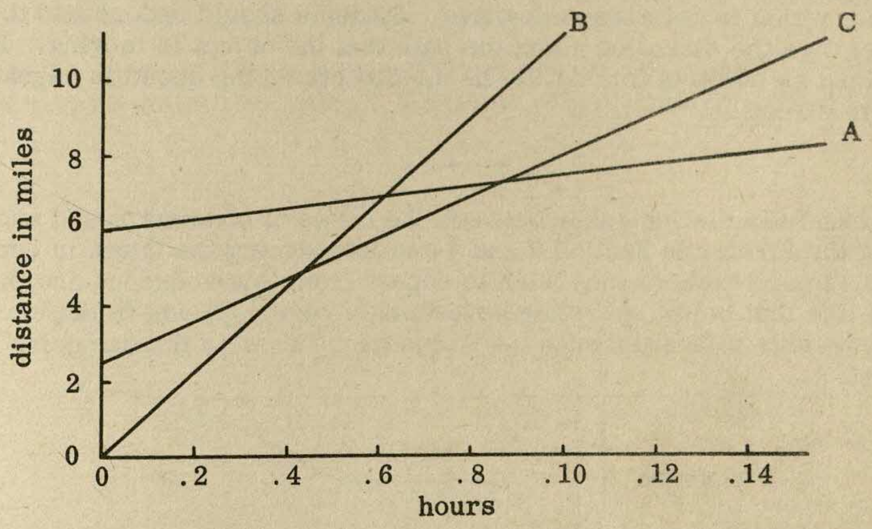
COMMENT Up to this point, little attention has been paid to the kind of trips being considered. Unless instructed otherwise, students may think of the various cars as traveling in straight lines. It will be easy to displace this idea by drawing a diagram like that below, showing that the distance d is not necessarily along a straight line, but rather is simply a distance measured by mileposts along a road which may be curved.



The idea of mileposts as a distance scale is introduced to develop the concept that distance can be measured in terms of a coordinate system with an arbitrary origin. The distance covered during a given time interval need not be measured from some established zero, but can be deduced from the difference between two distance markers, each of which is measured from a common zero. Students may bring up the idea that the distance traveled during a time interval can be measured by the car's mileage indicator (odometer) as well as by mileposts on the road. This is correct. However, if we extend the discussion to the relative positions and motions of two or more cars (When will they pass? Where will they pass? Will the car behind catch up before the one ahead reaches Blankville?), we need a single distance scale which will apply to all cars. Odometers are not synchronized. Mileposts are simpler.

DEVELOPMENT Experiment I-5 should be worked in any time after Section 3 is completed.

The construction of distance-time graphs should go more easily than the construction of speed-time graphs. Care may be needed to distinguish between these new distance-time graphs and the previous speed-time graphs. This is probably best done by frequently asking exactly what is happening to the car at each position represented on the graph. Intensive and frequent classroom work with graphs is the best way to get this material across. You can use a blackboard graph like that shown below as a basis for asking questions about the physical situation described by the graph:



Which man is traveling fastest? (B).

Are all three ever at the same point on the road? (No).

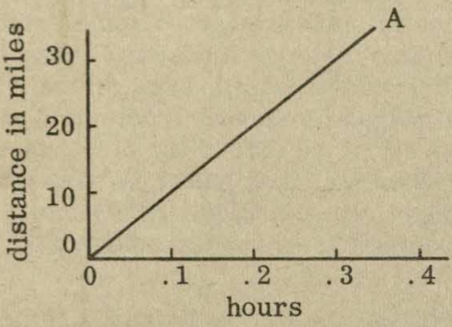
When B passes A, where is C? (About 1 mile back).

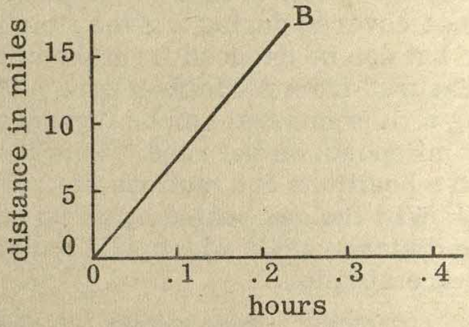
How far did B travel between the times he passed C and A? (About 2 miles).

Which men are on bicycles? (B must be riding to cover 9 miles in 0.8 hours. Since C's speed is a little over 5 miles per hour, he at least needs to trot. Since A's speed is about 1 1/2 miles per hour, he should be able to walk).

After the class has worked with distance-time graphs, you might put on the board a graph using numbers but no physical units (miles or miles per hour) indicated on the vertical scale. Ask for the speed of the car at a certain time. Some students will be prepared to answer following the previous pattern, while the more cautious ones will demand that you complete the graph by telling them what is represented by the vertical scale.

Some classes may have trouble with graphs plotted with different scales, as discussed on page 58. To clarify this point, plot two separate graphs, such as those shown below, and ask which car goes faster:



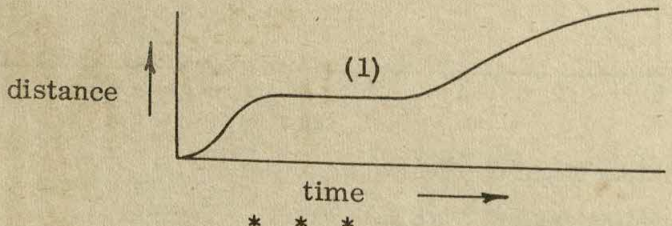


Car B has the "steepest" graph, but the car travels only 15 miles in .2 hours, hence it has a speed of 75 mph, whereas car A goes 30 miles in .3 hours with a speed of 100 mph.

COMMENTS The word "speed" is often used to represent the magnitude of the velocity vector, and as such is a positive quantity, like the reading of a speedometer. However, since the word "velocity" is not introduced in its vector sense until Chapter 6, "speed" means a one-dimensional "velocity" in Chapter 5. Because speed is identified with the slope of a graph, and because a slope can be positive or negative, our "speed" is a signed quantity which is not always positive. Students should understand that the sign of the speed specifies the direction along the path that the object is moving. If speed is arbitrarily defined as positive for motion in one direction, the speed is negative for motion in the opposite direction.

* * *

The text carefully distinguishes between the cases of constant speed and variable speed by discussing the former in Section 3 and 4 and introducing the latter in Section 5. With an able class, some teachers may wish to depart from this sequence and present from the first a graph like that below, possibly containing a region (1) where the car is stopped. Students may be able to answer such questions as: "When is the car going fastest? When slowest?", etc.



* * *

The equation $v = \frac{d_2 - d_1}{t_2 - t_1}$ emphasizes the need for the milepost and clock ideas. The speed is expressed as a difference between two distances divided by a difference in the two corresponding times: a distance interval divided by a time interval. Note that on the graph of distance vs. time, the slope represents a physical quantity, speed, and has units of distance/time.

If the previous sections have been thoroughly understood, students should have no trouble with the basic ideas presented in Section 5. The trip of Figure (a) leads directly to Figure (b), and students may even be led to propose the tangent idea without previous work with the idea of the "magnifying glass". Certainly they should be able to decide at which times in Figure (b) the car is going fastest and slowest. In order to decide between two points such as (1) and (2), they may be led to draw the tangent line.

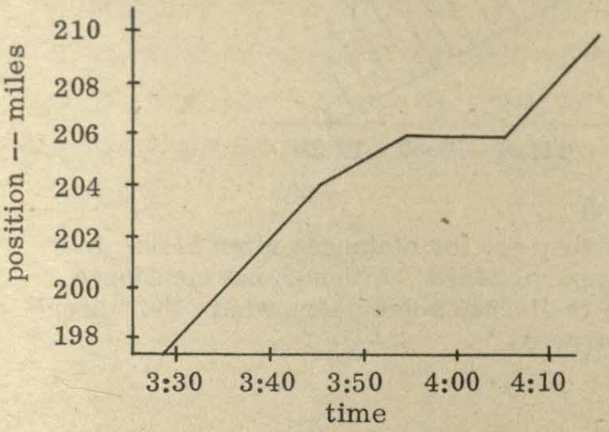


Figure (a)

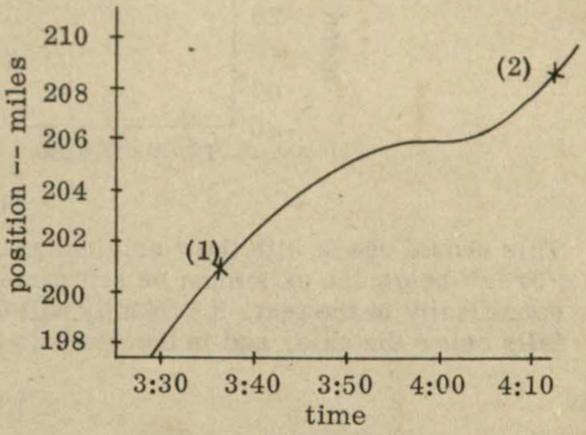


Figure (b)

A possible difficulty with a limiting procedure such as the "magnifying glass" technique, is that students may get the idea that the result is only approximate. The procedure outlined is a proper method for determining the slope, and can be done with any desired degree of accuracy (within the limits of error of the experimental data). (For a given mathematical function, the slope can be found exactly, by the methods of calculus.)

* * *

At this point the class will have a powerful set of graphical tools. Problems can be invented which may be quite interesting. A certain trip may be described by Figure (c).

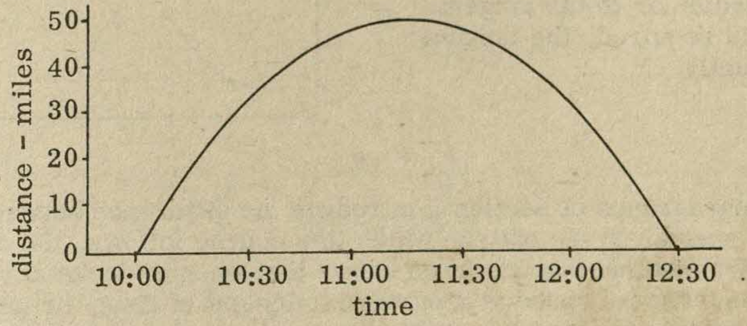


Figure (c)

This might be an automobile trip. With different distance and time scales it could be a rock thrown into the air. It may be drawn carefully, or it may be just a quick sketch (with no very violent changes in the slope). Notice that Figure (c) represents an "out and back" trip. By measuring slopes the students can construct a speed-time graph, something like Figure (d). Then by counting squares they can measure, say, the distance traveled from 10:00 to 11:00.

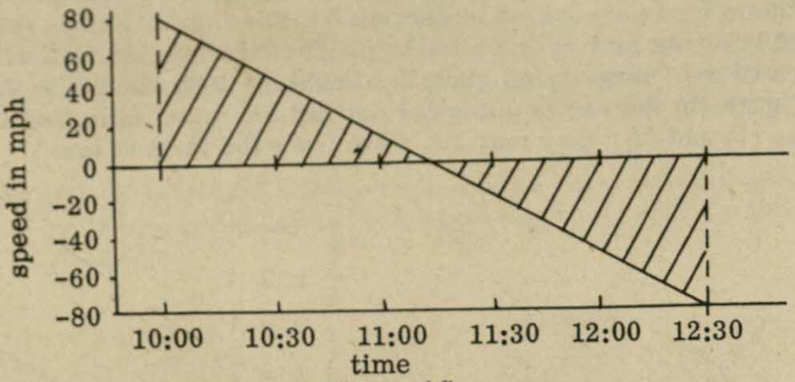


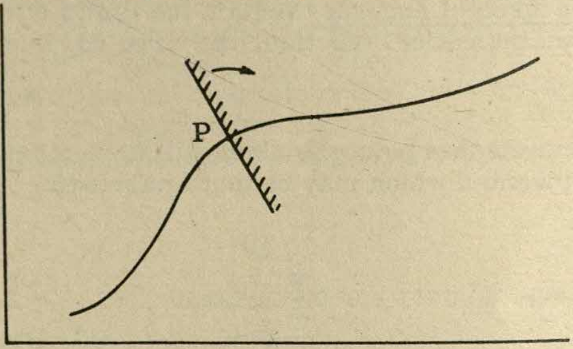
Figure (d)

This should check with their original graph. If they ask for distances after 11:15, the "area" below the axis must be subtracted, instead of added. Although not mentioned specifically in the text, it probably will be well to discuss some cases where the "area" falls below the axis, and is therefore to be subtracted.

* * *

One problem which has bothered some classes is how to draw a tangent to a curve. It should be understood that we do not require an exact geometric construction.

A method of drawing the tangent which may interest some students is the following: (This method defines the perpendicular to the curve at a given point. Then the tangent is drawn at a right angle to the perpendicular.) Place a plane mirror over the graph approximately perpendicular to the curve at the point P where the tangent is desired. Rotate the mirror about P until the graph and its image form a smooth, continuous curve. Draw a line along the edge of the mirror. This line is perpendicular to the tangent. Unless precision is required, the tangent may be drawn visually.



* * *

The final two paragraphs of Section 5 introduce the difference between average speed and instantaneous speed. In the pattern which this course follows, the distinction is not very important. We assume that a person knows that when he looks at the speedometer of a car he is measuring the speed at a particular instant of time, or instantaneous speed; whereas when he says he averaged 40 miles per hour on a four hundred mile trip, he means he traveled for ten hours, probably very little of the time at exactly 40 miles per hour. (If your students like problems like this, ask them if, in averaging 40 mph on a 400 mile trip a car ever has to travel at exactly 40 miles per hour. The answer is yes if the car cannot change its speed discontinuously.) The instantaneous speed is just the average speed determined for a very short time interval, so short in fact that any changes in speed during the time interval are so small that they are negligible.

A classic question dealing with average speed is the following: On a 60 mile trip a man travels the first 30 miles at 30 miles per hour. How fast must he go for the next 30 miles in order to average 60 miles per hour for the trip? The answer is, of course, an infinite speed, since he has already taken the one hour needed to average 60.

QUIZ QUESTIONS

1) A car passes mileposts at the indicated times and travels with constant speed in between.

(a) Between which mileposts did the car travel at the greatest speed? What was the greatest speed?

(b) How long did the car stop?

(c) At which milepost did the car not change speed?

(d) When did the car start its return trip?

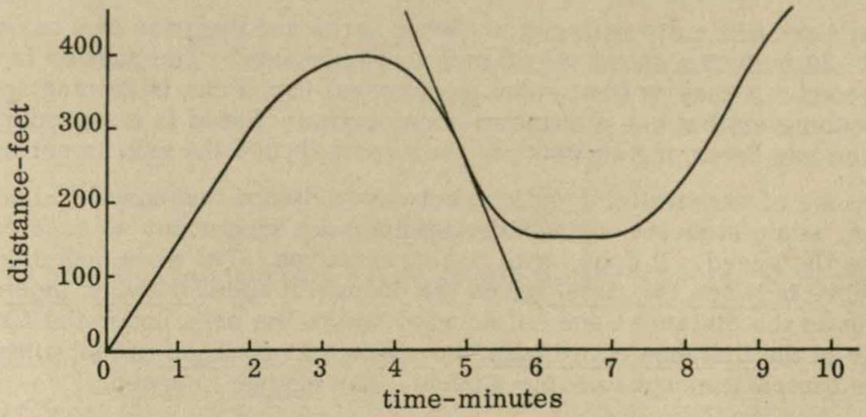
(e) What was the average speed on the outgoing trip? On the return trip?

Milepost	Time
0	3:15:04
10	3:30:04
15	3:37:34
15	3:57:34
25	4:07:34
20	4:17:34
10	4:32:34
0	5:02:34

Answers: (a) 15-25; 60 mi/hr
(b) 20 min.
(c) 10 on outward trip
(d) 4:07:34
(e) 28.5 mi/hr; 27.3 mi/hr or -27.3 mi/hr

To give problems with continuously varying speeds, you almost have to reproduce the graph for the problem. The variations are limitless. Here is one example:

2) A man walks along a path such that his position at various times is given by the accompanying graph.



(a) Between what times were his speeds constant? What were these constant speeds? [0 to 2 minutes, 150 ft/min; 6 to 7 minutes, 0 ft/min]

(b) At what times was he stopped? [At 3.6 minutes, and between 6 and 7 minutes]

(c) At what time was the magnitude of his speed the greatest? What was the speed at that time? [At 5 min; -240 ft/min]

Section 6 - Acceleration Along the Path

PURPOSE To introduce the concept of acceleration for one-dimensional motion, and to show that the acceleration at any instant can be determined from the slope of the speed-time graph for that time.

EMPHASIS A good understanding of acceleration is important. This section should be covered thoroughly before proceeding to Section 7. If students understand Sections 3, 4, and 5, this section should go rather quickly. Be certain that students understand the difference between "speed" and "acceleration", since these words are frequently confused in ordinary conversation.

COMMENT Many students will be foggy about the nature of acceleration. They may confuse speed and acceleration and use speed units to describe accelerations. A "fast" car may also be able to accelerate rapidly, but not necessarily. (The Indianapolis racers don't.) To clarify the distinction between speed and acceleration, this section uses mixed time units to describe acceleration: miles per hour per second. The hope is that this will focus attention on the fact that acceleration is related to how fast the speed is changing. Attention to units in this section is of first importance. You may wish to refer to the comments on the changing of mixed to consistent units in Section 7 of both the text and the Teacher's Guide.

* * *

The transition from a qualitative idea to the equation

$$a_p = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t},$$

should be fairly easy since speed-time situations have already been considered.

The transition from constant accelerations to continuously variable accelerations is brief in the text and may need amplification in class. The "magnifying glass" procedure probably should be repeated in the classroom.

* * *

Students may have difficulty with signs. What is the acceleration of a car which goes from a speed of -20 mph to a speed of -30 mph in 10 seconds? The answer is -1.0 miles per hour per second. It may at first seem paradoxical that a car is gaining speed (as read on the speedometer) but has a negative acceleration. Speed is a signed quantity. A corporation which has debts of $10,000 one year and $20,000 the next is not making money.

CAUTION Because of the similar relations between distance and speed and between speed and acceleration, some students will wonder whether the area under an acceleration vs. time curve gives the speed. It does, with one reservation. The area under the acceleration vs. time curve between two times gives the change in speed between those two times. This is the same as the distance case, of course, where the area under the speed curve gives the change in the distance coordinate, but change in distance is just distance traveled which is a more natural concept to some students than change in speed.

The vector acceleration introduced in the next chapter is not always in the direction of motion, i.e., is not necessarily in a direction along the path. The "acceleration along the path" defined in this section is really the component of the vector acceleration in the direction of motion. Note a difference between the velocity and acceleration cases: The vector velocity always points along the path, whereas the vector acceleration may not.

QUIZ QUESTIONS

1) A car is going at a speed of 56 mi/hr. One second later it is going 58 mi/hr. What is its acceleration along the road? [2 mi/hr-sec]

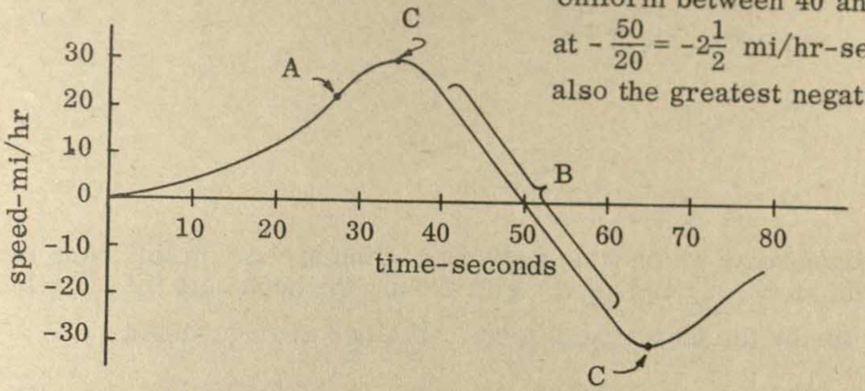
A car is going 56 mi/hr. One second later it is going 54 mi/hr. What is its acceleration along the road? [-2 mi/hr-sec]

If the car's speed were -56 mi/hr and then -58 mi/hr, one second later, what would be its acceleration? [-2 mi/hr-sec]

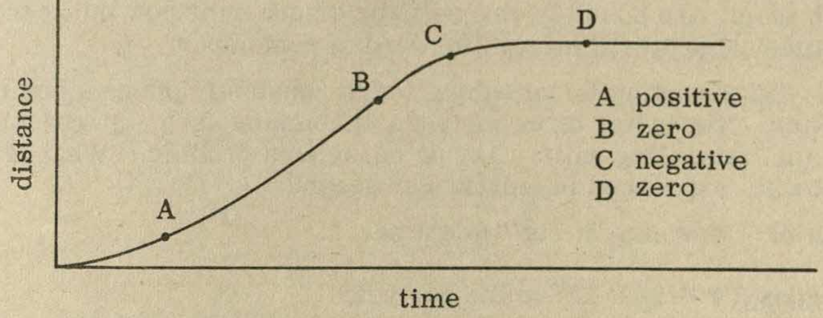
2) There are several uniform acceleration problems in the HDL. Simple variants are appropriate for quizzes.

3) Here is a plot of speed vs. time. Label the time where the acceleration is maximal and positive, (A); minimal and negative, (B); zero, (C). Between what times is the acceleration uniform? What is its value?

Uniform between 40 and 60 seconds at $-\frac{50}{20} = -2\frac{1}{2}$ mi/hr-sec. This is also the greatest negative acceleration.



4) Here is a distance vs. time curve. At the points labelled A, B, C, and D tell whether the acceleration is positive, negative, or zero.



A positive
B zero
C negative
D zero

Section 7 - Useful Results for Motion With Constant Acceleration Along the Path

PURPOSE To develop the algebraic equations for motion when the acceleration along the path is constant.

CONTENT a. $v_f - v_i = a_p (t_f - t_i) = a_p T$.

b. $d = \frac{1}{2} (v_f + v_i) T$.

c. $v_f^2 = v_i^2 + 2 a_p d$.

d. $d = v_i (t_f - t_i) + \frac{1}{2} a_p (t_f - t_i)^2 = v_i T + \frac{1}{2} a_p T^2$.

e. Unit changes.

EMPHASIS Understanding, not memorization is important. Use one full period to make sure the derivations are understood.

CAUTION Although these "formulas" for motion with constant acceleration are very useful, they are not the high point of the chapter. All too often students carry away $d = \frac{1}{2} at^2$ as one of the several things they remember about physics without remembering in which cases it may be applied. Since constant acceleration problems are encountered frequently, memorization of the formulas does the student no harm if it comes through understanding and use rather than by rote.

COMMENT A special case of the above equations is that in which the object starts from rest at time $t = 0$, i.e., $v_i = 0$, $t_i = 0$. Then,

$$v_f = a_p t_f = a_p T,$$

$$v_f^2 = 2a_p d,$$

$$d = \frac{1}{2} a_p t_f^2 = \frac{1}{2} a_p T^2$$

It is often useful to emphasize which time goes with which speed. In this case the formulas are usually written as $t_f - t_i$, as above. For brevity the book sets the time interval $t_f - t_i = T$. This may often be the most useful form. The two are equivalent.

DEVELOPMENT Formulas are useful and they are very attractive to students. To avoid the danger of substituting formulas for understanding, help students to remember the basic ideas of the chapter by working some of the HDL problems which are obviously constant acceleration problems, using graphical techniques. Problem 22 is good for this purpose. You may surprise them, and possibly yourself, by discovering how quick the graphical techniques are, and how easy and straightforward to remember.

UNIT CHANGING Unit changing is something which constantly plagues beginning (and advanced) physicists. There are three methods in common use: (a) reasoning it out, (b) substitution, (c) cancelling units. Let us consider a problem: What is the speed of the earth, in its orbit, expressed in meters per second.

(a) An example of "reasoning it out" might be:

The Earth goes $2\pi \times 93 \times 10^6$ miles per year.

Therefore it goes $2\pi \times 93 \times 10^6 \times 5280$ feet per year,

or $2\pi \times 93 \times 10^6 \times 5280 \times 12$ inches per year,

or $2\pi \times 93 \times 10^6 \times 5280 \times 12 \div 39.37$ meters per year,

or $\dfrac{2\pi \times 93 \times 5.28 \times 10^9 \times 12}{39.37 \times 365.25}$ meters per day,

or $\dfrac{2\pi \times 93 \times 5.28 \times 10^9 \times 12}{39.37 \times 365.25 \times 24}$ meters per hour,

or $\dfrac{2\pi \times 93 \times 5.28 \times 10^9 \times 12}{39.37 \times 365.25 \times 24 \times 60}$ meters per minute,

or $\dfrac{2\pi \times 93 \times 5.28 \times 10^9 \times 12}{39.37 \times 365.25 \times 24 \times 60 \times 60}$ meters per second,

or 2.98×10^4 meters per second ($\approx$ 30 km/sec).

The problem is figuring out when to multiply and when to divide. The decision is made intuitively, i.e., should the quantity be larger or smaller?

(b) An example of the substitutional method (used in the text) is:

The Earth goes $2\pi \times 93 \times 10^6 \dfrac{\text{miles}}{\text{year}}$.

One then considers that this physical quantity, a speed, is made up of a number, $2\pi \times 93 \times 10^6$, times a primitive unit of speed, 1 mile/1 year:

$$2\pi \times 93 \times 10^6 \times \frac{1\text{ mile}}{1\text{ year}}.$$

But, 1 year = 365.25 days = 365.25 × 1 day

1 day = 24 hours = 24 × 1 hour

1 hour = 3600 seconds = 3600 × 1 second.

Substituting gives 1 year = 365.25 × 24 × 3600 × 1 second.

Similarly, 1 mile = 5280 × 12 × 1 inch.

But 1 meter = 39.37 × 1 inch, or $1\text{ inch} = \frac{1}{39.37} \times 1\text{ meter}$,

hence, $1\text{ mile} = 5280 \times 12 \times \frac{1}{39.37} \times 1\text{ meter}$.

Finally, the speed of the earth is

$$2\pi \times 93 \times 10^6 \times \frac{5280 \times 12 \times \frac{1}{39.37} \times 1\text{ meter}}{365.25 \times 24 \times 3600 \times 1\text{ second}} = \frac{2\pi \times 93 \times 10^9 \times 5.28 \times 12}{39.37 \times 365.25 \times 24 \times 3600}\ \frac{\text{meters}}{\text{second}}$$

(c) The third method, cancelling units, is based on the idea that any physical quantity can be multiplied by 1 without changing it. If 1 meter = 39.37 inches then

$$\frac{39.37 \times 1\text{ inches}}{1\text{ meter}} = \frac{1\text{ meter}}{39.37 \times 1\text{ inch}} = 1.$$

The change in units is then effected by multiplying by a string of 1's:

$$2\pi \times 93 \times 10^6 \times \frac{1\text{ mile}}{1\text{ year}} \times \frac{1\text{ year}}{365.25 \times 1\text{ day}} \times \frac{1\text{ day}}{24 \times 1\text{ hr}} \times \frac{1\text{ hour}}{3600 \times 1\text{ sec}} \times \frac{5280 \times 1\text{ foot}}{1\text{ mile}}$$
$$\times \frac{12 \times 1\text{ inch}}{1\text{ foot}} \times \frac{1\text{ meter}}{39.37 \times 1\text{ inch}}.$$

Finally the like units are cancelled leaving

$$\frac{2\pi \times 93 \times 10^6 \times 5280 \times 12}{365.25 \times 24 \times 3600 \times 39.37}\ \frac{\text{meters}}{\text{seconds}},\ \text{as before.}$$

Usually the computation is shortened by merely writing

$$2\pi \times 93 \times 10^6\ \frac{\text{miles}}{\text{years}} \times \frac{1\text{ year}}{365.25\text{ days}} \times \frac{1\text{ day}}{24\text{ hrs.}} \times \frac{1\text{ hour}}{3600\text{ seconds}} \times \frac{5280\text{ feet}}{1\text{ mile}} \times \frac{12\text{ inches}}{1\text{ foot}}$$
$$\times \frac{1\text{ meter}}{39.37\text{ inches}},\ \text{etc.}$$

Each method has its advantages and disadvantages. "Reasoning it out" is the fastest for a simple problem, but most subject to error. A student certainly should be able to do simple problems this way before worrying about the other methods. "Substitution" is less subject to error and gives a good feeling for what is happening. "Cancelling units" is a rather formal process but is least subject to error and probably fastest for a complicated problem.

Some time must inevitably be spent on converting units, and perhaps this is as good a place as any to do it.

Section 8 - Ranging by Speed: Radar and Sonar

PURPOSE To provide an illustration of the technique of measuring distance by timing a signal which travels to an object, then back to an observer.

CONTENT a. Measuring distance by measuring the time it takes a signal of known velocity to traverse that distance.

b. Application of this principle by bats and radar operators!

EMPHASIS An interesting aside. Treat briefly. Can be used as a part of your chapter summary or as an extension of Sections 1 and 2.

COMMENT This section is intended simply as a reading assignment, although it can be used as the basis for interesting problems on simple uniform motion. For example: How long does it take a radio signal ($v = 3 \times 10^8$ meters/sec) to go to the moon ($d = 3.8 \times 10^8$ meters) and back?

Chapter 5 - Motion Along a Path

For Home, Desk and Lab – Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Problems 16 and 17 comprise a pair which tests the key concepts of the chapter.

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 5-18 to 5-39 .

Section	Easy	Medium	Hard	Class Discussion
1	1, 2*, 5*	3*, 4	6	5
2	7a*, b	8*	9*, 17*	7, 9
3	7c, 10, 11			
4, 5	14	12, 13*		12
6	15*		16*, 26, 29*	[16*, 17*], 26, 29
7		18*, 19, 20, 21 22*, 23*, 24*, 25	27	18, 22, 23
9			28	

SHORT ANSWERS

1. 6.6 m/sec.

2. 1 hour.

3. 400 mi trip: 80/9 hr.
   480 mi trip: 78/9 hr.

4. (a) See graph on page 5-19.
   (b) Cadillac.

5. (a) 2 mi.
   (b) 30.0 mi.
   (c) See graph on page 5-20.
   (d) 0.3 hr.
   (e) See solution on page 5-20.

6. (a) See graph on page 5-20.
   (b) 17 VT.
   (c) 5/2 T.
   (d) 23/8 T.
   (e) 34/15 V.

7. (a) 16.2 m/sec, 21.5 m/sec.
   (b) 41.4 meters.
   (c) $2.4\ m/sec^2$.

8. 4.53 ± .04 mi.

9. (a) 2/300 = 0.0067 hrs.
   (b) 2/15 = 0.13 mi.
   (c) B by 0.10 mi.
   (d) 0.015 hrs.
   (e) 0.6 mi.

10. (a) 60 mi/hr.
    (b) 2/3 (no units).
    (c) 15 miles per gallon.
    (d) -4 pounds per week.

11. See graph on page 5-24.

12. (a) 0.45 hr.
    (b) about 75 mi/hr.
    (c) about 8 mi/hr.
    (d) 27 mi/hr.

13. (a) See graph on page 5-26.
    (b) Between mileposts 52 and 62.
    (c) 29.6 mi/hr or -29.6 mi/hr.
    (d) 56 or -56 mi/hr.
    (e) Between mileposts 66 and 68.

14. 60 mi/hr.

15. 6.7 mi/hr/sec.

16. See solution on page 5-27.

17. See graph on page 5-28.

18. (a) 10 m/sec.
    (b) 25 m.
    (c) 5 m/sec.
    (d) 400 m.

19. 9.8 m/sec.

20. (a) 300 cm/sec^2.
    (b) See graph on page 5-30.

21. (a) 2/35 hr.
    (b) -1225 mi/hr^2.
    (c) See graph on page 5-30.

22. (a) 9.6 km/hr sec.
    (b) 96 km/hr.
    (c) 0.13 km.
    (d) 20 m.
    (e) d (during nth sec) = (n - 1/2)a.

23. (a) Car which started at 34 km/hr travels farther.
    (b) 3.3 sec.

24. (a) 3 mi/hr/sec.
    (b) 1/8 mi.

25. (a) 236 km/hr/sec = 65.5 m/sec^2.
    (b) 236 km/sec = 8.5×10^5 km/hr.
    (c) 4.24×10^5.

26. See solution on page 5-34.

27. (a) 1000 m/sec.
    (b) 6.67 sec.
    (c) 5.0 km, 3.3 km, 13.3 km.
    (d) See solution on page 5-36.
    (e) See solution on page 5-36.

28. See discussion on page 5-37.

29. (a) See graph on page 5-38.
    (b) See graph on page 5-39.
    (c) 12 m/sec^2.

COMMENTS AND SOLUTIONS

PROBLEM 1

A runner makes one complete lap around an 800-meter track in 2 min 1.2 sec. What was his average speed in m/sec?

If the runner covers 800 meters in 121.2 seconds, his average speed is 800 m ÷ 121.2 sec = 6.6 meters/sec.

PROBLEM 2

Two cars, one traveling at 40 mi/hr and the other at 60 mi/hr, start a trip of 120 miles at the same time. How much sooner does one car arrive at its destination than the other?

The car traveling 40 miles per hour covers 120 miles in 3 hours. The car traveling 60 miles per hour takes 2 hours. Therefore the faster car arrives 1 hour sooner.

PROBLEM 3

A man knows that by one route between two towns he must go 400 miles, and can average 45 mi/hr. By another route he can travel on a 400-mile stretch of superhighway, averaging 60 mi/hr, but he must travel an extra 80 miles on roads where he can average only 40 mi/hr. Which route takes the shorter time?

The 400 mile trip at 45 mph takes $\frac{400 \text{ mi}}{45 \text{ mi/hr}} = \frac{80}{9}$ hrs.

The 480 mile trip takes $\frac{400 \text{ mi}}{60 \text{ mi/hr}} + \frac{80 \text{ mi}}{40 \text{ mi/hr}} = \frac{26}{3} = \frac{78}{9}$ hrs.

Thus the 480 mile trip takes less time.

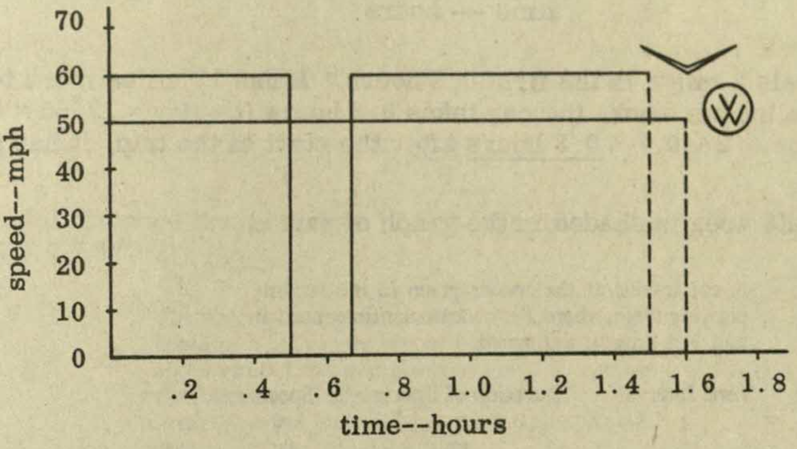
PROBLEM 4

A Volkswagen and a Cadillac are driven on the same 80-mile trip. The Volkswagen travels at 50 mi/hr all the time. The Cadillac starts at the same time, driving at 60 mi/hr, but the driver stops for ten minutes after he has traveled for half an hour.

(a) Make a graph of speed versus time for the two cars.

(b) Which car is the first to arrive at the destination?

a)



b) The Volkswagen travels 80 miles at 50 mi/hr, taking 80/50 = 1.6 hours for the trip. The Cadillac travels 80 miles at 60 miles per hour, taking 80/60 = 4/3 hours <ins>driving time</ins>. However, it stops for 1/6 hour, and hence takes a total time of $4/3 + 1/6 = 1.5$ hours. The Cadillac beats the Volkswagen; an amoral story indeed.

PROBLEM 5

Data for a trip made in a car is given in the accompanying table:

Time Interval	Duration of Interval (hr)	Speed (mi/hr)
1	0.10	20
2	0.40	60
3	0.20	20

(a) How far does the car go in the first interval?

(b) What is the total distance of the trip?

(c) Make a graph of speed versus time for this trip.

(d) How long does it take the car to travel the first 14 miles?

(e) Indicate the area on your graph which corresponds to the first 14 miles of travel.

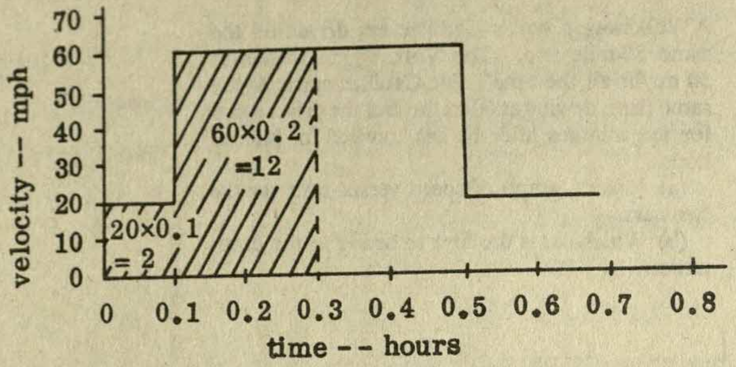
In discussing part (e), it would be well to show that the hatched area is really 14 miles, by performing the multiplication shown on the graph.

a) $d = vt$; $d = 20 \text{ mph} \times .1 \text{ h} = \underline{2 \text{ miles}}$.

b) $d = vt$; $d_1 = 2$ miles, $d_2 = 0.4 \times 60 = 24$ miles, $d_3 = 0.2 \times 20 = 4$ miles.

$d = d_1 + d_2 + d_3 = \underline{30 \text{ miles}}$.

c)



d) The car travels 2 miles in the first 0.1 hours. It has 12 miles more to go. To go 12 miles at 60 miles per hour, the car takes 0.2 hours ($t = d/v = 12/60 = 0.2$). Therefore, at a time 0.2 + 0.1 = 0.3 hours after the start of the trip, it has gone 14 miles.

e) The appropriate area is shaded in the graph of part c).

PROBLEM 6

A car travels at the speeds given in the accompanying table, where *T* is a particular time interval and *V* is a particular speed.

Time Interval	Duration of Interval	Speed
1	T	V
2	3T	4V
3	T	2V
4	2T	V/2
5	T/2	2V

(a) Make a graph of speed versus time for this trip.

(b) What is the total distance of the trip?

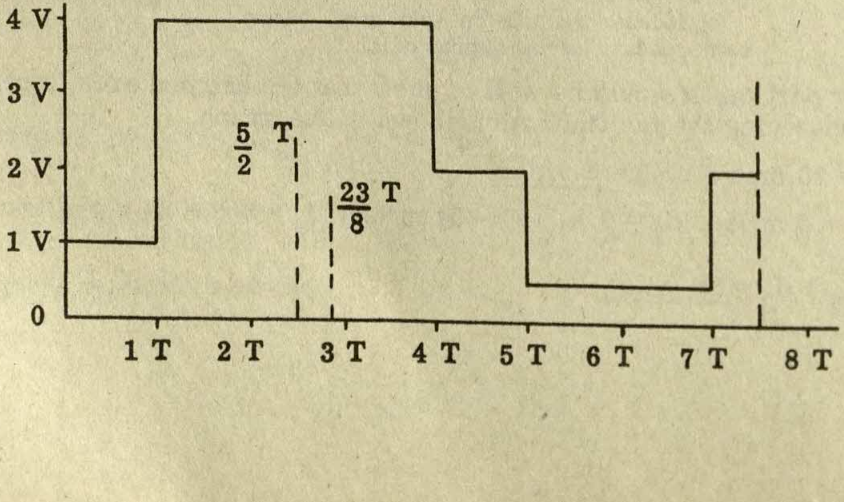
(c) How long after the start of the trip is needed for the car to travel a distance of 7VT?

(d) How long a time does it take for the car to make the first half of the trip?

(e) What is the average speed for the entire trip?

This problem on interpreting speed vs. time graphs is intended as an exercise in the use of abstract symbols. It can also be used to illustrate that the choice of scales for graphing is arbitrary. The principal difficulty in this problem will be seeing that the symbol T is not the time during which the car travels, but rather, an arbitrary unit of time, say 6 seconds. It is as though the man timing the car had a watch which read 0, T seconds, 2T seconds, 3T seconds, etc. Similarly the speedometer is calibrated 0, V, 2V, 3V, 4V, etc., miles per hour.

a)



b) $d = vt$

First interval d = VT, second interval d = (4V) (3T) = 12 VT, third d = 2 VT, fourth d = VT, fifth d = VT. Total distance = 17 VT. During discussion be sure to point out the "areas" of each interval of the trip.

c) After T it has traveled VT. To travel 7 VT, it still has 6 VT to go. At a speed of 4 V this takes a time (6 VT) ÷ (4 V) = (3/2) T. This added to T gives (5/2) T.

d) Since the distance of the total trip is 17 VT we need to find the time required to travel the distance 8 1/2 VT. At the end of the first interval the car will have gone 1 VT. Thus to the time T, we need to add the time required for the car to travel 7 1/2 VT during the second interval. We must find the time T which multiplied by 4 V gives 7 1/2 VT.

$$\frac{15}{2}\ VT \div 4\ V = \frac{15}{8}\ T$$

$$\frac{15}{8}\ T + T = \underline{\frac{23}{8}\ T}$$

e) The total distance traveled was 17 VT, the total time 7 1/2 T. The average speed is then 17 VT ÷ 7 1/2 T = 34/15 V.

PROBLEM 7

The accompanying data show the instantaneous speed of a car at intervals of 1 second. Plot the speed versus time, and use your graph to answer the following questions.

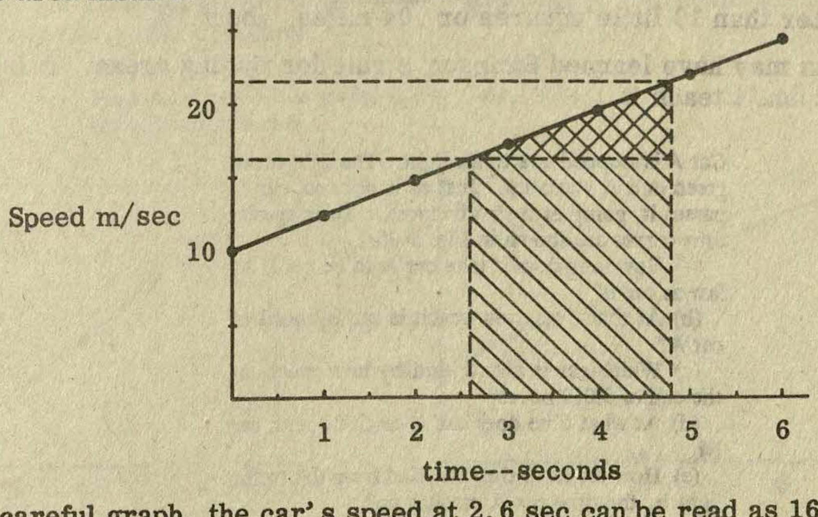
(a) How fast is the car going at 2.6 sec? At 4.8 sec?

(b) How far did the car travel between the two instants in part (a)?

(c) What is the slope of the graph?

Time (sec)	Speed (m/sec)
0.0	10.0
1.0	12.4
2.0	14.8
3.0	17.2
4.0	19.6
5.0	22.0
6.0	24.4

This problem gives practice in plotting graphs of speed vs. time and finding a distance traveled by the area method.



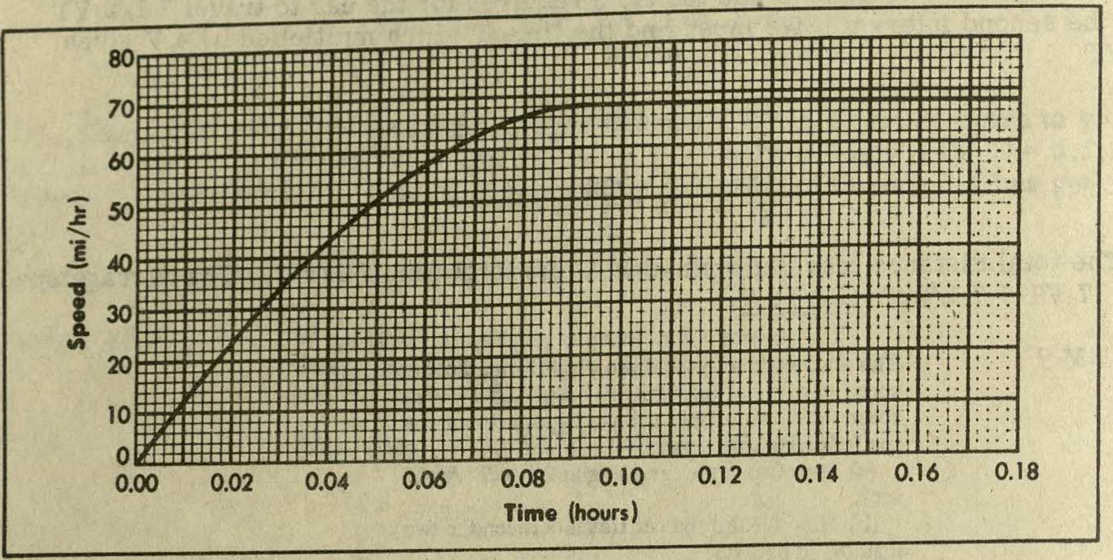
a) From a careful graph, the car's speed at 2.6 sec can be read as 16.2 m/sec, and 21.5 m/sec at 4.8 sec.

b) By breaking up the area as shown, the total distance traveled (shaded area) between the two instants is found to be 16.2 × (4.8 - 2.6) = 35.6 meters (the rectangle) plus 1/2 × (4.8 - 2.6) × (21.5 - 16.2) = 5.8 meters (the triangle), or 41.4 meters in all.

c) The slope of the graph is 2.4 meters per second per second. This is, of course, the acceleration of the car.

PROBLEM 8

A train speeds up according to the speed-time graph shown in Fig. 5-29. How far does it travel in the first six minutes?



This problem forces the student to perform a square-counting calculation in a problem in which he must compute a distance from a speed vs. time graph.

Six minutes = 0.10 hours. The area under the graph from 0 to 0.10 hours is 4.53 miles.

Perhaps the most straightforward way to count these squares is to count the number of large ones (38) and then the left over small ones. The area of each large square is 0.1 miles which gives 3.8 miles. There are then 161 small squares completely under the curve and an estimated 21 more in partial small squares. Since each small square is .004 mi this is .73 mi. Then 3.8 + .73 = 4.53 mi in all. This counting procedure seems to be good to better than 10 little squares or .04 miles, about 1%.

Some students may have learned Simpson's rule for finding areas. It is fine if they know it, but don't teach it.

PROBLEM 9

Car A is stopped at a traffic light. The light turns green and A starts up. Just as it does so, car B passes it, going at a steady speed. Their speed-time curves are shown in Fig. 5-30.

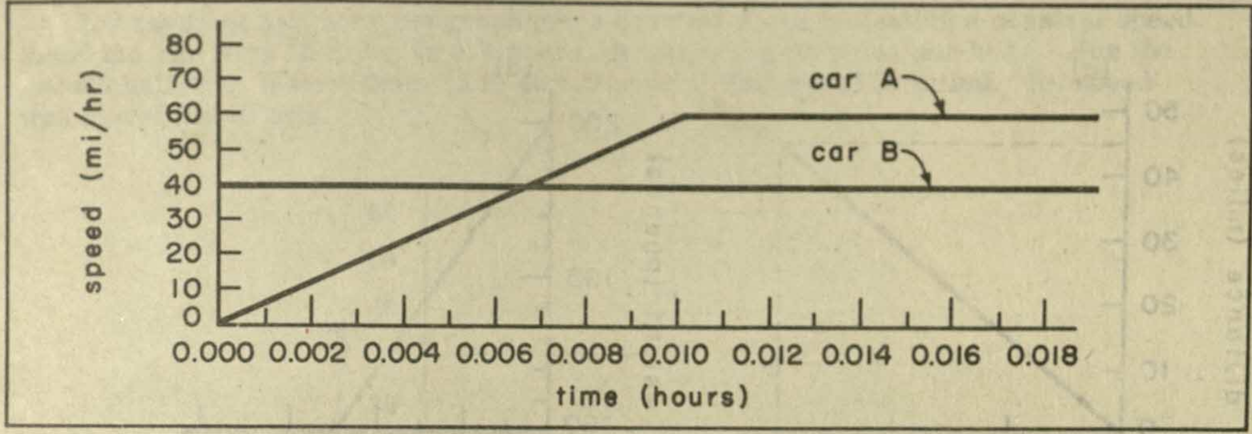
(a) How long does it take car A to be going as fast as car B?

(b) At that time, how much is car B ahead of car A?

(c) Which car is ahead, and by how much, at the end of 0.010 hours?

(d) At what time does car A catch up with car B?

(e) How far have they traveled from the traffic light by the time car A catches up?



This is a problem in interpreting speed vs. time graphs. It is somewhat more complex than the previous ones. Be sure the students can tell exactly what is happening.

a) The cars have the same speeds where the graphs cross at 0.0067 or 2/300 hrs after car A left the stoplight.

b) First point out that the area under car B's graph is larger than that under A's. B has therefore traveled farther, in fact B has traveled twice as far as A. B has traveled 40 × 2/300 = 4/15 miles. A has traveled 1/2 × 40 × 2/300 = 2/15 miles. Therefore B is 2/15 = 0.13 miles ahead.

c) At the end of 0.010 hours it is still clear that the area under B's graph is greater than that under A's. B has traveled 0.010 × 40 = 0.40 miles; A has gone 1/2 × 0.010 × 60 = 0.30 miles. B is therefore 0.10 miles ahead of A.

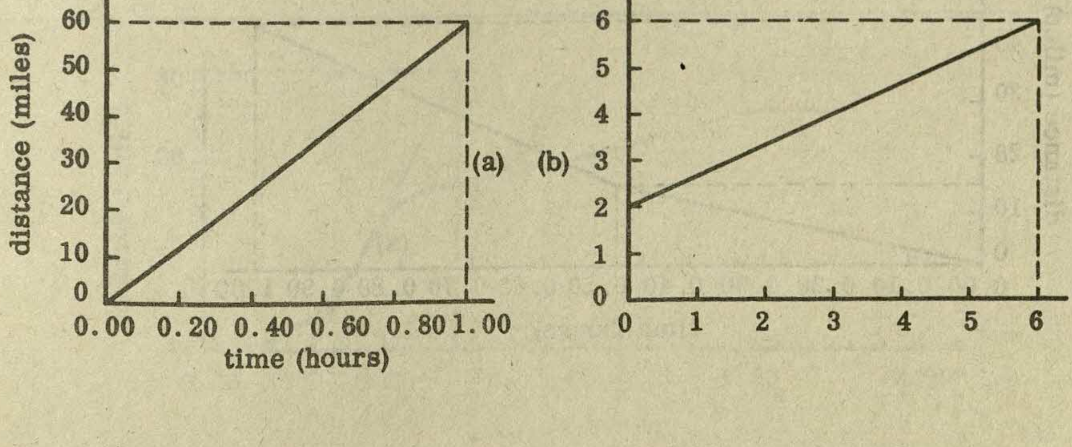
d) This can be done in many ways, but perhaps the easiest way is the following: A has caught up when the areas under the graphs are equal. At 0.010 hours the area under B was 0.10 miles greater than that under A. At later times the area between the two graphs is 20 mph × (time interval). For this to be 0.10 miles the time interval must be

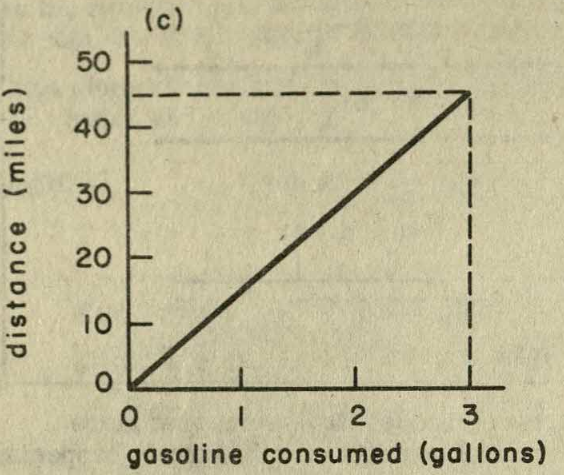
$$t = \frac{0.10 \text{ mi}}{20 \text{ mi/hr}} = 0.005 \text{ hrs.}$$

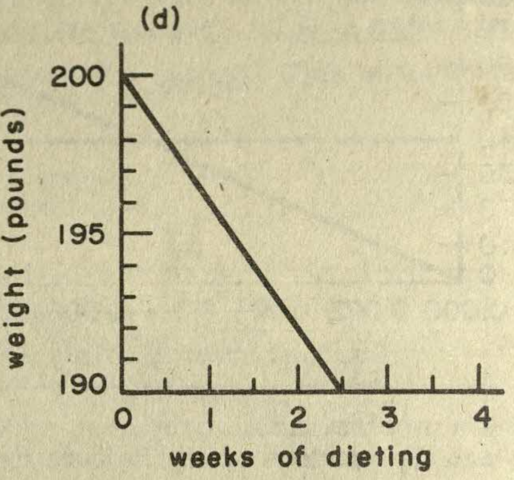
Thus A catches B at 0.010 + 0.005 = 0.015 hrs.

e) At 0.015 hours B has traveled 0.015 × 40 = 0.6 miles. A, if we have done things correctly, has gone the same distance.

PROBLEM 10 Find the slopes of the graphs in Fig. 5-31. State the units in each case.







This problem gives practice in finding slopes of straight lines. Particularly, this problem emphasizes units:

a) This car has traveled 60 miles in one hour. The slope is thus 60 mph. It may be worthwhile to point out that it went between 12 and 30 miles between 0.2 and 0.5 hours, a distance of 18 miles in 0.3 hours or again a slope of 60 mph.

b) Here no units are given. The curve rises from 2 to 6 units in 6 units along the horizontal, or 4 units in 6 units or 2/3 "units per unit" or a slope of 2/3.

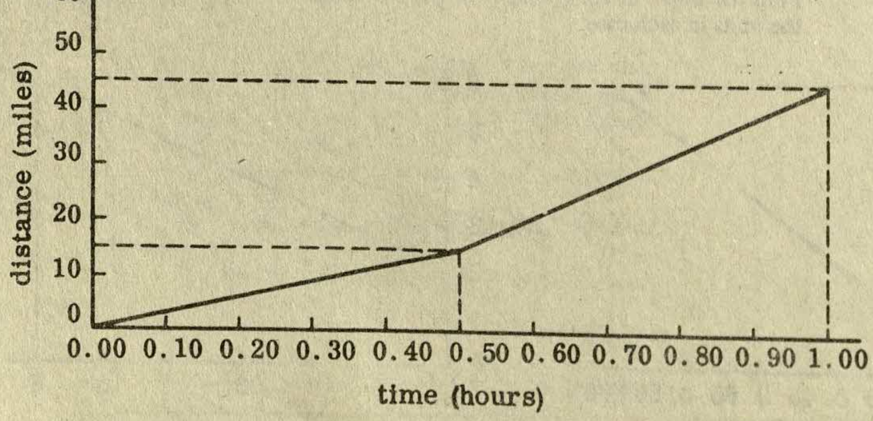
c) An interval of 3 gallons gives a rise of 45 miles. This graph then has a slope of 15 miles per gallon. Presumably this is the distance a car traveled plotted against the gasoline used, and the slope is the rate at which this car uses gasoline in miles per gallon.

d) This dieter (man or woman?) went from 200 to 190 pounds in 2 1/2 weeks. The slope of the graph of weight vs. time is -4 pounds per week. Notice that in common usage, one would say he lost 4 pounds per week. The negative sign is equivalent to the word lost in this context.

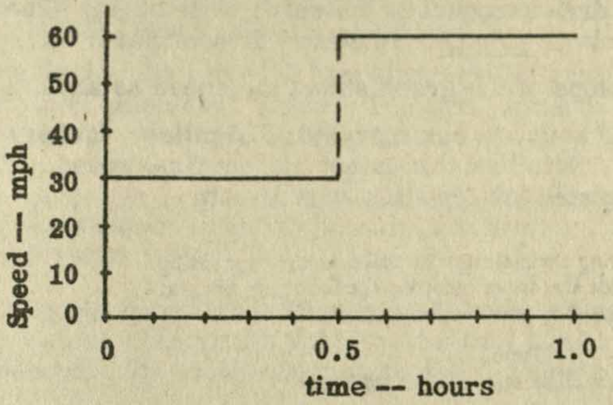
It should be pointed out how commonly the concept of slope is used in everyday affairs.

PROBLEM 11

The distance-time graph of a car traveling along a road is shown in Fig. 5-32. Make a graph of its speed versus time.



For the first half hour the graph has a constant slope indicating a constant speed. Since the car went 15 miles in 0. 5 hours, it was going 30 miles per hour. For the second half hour it went from 15 to 45 miles or a distance of 30 miles. Its speed was therefore 60 mph.



PROBLEM 12

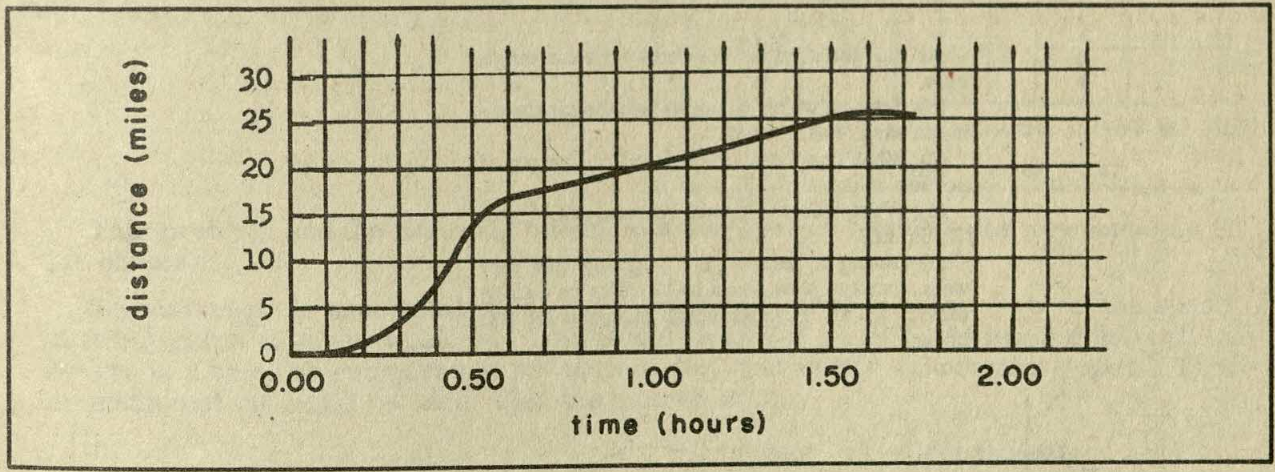
The distance-time graph of a car is shown in Fig. 5-33.

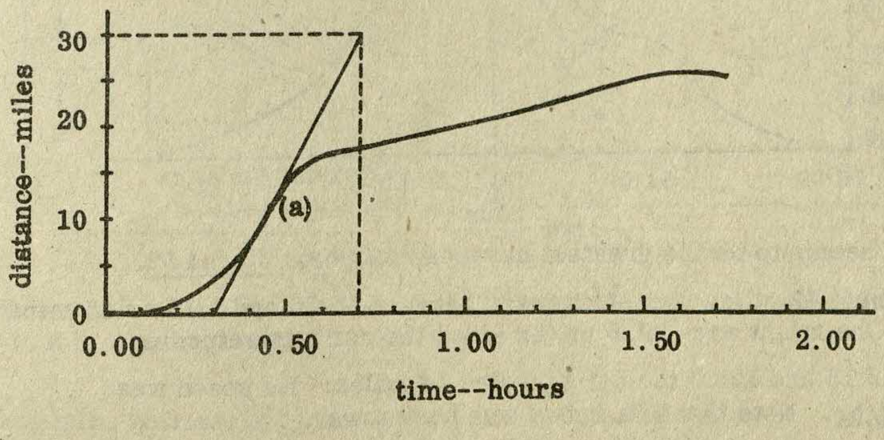
(a) At what time is the car going at the greatest speed?

(b) How fast is it traveling at that time?

(c) How fast is the car going at 0.70 hours?

(d) What is the average speed for the first half hour?





This problem requires the interpretation of a distance vs. time graph where the speed is constantly changing. Some will find this difficult.

a) The point (a) is found by inspection as that point having the steepest slope. The time at which the car is going at the greatest speed is about 0.45 hours.

b) The sloping line is drawn tangent to the curve at point (a). The slope is about 30 miles in 0.4 hours or about 75 mph. Accuracy is poor here.

c) At 0.70 hours the slope of the graph shows the speed as about 8 mph.

d) During the first half hour the car traveled 13.5 miles. It therefore had an average speed of 27 mph. Note that this is not half its final speed. That is only the case for uniformly accelerated motion (this case is not).

PROBLEM 13

During a trip along a turnpike, a passenger sitting beside the driver recorded the following times at which they passed various mileposts.

Time hr:min	Milepost
10:05 a.m.	40
10:25	45
10:40	52
10:50	62
11:00	66
11:10	68
11:25	78
11:40	82
11:50	82
12:05 p.m.	76
12:15	70
12:30	56
1:15	40

(a) Plot the distance traveled by the car versus time.

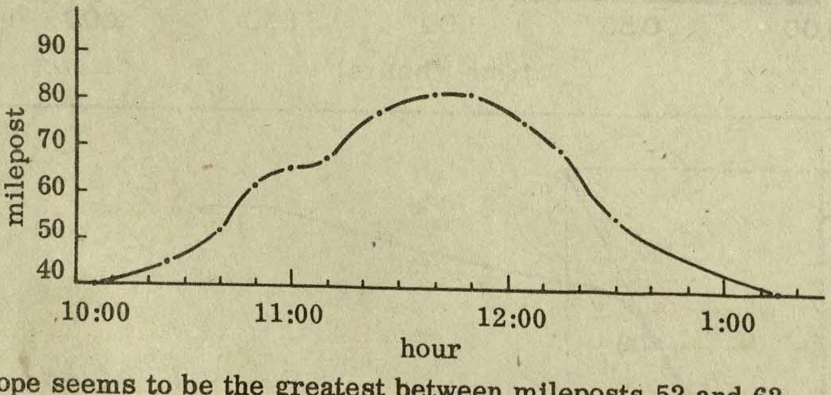
(b) Between which mileposts was the car traveling at the greatest speed?

(c) What was the average speed on the return portion of the trip?

(d) What was the speed between 12:15 and 12:30 p.m.?

(e) During a part of the outgoing trip they were forced to slow down to 12 mi/hr by a slow-moving truck. Between which mileposts did this occur?

a)



b) The slope seems to be the greatest between mileposts 52 and 62.

c) The returning 42 miles were traversed between 11:50 and 1:15 or 85 minutes or at 29.6 mi/hr. One might say -29.6 mi/hr since the car was returning.

d) Between 12:15 and 12:30 the car traveled 14 miles. Its speed was 14 ÷ 1/4 = 56 mi/hr. Note that this speed was back toward the starting point and might therefore be called -56 mi/hr.

e) Between mileposts 66 and 68.

PROBLEM 14

A train travels 60 mi/hr for 0.52 hr, 30 mi/hr for the next 0.24 hr, and then 70 mi/hr for the next 0.71 hr. What is its average speed for the trip?

A train which travels 60 mph for 0.52 hours covers 31.2 miles. In the next time period it covers 7.2 miles, then 49.7 miles. The total distance traveled is 88.1 miles in 1.47 hours. The average speed for the trip is 88.1 miles ÷ 1.47 hours = 60 mph.

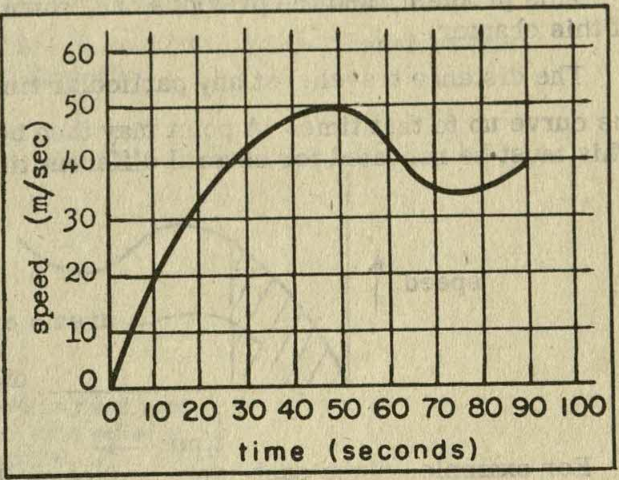
PROBLEM 15

A car going at 20 mi/hr accelerates to 60 mi/hr in 6.0 seconds. What is the average acceleration along the road?

The car, in changing its speed from 20 mph to 60 mph, changes its speed by 40 mph in 6.0 seconds. It therefore changes its speed at the rate of 40 mph ÷ 6 seconds = 6.7 miles per hour per second. Its acceleration is then 6.7 miles per hour per second.

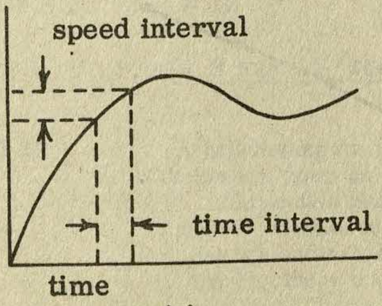
PROBLEM 16

From the graph of *speed* versus *time* for a car shown in Fig. 5-34, deduce the graph of *acceleration* versus *time*.

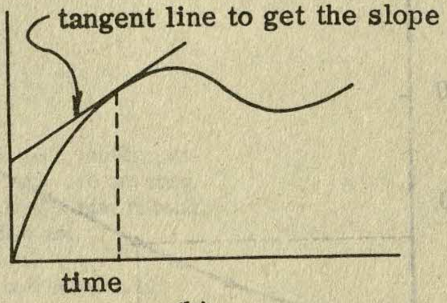


This problem and the following one form a good pair for testing the key concepts of this chapter.

To construct the acceleration vs. time graph, you must take the slope of the speed vs. time graph at several places. This would be a good place to review the interval-on-the-graph method as compared to the tangent-line method for calculating slopes. Thus the method at (a) might be used, then the method at (b).

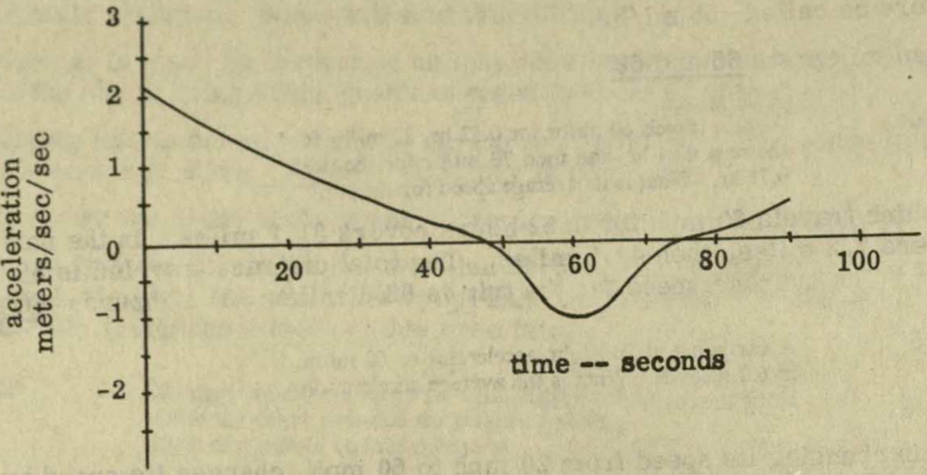


(a)



(b)

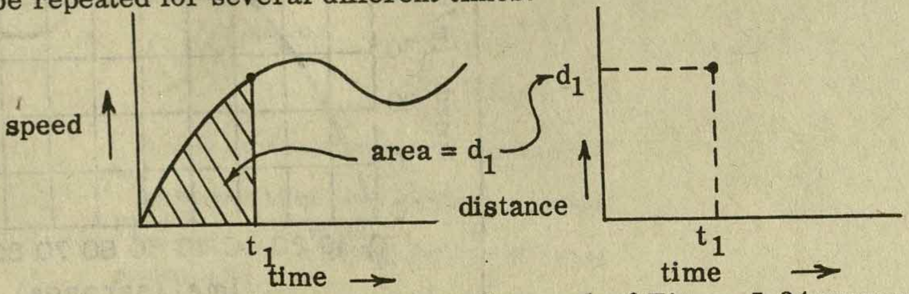
The resulting acceleration vs. time graph is shown below. Note the negative acceleration as the car is slowing down.



PROBLEM 17 Make a graph of *distance* traveled versus *time* for the car of Problem 16.

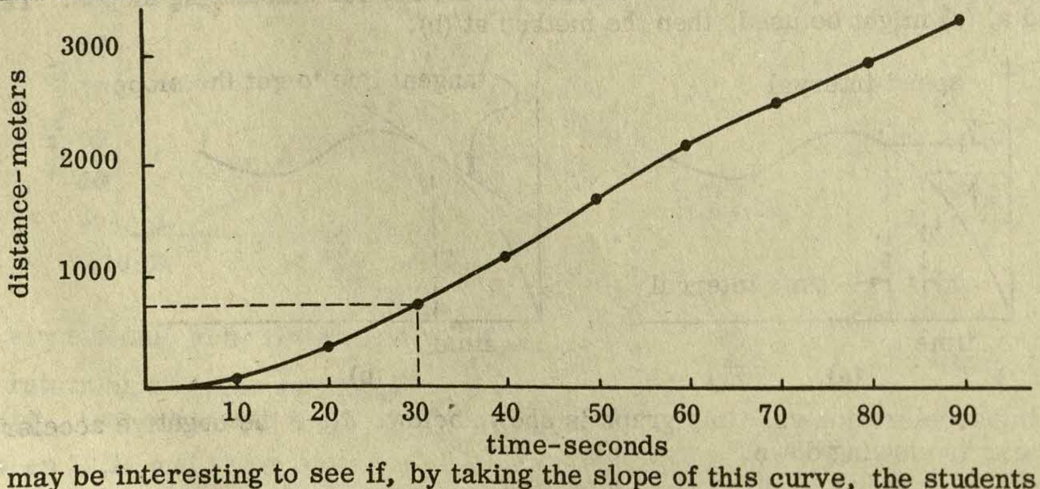
This problem, and the previous one, form a good pair for testing the key concepts of this chapter.

The distance traveled at any particular time t_1 is computed by taking the area under the curve up to that time. A point may then be plotted on the distance vs. time graph. This must be repeated for several different times.



For example, since each large square on the graph of Figure 5-34 represents 10 seconds × 10 m/sec = 100 meters, the distance traversed after 30 seconds is about 7.5 "squares" or 750 meters. This gives one point plotted below. Other points are obtained in the same way.

Since the speed along the road never goes negative, the distance constantly increases. The final result is



It may be interesting to see if, by taking the slope of this curve, the students can get back the original one. It is easy to see that the result will be quite inaccurate.

PROBLEM 18

A bobsled has a constant acceleration of 2.0 m/sec² starting from rest.

(a) How fast is it going after 5.0 seconds?

(b) How far has it traveled in 5.0 seconds?

(c) What is its average speed in the first 5.0 seconds?

(d) How far has it traveled by the time its speed has reached 40 m/sec?

a) $v = a\,(t_f - t_i)$

$$v = 2\,\frac{m}{sec^2} \times (5 - 0)\ sec = \underline{10\ m/sec}.$$

b) $d = \frac{1}{2}\,a\,(t_f - t_i)^2 + v_i\,(t_f - t_i) = \frac{1}{2} \times 2 \times 5^2 + 0 = \underline{25\ m}.$

c) Its average speed is the distance traveled divided by the time or 25/5 = 5 meters/sec. Note that this is just half the velocity at the end of a 5 second interval—a general result for constant acceleration from rest.

d) During the time the sled is accelerating to 40 m/sec, its average speed is 20 m/sec. But 40 m/sec = 2 m/sec^2 × t or t = 20 seconds. It has therefore gone 20 seconds × 20 m/sec = 400 meters. More directly:

$$v = at + v_i$$

$$40 = 2t + 0;\ t = 20\ seconds$$

$$d = \frac{1}{2}\,a\,(t_f - t_i)^2 + v_i\,(t_f - t_i)$$

$$= \frac{1}{2}\,a\,t^2 + 0 = \frac{1}{2} \times 2 \times 20^2 = \underline{400\ meters}.$$

PROBLEM 19

A car, initially traveling at uniform speed, accelerates at the rate of 1.0 m/sec² for a period of 12 seconds. If the car traveled 190 meters during this 12-second period, what was the speed of the car when it started to accelerate?

$a = 1\ m/sec^2,\ t_f - t_i = 12\ sec,\ d = 190\ meters.$

$$d = \frac{1}{2}\,a\,(t_f - t_i)^2 + v_i\,(t_f - t_i)$$

$$190 = \frac{1}{2} \times 1 \times 12^2 + v_i \times 12$$

$$190 = 72 + 12\,v_i;\ v_i = \underline{9.8\ meters/sec}.$$

PROBLEM 20

A ball starting from rest rolls with uniform acceleration down an inclined track 216 cm long. A multiple-flash photograph shows that it took the ball 1.2 sec to cover this distance.

(a) What was the acceleration?

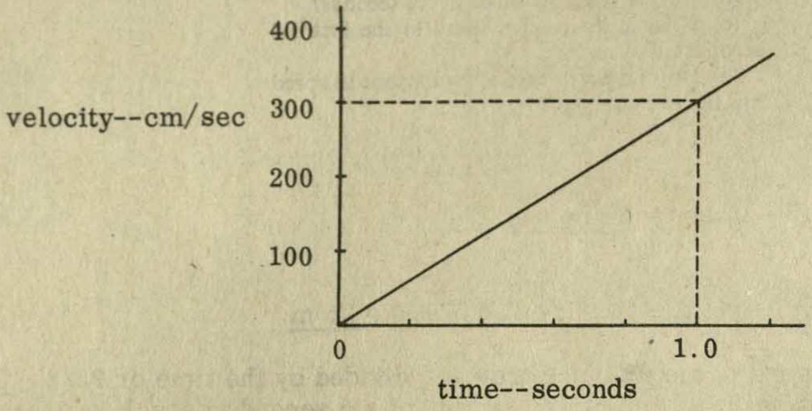
(b) Plot the velocity of the ball versus time.

Since d = 216 cm, and $t_f - t_i = 1.2$ sec

a) $d = \frac{1}{2}\,a\,(t_f - t_i)^2$

$$216\ cm = \frac{a}{2} \times 1.44\ sec^2;\ a = \underline{300\ cm/sec^2}.$$

b) $v = at$, so the velocity versus time graph is a straight line through the origin with a slope of 300 cm/sec^2.



PROBLEM 21 Assume that the express train mentioned at the very beginning of the chapter stops with uniform acceleration.

(a) How long does it take to stop?

(b) What is the acceleration along the path during the braking?

(c) Plot the position of each train versus time on the same graph and from this find whether or not a collision takes place.

a) $d = v_1 t + \frac{1}{2} a t^2$

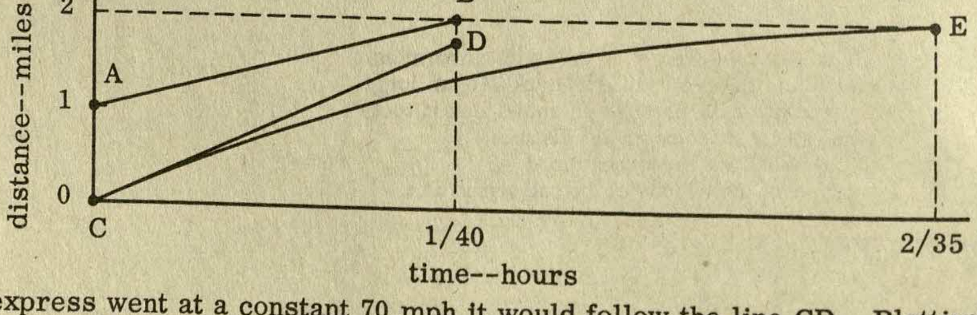
change in velocity $= 0 - v_1 = a\,t;\ a = -\frac{v_1}{t}$

hence $d = v_1\, t + \frac{1}{2}\left(-\frac{v_1}{t}\right) t^2 = \frac{1}{2} v_1\, t$

$2 \text{ mi} = \frac{1}{2} \times 70 \text{ mi/hr} \times t;\ t = \underline{\frac{2}{35}} \text{ hr}.$

b) $a = -\frac{70 \text{ mi/hr}}{\frac{2}{35} \text{ hr}} = \underline{-1225 \text{ mi/hr}^2}.$

c) The slow freight is much the easiest. At time $t = 0$ it is 1 mile down the track at point A. It has a uniform velocity along the track of 40 mph and therefore will go 1 mile in 1/40 hour to point B. Since the express stops in 2 miles we do not have to go beyond 2 miles on our graph.



If the express went at a constant 70 mph it would follow the line CD. Plotting this on the same graph shows that even if the express did not slow down, the trains would not hit within the two miles in which we are told it can be stopped. The trains do not crash.

However to plot the actual position of the express we note that

$$d = \frac{1}{2} a t^2 + v_1 t$$

$$= -\frac{1225}{2} t^2 + 70 t.$$

This graph is plotted as CE.

PROBLEM 22

An automobile starting from rest increases its speed (accelerates) at a constant rate for 10 seconds. Its speed at the end of 5.0 seconds is 48 km/hr.

(a) What is the acceleration?
(b) What will be its speed after 10 seconds?
(c) How far will it travel in 10 seconds?
(d) How far does it travel during the eighth second?
(e) Derive an equation for finding the distance a body, starting from rest and accelerating uniformly, travels during the *n*th second. ($n = 1, 2, 3$, etc.)

a) Acceleration along the path is the change in speed divided by the time interval:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{48 \frac{\text{km}}{\text{hr}} - 0}{5 \text{ sec} - 0}$$

$$= \underline{9.6 \text{ km/hr/sec}}.$$

b) $v_f - v_i = a\,(t_f - t_i)$

$v_f - 0 = 9.6\,(10 - 0)$; $v_f = \underline{96 \text{ km/hr}}$.

It might also be well to use

$v_f - 48 = 9.6\,(10 - 5) = 48$; $v_f = 96$ km/hr.

Of course it is always well to point out that this is what is meant by constant acceleration: If a car increases its speed by 48 km/sec in 5 seconds it will also increase its speed by 48 km/sec in the next 5 seconds.

c) For this part of the problem it may be well to have the accelerations and speeds in consistent units, i.e., meters/sec. Thus:

$a = 9.6$ km/hr/sec. 1 hr = 3600 sec. 1 km = 1000 m.

$$a = 9.6 \frac{\text{km}}{\text{hr sec}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{8}{3} \text{ m/sec}^2.$$

Now, $d = \frac{1}{2} a\,(t_f - t_i)^2 + v_i\,(t_f - t_i)$

$$= \frac{1}{2} \times \frac{8}{3} \text{ m/sec}^2 (10 \text{ sec} - 0 \text{ sec})^2 + 0\,(10 \text{ sec} - 0 \text{ sec})$$

$$= \frac{800 \text{ m}}{6} = 133\ 1/3 \text{ meters} = \underline{0.13 \text{ km}}.$$

d) There are at least two ways of working this.

1. $d = \frac{1}{2} a (t_f - t_i)^2 + v_i (t_f - t_i)$; and $v_f = a (t_f - t_i)$

Now let $t_f = 8$ sec; $t_i = 7$ sec in the first formula

$$d = \frac{1}{2} a (1 \text{ sec})^2 + v_7 (1 \text{ sec})$$

But v_7 is the speed after 7 seconds and is given by the second formula

$$v_7 = a (7 \text{ sec} - 0) = 7 \text{ sec} \times a.$$

Therefore

$$d = \frac{1}{2} a (1 \text{ sec})^2 + (7 \text{ sec} \times a) (1 \text{ sec})$$

$$= \frac{15}{2} a (1 \text{ sec})^2 = \frac{15}{2} \left(\frac{8}{3} \text{ m/sec}^2 \right) 1 \text{ sec}^2 = \underline{20 \text{ meters}}.$$

2. Calculate d at 7 seconds and d at 8 seconds and subtract.

$$d_8 = \frac{1}{2} a (8 \text{ sec} - 0)^2 + 0; \quad d_7 = \frac{1}{2} a (7 \text{ sec} - 0)^2$$

$$\Delta d = d_8 - d_7 = \frac{1}{2} a (64 \text{ sec}^2 - 49 \text{ sec}^2)$$

$$= \left(\frac{15}{2} \text{ sec}^2 \right) a = \left(\frac{15}{2} \text{sec}^2 \right) \left(\frac{8}{3} \text{ m/sec}^2 \right) = \underline{20 \text{ meters}}.$$

e) The equation can be derived as a generalization of part (d).

$$\Delta d = \frac{1}{2} a (n \text{ sec})^2 - \frac{1}{2} a \left((n - 1) \text{sec} \right)^2 = \frac{1}{2} a (2n - 1) \text{sec}^2 = \underline{(n - \tfrac{1}{2}) a \text{ sec}^2}.$$

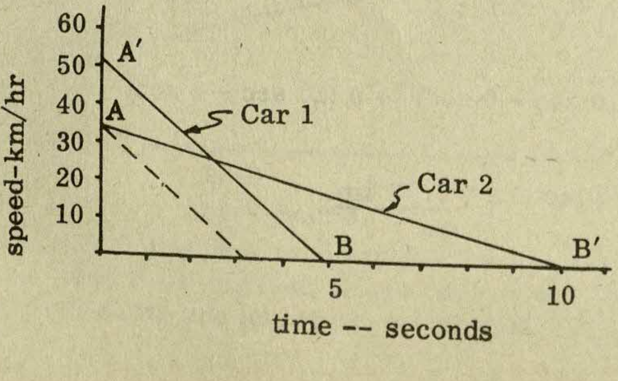
PROBLEM 23

A driver of a car going 52 km/hr applies the brakes, decelerates uniformly, and stops in 5 seconds. Another driver going 34 km/hr puts less pressure on his brakes and stops in 10 seconds. On the same graph plot speed versus time for each of the two cars.

(a) Which of the two cars traveled farther after the brakes were applied?

(b) Add a line to the graph which shows the second car decelerating at the same rate as the first car. How long does it take the car to stop at this rate of deceleration?

A speed vs. time graph for uniform acceleration is simply a straight line whose slope is the acceleration. Be sure the students realize that these graphs can be plotted by knowing that the cars start with a given speed at zero time—point A—and have zero speed at a later time—point B. The graph is obtained by connecting these points with a straight line.



a) The distance traveled by each car is the area under its curve. The areas are so nearly equal that simple inspection is probably not reliable. But (1/2) × 52 × 5 = area 1, and (1/2) × 34 × 10 = area 2. These "areas" are in peculiar units, kilometer seconds per hour, that is, in units of 1/3600 of a kilometer. They are, however, in the same units and can therefore be compared.

1/2 × 52 × 5 = 130 is less than (1/2) × 34 × 10 = 170, so Car 2, which started at 34 km/hr, travels farther.

b) "Deceleration at the same rate" indicates equal slopes. The dotted line shows Car 2 decelerating at the same rate as Car 1. Inspection of the graph indicates that it stops in about 3.3 seconds (actually (34/52) × 5 seconds).

PROBLEM 24

A car is traveling along a highway at 30 mi/hr. The driver then "steps on the gas" and accelerates at a uniform rate to a speed of 60 mi/hr in 10 seconds.

(a) What was the acceleration during this 10-second interval?

(b) How far did the car travel during these 10 seconds?

A "formula" problem in constant acceleration. It may, however, be done without formulas.

a) The car went from 30 to 60 mi/hr in 10 seconds. It changed its speed by 30 mi/hr in 10 seconds and therefore acceleration 3.0 mi/hr/sec.

b) In order to "plug into" the formulas it is safest to have consistent time units.

$$a = 3\,\frac{\text{miles}}{\text{hour seconds}} = 3\,\frac{\text{miles}}{3600 \text{ seconds seconds}} = \frac{3}{3600}\ \text{miles/sec}^2.$$

$$v_i = 30\,\frac{\text{miles}}{\text{hour}} = \frac{30}{3600}\ \text{miles/sec}$$

$$d = \frac{1}{2}\,a\,(t_f - t_i)^2 + v_i\,(t_f - t_i)$$

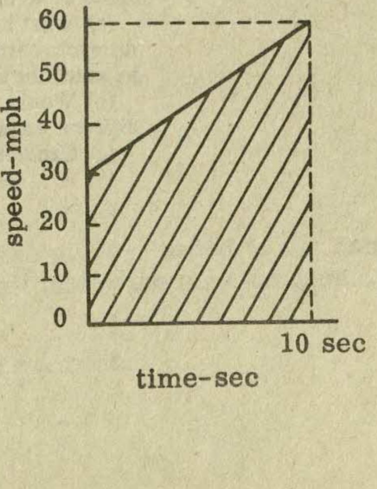
$$= \frac{1}{2} \times \frac{3}{3600}\,10^2 + \frac{30}{3600}\,10$$

$$= \frac{1}{8}\ \text{mile.}$$

On the other hand if the student remembers that the distance traveled during some time interval where the acceleration is uniform is just the time multiplied by the average of the initial and final speeds, we get

$$d = 10 \text{ sec} \times \frac{60 + 30 \text{ mi/hr}}{2} \times \frac{1 \text{ hr}}{3600 \text{ sec}} = \frac{1}{8}\ \text{mile.}$$

You may show this with the aid of a graph.



PROBLEM 25

A rocket which placed a satellite in orbit attained a speed of 2.90×10^4 km/hr in 2.05 minutes.

(a) What was the average acceleration in km/hr/sec? In m/sec/sec?

(b) If the rocket had enough fuel to maintain the same rate of acceleration for an hour, what speed would it have at the end of the hour starting from rest?

(c) How far would it travel during this hour?

a) The average acceleration is the change in speed divided by the time in which the change occurred.

$$a = \frac{29 \times 10^3 \text{ km/hr}}{2.05 \times 60 \text{ seconds}} = 236 \text{ km/hr/sec}$$

$$236 \frac{\text{km}}{\text{hr sec}} = 236 \times \frac{1000 \text{ meters}}{3600 \text{ sec sec}} = \underline{65.5 \text{ meter/sec}^2}.$$

$$\text{b)} \quad v = at = 236 \frac{\text{km}}{\text{hr sec}} \times 3600 \text{ sec} = \underline{8.5 \times 10^5 \text{ km/hr}}$$

$$\text{or,} \quad 236 \frac{\text{km}}{\text{hr sec}} \times 1 \text{ hr} = \underline{236 \text{ km/sec}}.$$

$$\text{c)} \quad d = \frac{1}{2} at^2 = \frac{1}{2} \times 65.5 \times 3600^2 = \underline{42.4 \times 10^8 \text{ meters}}$$

$$\text{or,} \quad \frac{1}{2} \times 236 \frac{\text{km}}{\text{sec}} \times 3600 \text{ sec} = \underline{4.24 \times 10^5 \text{ km}}.$$

This is just about the distance to the moon.

This whole problem "neglects" the effect of gravity. If a student asks "What about gravity?" this would be a good time to emphasize the difference between kinematics and dynamics. Kinematics is concerned only with the description of the motion of the rocket, not with why the rocket accelerates as it does. It would take fuel to overcome gravity as well as to accelerate the rocket. It would be very difficult to make a rocket which would actually accelerate uniformly regardless of whether gravity were acting on it or not. Here we must just assume the rocket to be uniformly accelerated!

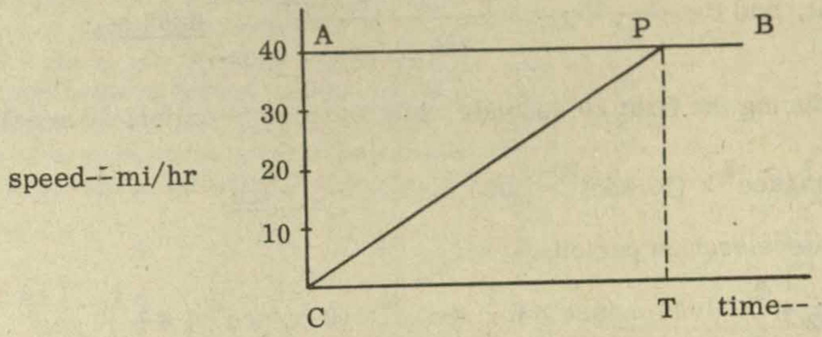
PROBLEM 26

Plot the speed-time graph of a car traveling at a uniform speed of 40 mi/hr. On the same graph plot the speed-time graph for a second car starting from rest and uniformly accelerating to a maximum speed of 40 mi/hr.

(a) What relationship do you find between the distances covered by the two cars during the time in which the second car is accelerating?

(b) Would this relationship hold if you chose a different acceleration for the second car?

(c) Can you prove this mathematically?



Some students may have difficulty plotting the simple curves shown since they are not given the acceleration of the second car or the time it takes to reach 40 mi/hr. This is the point of the problem; any uniform acceleration will do.

a) The distance covered by the first car is the area of the rectangle APTC, while the distance covered by the second is the area of the triangle CPT. Clearly the first car covers twice the distance of the second in the time T.

b) Since we chose no particular acceleration for our first plot, the same relation will hold for any acceleration.

c) The foregoing proof is really quite rigorous, but some students will prefer to do it algebraically. If so, it should be pointed out that if they use formulas developed in the text for uniform acceleration to prove this statement, they have gone full circle, since those formulas were derived from graphical considerations. However,

$d_1 = vt$

$d_2 = \frac{1}{2} at^2 + v_i t = \frac{1}{2} at^2 + 0$, since $v_i = 0$. But $a = \frac{v}{t}$, so $d_2 = \frac{1}{2} \frac{v}{t} t^2 = \frac{1}{2} vt$.

Finally, $d_1 = 2d_2$.

PROBLEM 27

A rocket-test sled is accelerated along a track for 10 sec with an acceleration of 100 m/sec². It then moves without acceleration for 5.0 sec, after which it moves with an acceleration of -150 m/sec² until it comes to rest.

(a) What is the maximum speed of the sled?

(b) How long does it take the sled to come to rest from the time it starts decelerating?

(c) What is the distance traveled during the first 10 sec? During the period of deceleration? During the entire trip?

(d) Plot a graph of the acceleration of the sled versus time. Find the "area" under the graph during the first 10 sec. How does this "area" compare with the maximum speed of the sled?

(e) Show that the change of speed of a body during any time interval is equal to the "area" under the acceleration-time graph for the same interval.

This is a graphical problem on acceleration in which the student is led to the conclusion that the velocity of an object is given by the "area" under the acceleration vs. time graph.

a) v = at for a uniformly accelerated object starting from rest.

$v = 100\ \text{m/sec}^2 \times 10\ \text{sec} = \underline{1000\ \text{meters/sec}}.$

b) $v_f = v_i + at$, and $t = \frac{v_f - v_i}{a}$; $t = \frac{0 - 1000\ m/sec}{-150\ m/sec^2}$, $t = \underline{6.67\ sec}$.

c) $d = \frac{1}{2}at^2$ during the first 10 seconds while the sled is uniformly accelerating.

$$d = \frac{1}{2}\,100\ m/sec^2 \times (10\ sec)^2 = 5000\ meters = \underline{5\ 0\ km}.$$

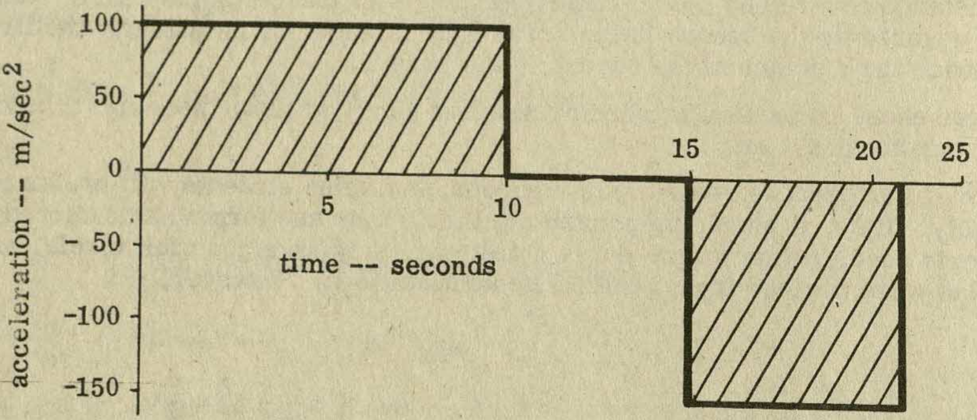
During the final deceleration period,

$$d = v_o\,t + \frac{1}{2}at^2 = 1000\ m/sec \times 6\frac{2}{3}\ sec - \frac{1}{2}\,150\ m/sec^2\left(6\frac{2}{3}\right)^2 sec^2$$

$$= 6,666\ m - 3,333\ m = 3,333\ meters = \underline{3.3\ km}.$$

The entire trip is 5.0 + 5.0 + 3.3 = $\underline{13.3\ km}$.

d)



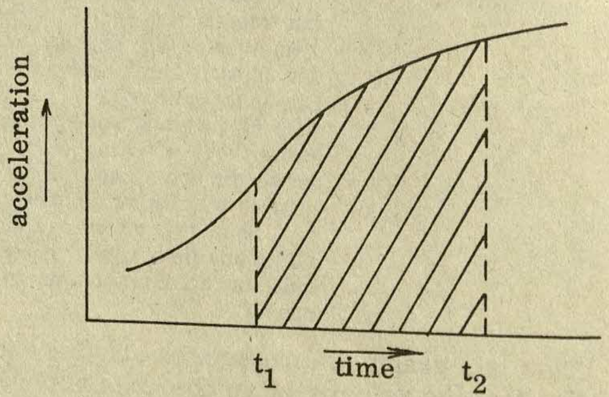
The "area" under the graph for the first 10 seconds is

$$100\ m/sec^2 \times 10\ sec = 1000\ m/sec$$

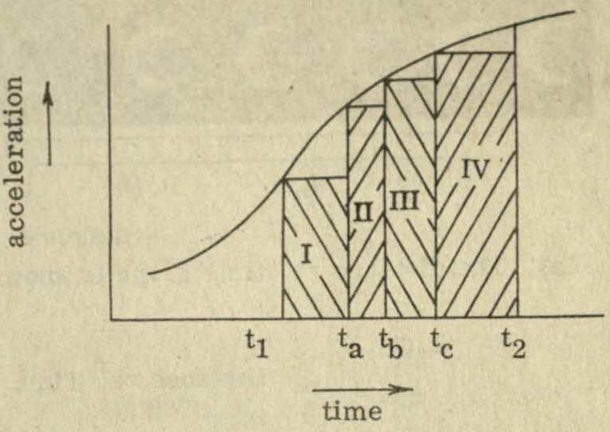
which is the same as the speed of the sled after the 10 second acceleration.

e) Suppose we have an acceleration vs. time graph as shown. The proposition is that the velocity at time t_2 minus the velocity at time t_1 is equal to the shaded area under the curve.

To prove this proposition, proceed exactly as the text does in showing that the area under a velocity curve and between two times is the distance traveled between those times. Note that "distance traveled" is just another way of saying "change in position".



The first part of (d) should have convinced students that if the acceleration is constant, the change in velocity is the area under the acceleration graph. Consider an object undergoing successive constant accelerations always such as to be accelerated less than the object in question. Clearly



$v_a - v_1 \cong$ area I;

$v_b - v_a \cong$ area II;

$v_c - v_b \cong$ area III;

$v_2 - v_c \cong$ area IV.

The total area under the step-curve is

area I + area II + area III + area IV $\cong v_a - v_1 + v_b - v_a + v_c - v_b + v_2 - v_c \cong v_2 - v_1$.

By considering more and more intervals, the area under the stepwise curve can be made to be equal to the area under the actual curve and the area then is the change in velocity. Note: it is the change in velocity not the velocity. As presented here the argument is shorter and less rigorous than that given in the text for the velocity vs. time graph.

PROBLEM 28

Look through the chapter and list all the places where the discussion must be limited to motion in a straight line — all the places when motion along a curved path would not do equally well.

This question is a difficult one to answer exactly. It is meant to be a thought provoking introduction to the next chapter rather than a test of understanding of the present chapter.

A careful reading of the chapter will show that the authors have been careful to talk always about motion along a particular path. Some students may have thought that a straight line path was assumed. Such was not the case. All distances could have been measured along a twisting road and all speeds could have been measured as miles (along the road) per hour. Even in the case of acceleration, the text says acceleration along the path. If the path is twisting, the acceleration along the path is not the only acceleration, but it is none-the-less calculated as defined in this chapter.

The correct answer to this question would seem to be that there are no places where this chapter is limited to straight line motion. We will see in the next chapter that the formulas derived in this chapter will take on new meaning for curvilinear motion.

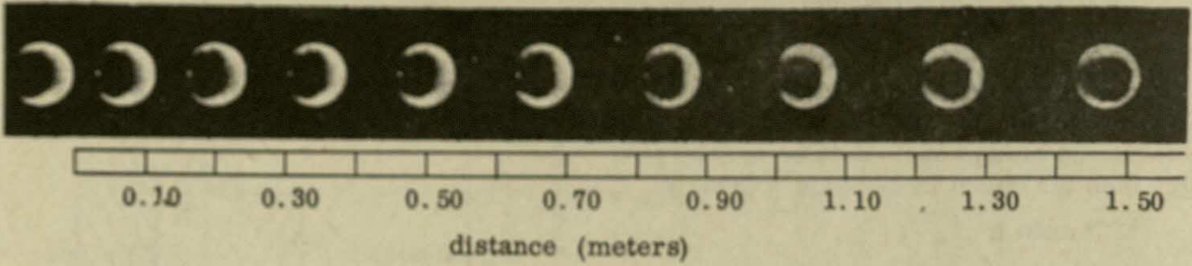
PROBLEM 29

Fig. 5–35 is a multiple-flash photograph of a moving ball taken at 1/30 sec intervals. The ball is moving from left to right and the zero point on the scale lines up with the right-hand edge of the ball's initial position.

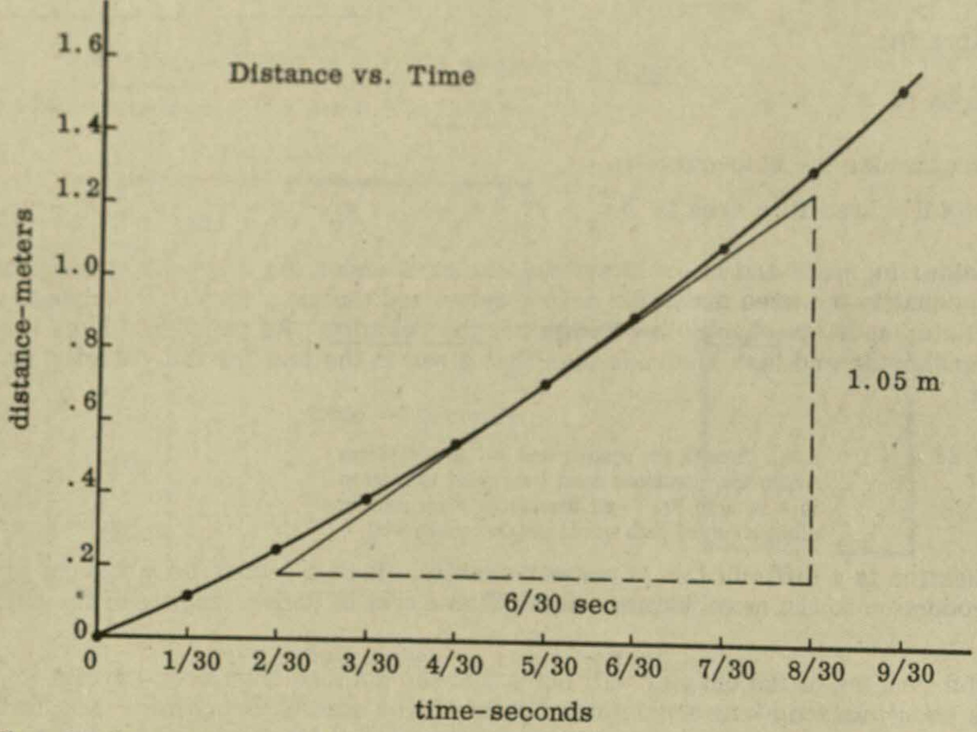
(a) Measure the distance from the zero point for each position of the ball, and plot a graph of distance against time to describe this motion.

(b) From your graph in (a), construct a speed-time graph.

(c) What does your speed-time graph tell you about the acceleration of the moving ball?



a) The distance vs. time graph is shown below.



b) The speed vs. time graph can be constructed in at least two ways. The first way is to measure the slope of the distance vs. time graph at various times. A sample measurement at t = 5/30 sec is shown. This gives a slope of

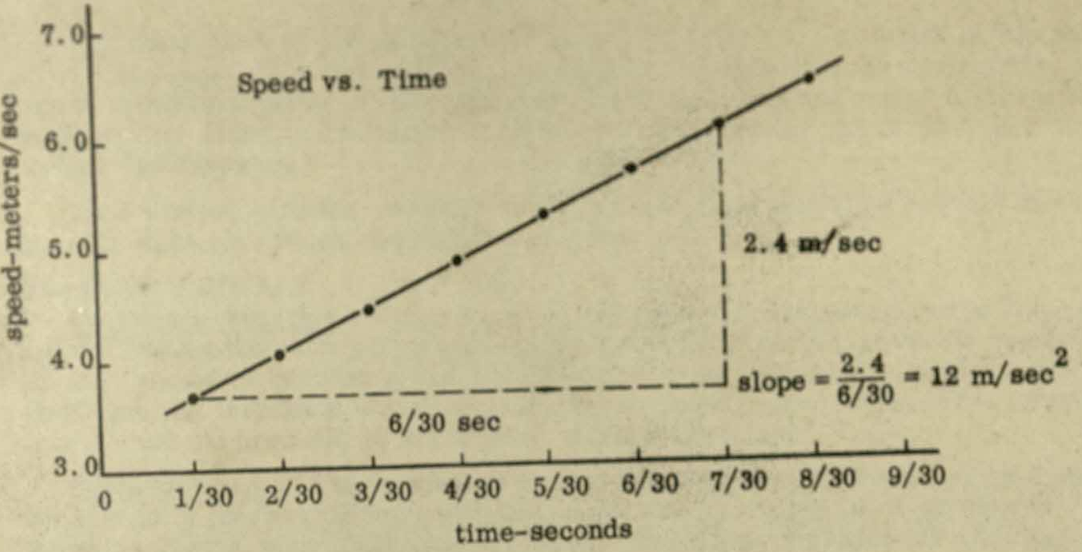
$$1.05 \text{ m} \div 6/30 \text{ sec} = 5.25 \text{ m/sec}$$

This value is plotted on the graph below. Note that the slope of the tangent line is very nearly the slope of the line drawn through the 4/30 sec and 6/30 sec points. This affords a second method for finding the speed at 5/30 sec.

$$\text{speed at } 5/30 \text{ sec} = \frac{\text{distance traveled from } 4/30 \text{ to } 6/30 \text{ sec}}{2/30 \text{ sec}}$$

$$= \frac{0.89 \text{ m} - 0.54 \text{ m}}{2/30 \text{ sec}} = 5.25 \text{ m/sec}$$

The entire speed vs. time graph, found by this second method is shown below.



c) The points of the speed-time graph scatter slightly as shown. However, the scatter is probably due to measuring difficulties and appears to be random. We therefore conclude the speed increases uniformly with time. The acceleration is constant and is 12 m/sec^2, from the measured slope.

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Chapter 6 - Vectors

The kinematics of one-dimensional motion (Chapter 5) is extended in this chapter to motion in space. For this purpose, the chapter introduces vector quantities. The chapter has three natural parts: 1) the nature of vector quantities and vector algebra; 2) vector acceleration; and 3) a discussion of frames of reference and the distinction between kinematics and dynamics.

This chapter includes centrally-important ideas and should be covered thoroughly through class discussion and problem-solving.

CHAPTER SUMMARY

Sections 1 through 4 Vectors are introduced by considering directed displacements and developing the rules for graphical addition and subtraction of vector quantities. The velocity vector is introduced and a number of simple navigational problems are discussed. The algebraic treatment of vector problems is considered in terms of the components of vectors and the products of multiplying vectors by scalars.

Sections 5 and 6 The concept of a vector acceleration is introduced as a measure of the change in (vector) velocity with time. An acceleration is involved whenever the velocity changes either in magnitude or in direction. Two kinds of motion are considered: constant vector acceleration, and changing acceleration.

Sections 7 through 9 Kinematic descriptions depend on the choice of a frame of reference, and a particular motion may often be more simply described by proper choice of a reference frame. Attention is called to the fact that kinematics does not tell why objects move as they do, and that Newtonian kinematics, while valid over large ranges, breaks down for objects moving with speeds close to the speed of light.

SCHEDULING CHAPTER 6

This chapter, like the preceeding one, is critically important. While Sections 1 through 6 should be covered thoroughly, most of the important concepts will be encountered again in Part III.

Some teachers will feel that considering the kinematic aspects of circular motion here, and the dynamic aspects in Part III, is an artificial division. This choice was made a) because it is logical to combine space and time measures in kinematics, b) to avoid an overlong analytic flavor in Part III, and c) because many teachers have found that the natural review provided in dynamics makes such a split effective. Considering the kinematics of circular motion at this point emphasizes that constant speed does not always imply zero acceleration.

While you will want to seek a good understanding of vectors, you should remember that Part III will make its contribution. Avoid getting bogged down in excessive detail at this point.

The following table suggests possible schedules for this chapter, consistent with the schedules outlined in the summary section for Part I. Sections which are enclosed with brackets [ ] can be deemphasized or skipped in class discussion without loss in continuity. Even though these sections may be deemphasized in class, they should be assigned for home reading.

	10-week schedule for Part I			6-week schedule for Part I		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2, 3, 4	4	0	-	4	0	-
Secs. 5, 6	2	0	-	2	0	-
Secs. [7], [8], [9]	1	0	-	0	0	-

RELATED MATERIALS FOR THE CHAPTER

Laboratory. While there are no experiments specifically intended for Chapter 6, laboratory time can be used to excellent advantage for supervised work in ruler-protractor solution of vector problems.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 6-17 ; detailed comments and solutions on page 6-18 to 6-32.

Section	Easy	Medium	Hard	Class Discussion
1	1, 2*, 3	4*, 5*, 6*, 7		4 and 5 together 2, 6, 7
2	8, 9*, 11*	10, 13*	12, 14*	9, 11, 14*
3	15, 16*, 18* 19	17, 20*, 27*	21*	16*
4	8	10, 22*	23, 24	10
5, 6		26*	25*	25, 26*

Film. A film on vector kinematics is being prepared but is not available at the time this guide is being written.

"Vectors", by Dr. A. V. Baez. This film records a lecture demonstration on vectors before a high-school class. Rubber-band models are used to show vector displacement in two and three dimensions. Vector addition, scalar multiplication of a vector, and other concepts are introduced. Running time: 28 minutes.

Demonstrations. Standard vector model demonstration apparatus may be somewhat useful in introducing the ideas of this chapter. Much more important is that students learn to solve vector problems at their desks with pencil, ruler, and protractor. A large number of vector problems should be used as class work to develop a working mastery of the vector character of motion.

Section 1 - Trips and Vectors; Addition and Subtraction of Vectors

PURPOSE To study how displacements add, as an introduction to the addition and subtraction of vectors.

CONTENT a. Trips along a line add like ordinary positive and negative numbers.

b. Trips in two dimensions add geometrically.

c. For most purposes in physics we are interested in the distance and direction of a displacement without regard to the location of the starting point.

d. In subtracting vectors, $\vec{C} = \vec{A} - \vec{B}$ is so defined that $\vec{A} = \vec{B} + \vec{C}$.

e. Scalars are quantities which add like numbers.

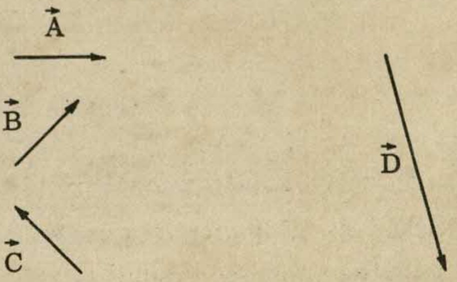
EMPHASIS Treat thoroughly. Graphical procedures for addition of vectors should be developed before tackling the more complicated physical situations discussed in Section 2.

DEVELOPMENT As a beginning, you can describe trips between local points familiar to the students as examples of quantities having magnitude (distance) and direction. Then describe a two or three-part trip and calculate the resultant displacement. Example: An air-line trip from Urbana, Illinois of 39.9 miles, 3.0° north of west, takes you to Clinton, Illinois. A subsequent trip of 43.6 miles in a direction 32.5° south of west takes you to Springfield. The total distance traveled is 73.5 miles. The resultant displacement, however, is only 69.3 miles in a direction 15.2° south of west.

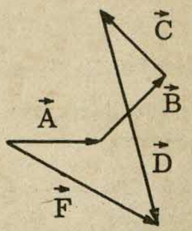
COMMENTS It is important for the students to draw vectors and to do vector additions graphically. Precision pays here. With sharp pencils and good rulers and protractors, two vectors can be added graphically to about 0.3% accuracy – better than can be done trigonometrically with a slide rule. Problem 6 in HDL is intended specifically for this purpose. The emphasis on good equipment and careful scale drawings stems from an actual class experience. A class was making scale drawings in a relaxed, easy way and found the activity somewhat dull even though they were working on problems with which they needed experience. As soon as care and precision were demanded, and the class engaged in informal competition for the most accurate results, interest picked up and was maintained.

There is another important point about scale drawings. From time to time in laboratory work, students will record angles and distances directly as scale drawings. After recording data in a drawing, some students who are comfortable with trigonometry will measure the drawing and turn to trigonometric computation. The accuracy will never be better than the original drawing! In most cases the computations can be carried out more easily by continuing graphically.

When $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$, $\overrightarrow{AC}$ is called the vector sum of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ (or resultant). Early in your class discussion you may want to consider some problems involving the sum of more than two vectors. Problem 1 in HDL is an example. Or, you can ask students to add vectors like those at the right.



(The vectors can be given pictorially as above or by specifying length and direction.) Each vector is moved parallel to itself so that its tail is at the head of the preceding vector. Notice that the vector sum is independent of the order of addition. Thus:



$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{F}$
$\vec{B} + \vec{A} + \vec{C} + \vec{D} = \vec{F}$
$\vec{D} + \vec{B} + \vec{C} + \vec{A} = \vec{F}$, etc.

Suggest that students verify the non-dependence on order. Different students can carry out the addition in different orders. Another example:

$\vec{A}$ = 10 miles east
$\vec{B}$ = 5 miles south
$\vec{C}$ = 10 miles 30° west of north
$\vec{D}$ = 5 miles north
$\vec{E}$ = 10 miles 30° west of south

Such problems (where the sum is zero) are a good introduction for the ideas of the opposite (or negative) of a vector and for vector subtractions.

Displacement vs. trip: A trip from London to Paris is not generally considered to be the same as a trip from Buffalo to Baltimore even though they are approximately the same direction and distance. If two pencils are at different spots on a desk and each is moved 1 foot northeast, the two pencils have not made exactly the same trips but something was certainly the same. The pencils have made the same displacement.

Vectors, as usually used by physicists, are like displacements; they are considered equal if they have the same magnitude and direction. It therefore makes no difference where a vector is moved if it is kept parallel to itself.

As a diversion, there is the old riddle about the hunter who traveled from his camp 10 miles south, 10 miles west, and finally returned to camp with a trip 10 miles north. Then he shot a bear. What color is it? Answer: white, because the man must be camping at the North Pole. (If the class is willing, this is a good place to drop this question. However, there is an extension of it which some student may know or think of on the spot. As well as starting at the North Pole, he could have started anywhere on the latitude circle 10 miles north of that latitude circle near the South Pole which has a circumference of 10 miles! Then the hunter's trip of 10 miles west takes him once around this latitude circle and he returns north to his starting place.) (If you get this far, someone may point out that the hunter could also start from a point on the latitude circle which is 10 miles north of any circle of latitude those circumference is $\frac{10 \text{ mi}}{n}$ where n is a whole number. . . . . ad nauseam.)

CAUTION It is natural, in discussing this section, to invent problems involving "map" trips between major cities in the United States. You can get strange results here if your trips are long ones and you insist on careful work. Since the earth is spherical, any flat map will introduce some distortions and these can make such problems messy. You can avoid this difficulty by keeping all the cities involved within a hundred miles of each other.

Section 2 - Velocity Vectors

PURPOSE To show that velocity is a vector quantity, and to illustrate the use of velocity vectors in solving navigational problems.

CONTENT a. Velocity is a vector.

b. Velocity must be "relative" to something.

c. The addition of two relative velocities.

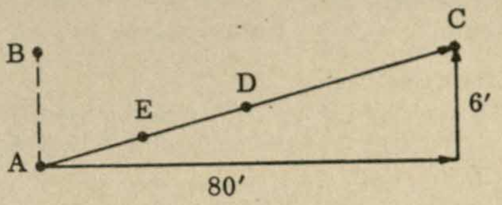
d. Course plotting: an exercise in vectors.

EMPHASIS Treat thoroughly: emphasis on the graphical solution of vector problems will develop insight.

DEVELOPMENT To introduce relative motion, you might ask (as you walk across the front of a classroom), "How fast am I going as I walk across the room? Four feet per second? Two?" Perhaps a student will propose a radical answer because he has considered that the earth is rotating. If there is no such proposal, ask how an answer of 800 mph might be justified. (This is the approximate speed at the surface of the earth due to the rotation of the earth about its axis.) Hopefully, someone will not be satisfied with that and will say that you are also going around the sun (66,500 mph). How about the motion of the sun through space? By now it should be apparent that velocities and speeds must always be measured relative to some other object or system. The system assumed to have "zero velocity" is chosen for convenience.

A more down-to-earth example is that of a person walking in a railroad car. If his velocity is 6 feet per second toward the engine relative to the train, and the train is going 80 feet per second relative to the ground, how fast is the person going relative to the ground? Clearly 86 feet per second. If he is walking toward the rear of the car, 74 feet per second. Sideways? Some students may invent the needed vector addition of the spot. Others may not see it for quite some time.

It may be profitable to go to a displacement picture for such train problems.



Suppose a man is at A and walks 6′ in one second so that he would go to B if the train were not moving. In this same second, however, the train has moved 80 feet down the track so that the man is not at B but at C. In one second he has moved from A to C. In 1/2 second he would be at D; in 1/4 second at E; and in fact he moves in a straight line from A to C. When he is at A, but moving; he is moving relative to the earth in a direction toward C. He is moving at a speed of $\sqrt{80^2 + 6^2}$ feet per second since he would go the distance $\sqrt{80^2 + 6^2}$ feet in one second. When we indicate the magnitude by "$\sqrt{80^2 + 6^2}$", we imply that the vector sum should be found by computation. Here is a fine place to use a scale drawing — or both methods. (Computing an answer is excellent practice for students, but we emphasize scale drawings because, aside from being a technique for solving problems, they display the relationships between the elements of a problem. A student who can solve a problem with a scale drawing almost surely "understands". He may be able to plug numbers into a formula without understanding.)

This whole process can be summarized as follows: To find how fast the man is moving relative to the ground add the velocity (velocity means vector velocity) of the train relative to the ground to the velocity of the man relative to the train.

Just as the man who walks around on the train walks relative to the train, so boats sail and swimmers swim relative to the water, and airplanes fly relative to the air. A good way to introduce these ideas is with a simple example such as the man who can swim 2 mph swimming upstream in a 2 mph current. He goes nowhere relative to the shore, although he is making good progress through the water.

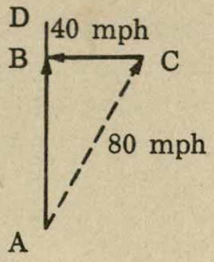
Problems of boats, swimmers, and airplanes are often complicated by the need to distinguish between the direction they are headed and the direction they are going. You might ask students to think about walking across the aisle of a train. They head directly across, facing the other side of the train. It is not necessary to scuttle across on a diagonal just because the train is moving. Just as a student who heads directly across the aisle has a velocity pointed directly across the aisle relative to the train, so an airplane heading north has a velocity directed north relative to the air. The plane may be flying in quite a different direction relative to the ground.

The navigation problem in the text (pages 78 and 79) is worth class time. When you do it, you probably should point out that from every vector equation in two dimensions, two facts can be found. The navigator was able to find his heading and the time the flight would take.

QUIZ PROBLEM If you need an extra problem for discussion or quiz, perhaps the following will be helpful.

1) A blimp, whose motors can push it through the air at 80 miles per hour tries to fly due north while a 40 mile per hour wind is blowing from the east. What heading must the blimp take? What will be its ground speed?

An unknown angle must be added to $\overrightarrow{BC}$ to obtain a vector sum along AD. The angle is 30° east of north. $\overrightarrow{AB}$ is 69 mph.



Section 3 - Components of Vectors

PURPOSE To point out that vectors can be represented by the sum of two or more components and that sometimes it is easier to think about the components of a vector than the vector itself.

CONTENT a. Illustration of how some results are dependent only on one component of a vector regardless of the value of the component perpendicular to it.

b. Finding the components of a vector along two mutually perpendicular directions.

c. Building a vector from its components.

d. The advantages of representing a vector by perpendicular components.

EMPHASIS Vector components are important, but should not take too much time because they are not difficult. It is more important for students to master the graphical constructions introduced earlier than to develop facility with algebraic methods.

COMMENTS By giving examples, the text leads to the idea of a component without defining "component" in words. Do not press students for such a verbal definition. The test is whether, given a vector and a line, as in Figure a, a student can sketch the component of the given vector along the given line. If pressed, he should be able to tell which angles are right angles, and through what points construction lines are drawn. You will sometimes find a student who, when given a line vector as in Figure b, will say that the component of the vector along the line is as shown in Figure c, and he will not be sure which of the two angles is the right angle, because the original vector and line were so nearly parallel.

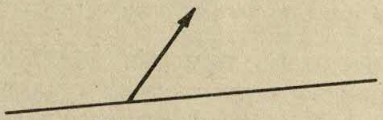


Figure a

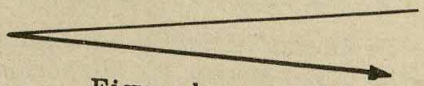


Figure b

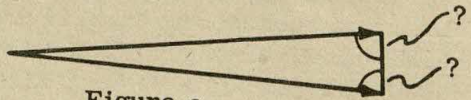
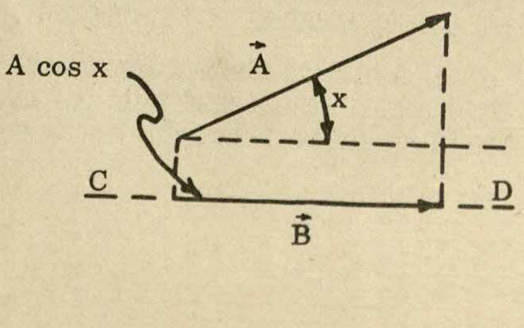


Figure c

You may want to discuss the two special cases: 1. The component of a vector along a line perpendicular to the vector is a zero vector, and 2. the component of a vector along a line parallel to the vector is the given vector itself.

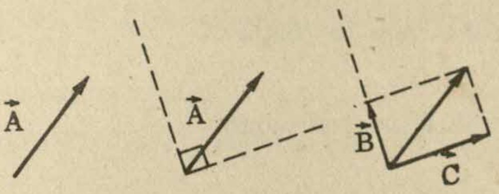
Students who have had trigonometry will see that if the angle between a vector $\vec{A}$ and a line CD is x, then $\vec{B}$ is the component of the vector $\vec{A}$ along the line CD and $B = A \times \cos x$, where A and B are the magnitudes of $\vec{A}$ and $\vec{B}$, respectively.



* * *

There are two distinct uses for components of vectors. One of these is to find the "effectiveness" of a vector in a certain direction. A plane flying 60° east of north at a speed of 300 miles per hour has an "effective speed" of 150 mph north, i.e., it is moving north at a speed of 150 mph.

The second use for components is to replace a vector by its components along two mutually perpendicular directions (for a two-dimensional vector). An example is given below.



$\vec{B}$ and $\vec{C}$ are the components of $\vec{A}$ taken along mutually perpendicular lines.

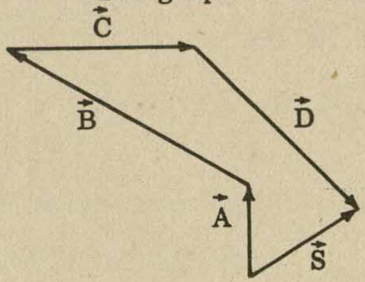
The components depend on which particular set of mutually perpendicular lines is chosen, but the vector sum of the components always gives back the original vector. Frequently, in adding vectors, it is easier to replace vectors by their components along a set of perpendicular lines, then add the components.

The most extensive use of components is in solving problems by analytical rather than graphical techniques. Consider the following problem:

Add the trips:

$\vec{A}$: 10 miles north.
$\vec{B}$: 30 miles 60° west of north.
$\vec{C}$: 20 miles east.
$\vec{D}$: 25 miles 45° east of south.

The graphical addition is shown below:



$$\vec{S} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Taking components will not facilitate the graphical solution of this problem. Taking components and adding them would be unnecessary work. We reiterate that a student with a good ruler, a good protractor and a sharp pencil can solve problems such as this more accurately and quickly than by using a slide rule. However, if we decide to do the problem by trigonometry, the east-west and north-south components of the vector can be separately summed and the vector sum of the sums of the components then found.

Let $\vec{A}_{EW}$ be the component of $\vec{A}$ along an east-west line. We have then:

$\vec{A}_{EW} = 10 \cos 90° = 0$

$\vec{B}_{EW} = 30 \cos 30° = -25.98$ miles (- means west)

$\vec{C}_{EW} = 20 \cos 0° = +20.00$ miles

$\vec{D}_{EW} = 25 \cos 45° = +17.68$ miles

Add these east-west components to obtain the total east-west component,

$\vec{S}_{EW} = +11.70$ miles.

Similarly for a north-south line:

$\vec{A}_{NS} = 10 \cos 0° = 10.00$

$\vec{B}_{NS} = 30 \cos 60° = 15.00$

$\vec{C}_{NS} = 10 \cos 90° = 0.00$

$\vec{D}_{NS} = 25 \cos 45° = -17.68$

$\vec{S}_{NS} = \text{sum of above} = 7.32.$

Then the length of S

$$S = \sqrt{S_{NS}^{\;2} + S_{EW}^{\;2}} = \sqrt{190.47} = 13.8 \text{ miles, and the angle of}$$

$$\vec{S} \text{ is } \theta = \text{arc tan}\,\frac{S_{EW}}{S_{NS}} = \text{arc tan}\,\frac{11.70}{7.32} = 58.0° \text{ east of north.}$$

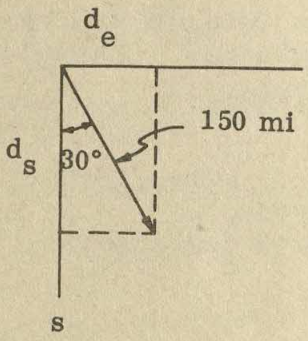
The teacher must consider carefully his students' in trigonometry before trying anything like this in class. If in doubt, treat components only graphically.

QUIZ PROBLEM If you need an extra problem for discussion or quiz, the following may be helpful. It can be solved either graphically or trigonometrically.

1) How far south will a motorist be if he travels 150 miles on a road that runs 30° east of south? How far east does he travel?

Solution: $d_s = 150 \cos 30° = 130$ mi.

$d_e = 150 \sin 30° = 75$ mi.



Section 4 - Multiplying Vectors by Numbers and Scalars

PURPOSE To point out that multiplying or dividing a vector by an ordinary number means multiplying or dividing the magnitudes while leaving the direction unchanged.

CONTENT a. The vector $k\vec{A}$ is a vector k times as long as $\vec{A}$ and in the direction of $\vec{A}$ (if k is positive).

b. The vector $k\vec{A}$ may have different units from $\vec{A}$ if k is a scalar having units instead of just a number.

EMPHASIS This section should go rather easily, and can be covered in a short time..

COMMENTS The scalar multiplying a vector may have units. In this case the product of the scalar and vector is a new vector quantity. If this quantity is to be graphed, it should be drawn separately from the previous vector. Suppose, for example, that a car goes 20 miles, 30° west of north, in 30 minutes. What is its velocity? Thirty minutes = 1/2 hour, hence the magnitude of the velocity vector is 20 mi/(1/2 hr) = 40 mi/hr, and its direction is 30° west of north. A displacement vector divided by a time interval (scalar) results in a velocity vector. Students should avoid plotting the velocity vector on the same diagram as the original displacement vector. They are different quantities.

The care required in dealing with two different quantities in such a problem may be emphasized with a nonsense example: A car goes 30 mph directly east, then goes 20 miles north. Does adding the two vectors make sense? (No!)

Here is another way of emphasizing that displacement and velocity are different kinds of quantities:

A man has a velocity of 8 feet per second northwest and walks for 45 seconds. What is his displacement? Draw his velocity and displacement.

The large factor of 45 will force the use of a new scale for displacement, which is what we want. There is not much point in a displacement vector of length 360 feet being drawn 45 times as long as a velocity vector of 8 feet per second.

QUIZ PROBLEM

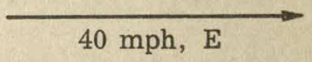
1) A car goes 30 mph northeast for 10 minutes and then goes 40 mph due east for an hour.

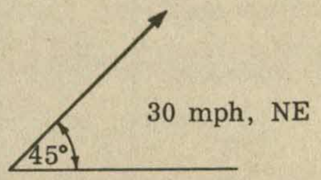
(a) Draw the two velocity vectors.

(b) On another diagram, draw the two displacements and the vector sum of the displacements.

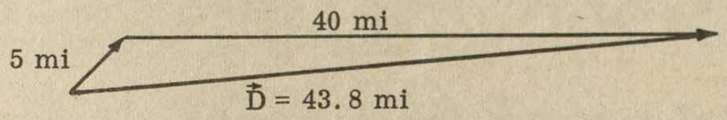
Solution:

(a) These velocity vectors are not to be added, since they do not occur simultaneously.





(b)



Section 5 - Velocity Changes and Constant Vector Acceleration

PURPOSE To introduce the concept of vector acceleration.

EMPHASIS It is important for students to grasp constant vector acceleration before going on to changing acceleration (Section 6). Use plenty of problems. A number of problems suitable for class discussion are given below. If your students' understanding is less than complete, do not abandon hope. They will meet vector acceleration again in Part III.

CONTENT To introduce acceleration as a vector quantity, a multiple-flash photograph of the fall of two balls is analyzed. One ball simply drops straight down. The other starts out with an initial horizontal velocity so that it falls along a curved path. The velocity vector of the first ball stays constant in direction, but changes in magnitude. The velocity vector of the second ball changes both in direction and magnitude.

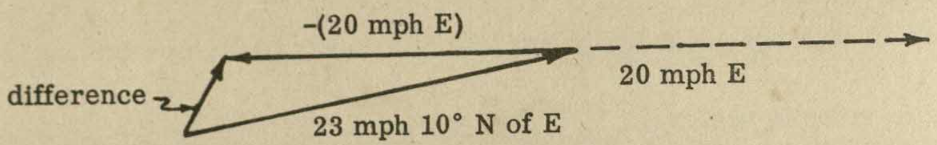
The change of magnitude of the first ball's velocity is described as an acceleration, $a = \frac{\Delta v}{\Delta t}$. Since the change in the velocity only affects its magnitude, the acceleration in this case can be treated as a scalar, but in fact it is a vector since it is the difference, $\Delta\vec{v}$, between two vectors (itself a vector) divided by a scalar, Δt.

The change of velocity of the second ball must be regarded as due to a vector acceleration since both the direction and the magnitude of the velocity change. A vector equation is therefore developed: $\vec{v}_f = \vec{v}_i + \vec{a}t$. The case described in the text involves constant acceleration and is drawn from common experience. The fact that this is a body falling under the force of gravity should not be stressed or developed since the behavior of bodies under the action of forces is intentionally postponed until Part III.

COMMENT The following paragraphs suggest supplementary examples in case students need more than the example in the text to grasp clearly the concept of vector acceleration. (There are no problems in HDL for this section.)

Supplementary Examples

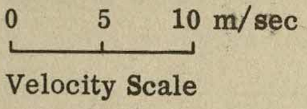
1. If a car were going 20 miles per hour at 3:15:33, and 23 miles per hour at 3:15:45, by how much did its speed change? 3 mph. Then: If a car were going 20 miles per hour due east at 3:15:33 and 23 miles per hour 10° north of east at 3:15:45, by how much did its velocity change? The answer is their vector difference. The answer to the proposed problem is shown graphically below. Note that the difference is a vector.

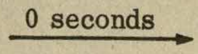


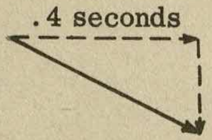
It has a magnitude (which is not 3 mph, but 4.8 mph) and a direction. It would be incorrect to say that the car's velocity increased. Vectors cannot be directly ordered except as regards their magnitude. The speed of the car increased by 3 mph. The velocity of the car changed by 4.81 mph, 33.5° east of north. It is often useful to know how rapidly the velocity is changing. In the foregoing example, the velocity changed by 4.81 mph, 33.5° E of N, in 12 seconds: or, $\frac{4.81}{12} = .40$ miles per hour per second, 33.5° E of N.

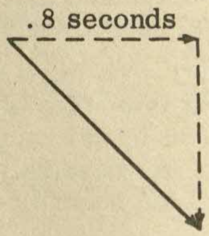
2. The equation $\vec{v}_f = \vec{v}_i + \vec{a}t$ is of key importance. The students will have little grasp of its meaning unless they go through graphical steps to develop it themselves. The graphical work will pay great dividends in helping them to grasp the next section. Here is one suggestion:

Suppose an object has an initial velocity of 10 m/sec to the right and that the velocity changes by adding 5 m/sec down every .4 seconds. Have the class draw the velocity vector at the end of 0, .4, .8 seconds.

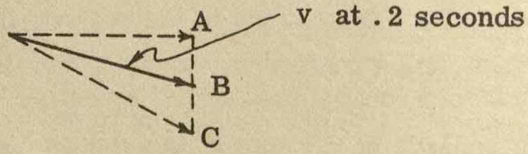








Now ask them to draw the velocity at .2 seconds. They should recognize that the tip of the velocity vector comes to the point B on the diagram midway between points A and C which represent 0 seconds and .4 seconds respectively. That is, .2 seconds is half-way between 0 and .4 seconds, and consequently we add only 1/2 (5 m/sec, down) to the initial velocity at 0 seconds. You can ask them for other fractional times, as well as for times such as 1.0 seconds ($\frac{1.0}{.4}$ = 2 1/2 basic time intervals, hence add 2 1/2 vectors down).



Now ask for a formula for t seconds. They should now be able to suggest that you add ($\frac{t}{.4}$) vectors of 5 m/sec length downwards.

Ask if they can give you an equation stating this. In all likelihood they will flounder badly, but some may say something such as

$$v_f = 10 + \frac{t}{.4} \times 5$$

leaving out the directions. Of course you can show them this equation is wrong by having them compute v_t from the equation for t = .8 seconds (20 m/sec) and compare it with their graphical answer (14 m/sec.)

You will then be able to present them with the equation:

$$v_f = (10 \text{ m/sec, right}) + (\frac{t}{.4}) (5 \text{ m/sec, down})$$

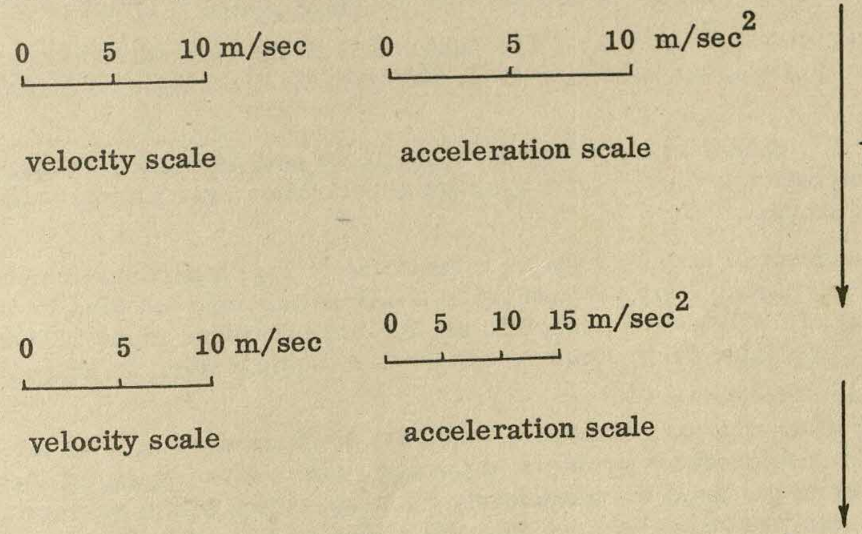
We may consider $\frac{t}{.4}$ (5 m/sec, down) to be a scalar, $\frac{t}{.4}$, times a vector, 5 m/sec, down, and rewrite this as $(\frac{t \times 5}{.4}$ m/sec, down); or $(12.5 \frac{m}{sec^2} \times t$, down); or $t \times (12.5 \frac{m}{sec^2}$, down). Therefore, $\vec{v}_f = (10$ m/sec, right) + t (12.5 m/sec^2, down), which is of the general form

$$\vec{v}_f = \vec{v}_i + \vec{a}t.$$

Be sure to emphasize the difference between the above equation and that for acceleration along the path, a_p,

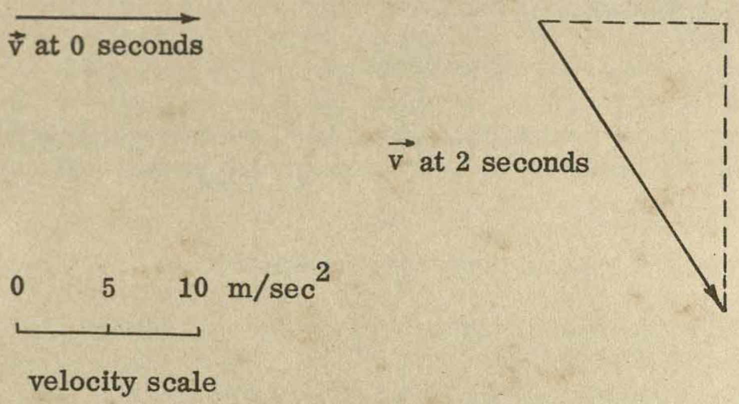
$$v_f = v_i + a_p t.$$

Now have your students plot the vector (12.5 m/sec^2, down). To do this they may need to specify the scale they will use for acceleration. Some may not realize either that they must choose a scale or that they are free to make any choice they want. Thus both of the solutions below are correct:



The development suggested above is intended to explore a number of hidden points which may trouble students in Section 6-6 on changing acceleration. In particular, troubles arise because we are dealing with vectors, and because the acceleration has different units from velocity and must therefore be plotted on its own scale.

At this stage you should turn the problem around and have the class compute the constant acceleration $\vec{a}$ that would change $\vec{v}_i$ into $\vec{v}_f$ in the time t. Once again introduce the problem with a graphical example:



Several new points now come up. You might first ask the class to find the velocity vector which has been added to the initial velocity to get the velocity at 2 seconds.

For our example, it is seen to be (15 m/sec, down). This vector must be $\vec{a}t$. Therefore, since t is 2 seconds, $\vec{a}$ has a magnitude of (15/2) m/sec^2 and a direction down. To plot it we must again choose our acceleration scale. We then draw a vector down and of length corresponding to 7.5 m/sec^2 on the acceleration scale.

Students should note that $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$.

Section 6 - Changing Acceleration and the Instantaneous Acceleration Vector

PURPOSE To extend the concept of acceleration to cases where it is not constant.

CONTENT a. In general $\vec{a} = \Delta\vec{v}/\Delta t$. The vector $\vec{a}$ is in the same direction as the vector $\Delta\vec{v}$ and need not be in the same direction as $\vec{v}$, nor need its direction and magnitude be constant.

b. An example of a speedboat speeding up on a curved path is used to develop the idea of a changing acceleration vector, of the average acceleration over an interval, and the instantaneous acceleration.

EMPHASIS The concept of acceleration as a vector is of key importance. All of the ideas in this section, particularly instantaneous acceleration, may not sink in on this first encounter, but students will have a chance to dig further into these ideas in Part III. At this point, the most valuable thing students can do is individual work on graphical solutions.

DEVELOPMENT Work out, or preferably, have the students work out, an accurate and detailed analysis of the speedboat problem in the text, using data measured from Figures 6-25 and 6-27. Try to get them to obtain fairly accurate tables of the average value of the acceleration $\vec{a}$ during each 10-second interval shown on Figure 6-25 and the average value of $\vec{a}$ during each 2-second interval shown in Figure 6-27. The two tables show approximately the same value for the acceleration during the same intervals but the second table should show some detail of the way $\vec{a}$ is changing.

Students should perform a similar graphical analysis of the circular motion portrayed in Figure 6-29. Other examples are given in HDL problems 25 and 26. Graphical analysis of these two problems is more instructive at this point than trigonometric solutions.

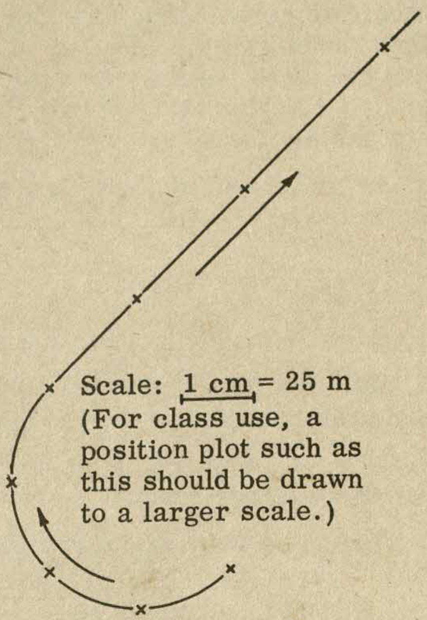
Do enough to make the concepts of average and instantaneous acceleration clear. The major treatment of uniform circular motion is in Part III. Here you may wish to use it simply as one example in which you can state a simple rule for the velocity vector at different times. Avoid being drawn into a complete discussion of circular motion. Above all, do not let forces creep into the discussion. If you find you are using forces to deduce the direction of an acceleration, you are doing the wrong thing. Students have a natural desire, in fact an almost irresistible urge, to talk about centrifugal force, but you must resist attempts to get into such matters. Acceleration, like displacement or velocity, is a geometrical concept. For example, from the fact that an object moves at 5 m/sec in a circle of radius 2 meters, we can compute its instantaneous acceleration without any mention of forces. In fact, in Part III we shall wish to deduce the instantaneous acceleration, $\vec{a}$, from measurements of velocity and time, and compare it with the force acting on a body of known mass. When the force is measured separately we can then study Newton's law $\vec{F} = m\vec{a}$, and see that it is really a vector law. The logic is spoiled if the students get the incorrect idea that $\vec{a}$ can only be found from the force.

SUPPLEMENT At the back of this volume as Appendix 1, there is a supplement on circular motion. The inclusion of the supplement in this volume is not meant to suggest that all of the included information about circular motion should be taught. This might be appropriate only in classes that are giving considerably more time to physics than the normal 5-6 periods per week for a year. You may, however, find the supplement helpful as background information for your classwork on circular motion. Also, some of the examples may be useful as quiz problems.

QUIZ PROBLEM

1) Give the students sheets of graph paper on which the positions of an object at four or more equally spaced instants of time are indicated. Have them compute the average velocity during each interval and the average acceleration from one interval to the next. Have the points represent a constant speed, curved course followed by a straight-line speeding-up segment. Such a quiz will take some time. While students are working, you should check to see if they are proceeding correctly.

Sample: A race car follows the path indicated on the paper. Successive time intervals of 1.0 sec are indicated by x's. Calculate graphically the average velocity during each 1.0 sec interval and the average acceleration between successive intervals. Choose a scale and draw a vector to represent each average acceleration.



Scale: 1 cm = 25 m
(For class use, a position plot such as this should be drawn to a larger scale.)

Section 7 - The Description of Motion; Frames of Reference

PURPOSE To point out that there are alternatives in choosing a frame of reference for the description of the motion of a physical system, and that the proper choice of a frame of reference may be extremely helpful in simplifying the vectorial description.

CONTENT a. The motion of an object may be described by the successive positions of a vector drawn from some reference point to the object.

b. Different frames of reference make the description of the motion very different.

c. A proper choice of a frame of reference may make the motion appear relatively simple.

EMPHASIS Reading assignment. A few minutes of class time should suffice.

COMMENT Although the proper choice of coordinate systems is very important in advanced physics, in this course there will be few occasions which will require students to exercise judgement in this regard. It is probably better to meet these situations as they arise, rather than attempt to discuss the problem in general. This topic will be reviewed extensively when the historical aspects of the problem of planetary motion are discussed in Part III.

Section 8 - Kinematics and Dynamics

PURPOSE To note that a kinematic description of motion is only a beginning to the study of motion, and that the study of why objects move as they do will come in Part III.

CONTENT a. Definition of kinematics as a description of motion.

b. Discussion of why physicists prefer to study simple systems and why engineers, of necessity, study complex ones.

EMPHASIS Light. Reading assignment followed by short class discussion (five minutes ought to do).

COMMENT A simple example of the distinction between kinematics and dynamics might be the following one. Swing a stone on a string around your head. The students can now describe the stone's displacement from some reference point at any time; they can find its vector velocity if they have stop watches; they can compute its average acceleration. In other words, they could do a lot of describing. But how hard must you pull on the string to keep the stone from flying away? How fast can you whirl it before the string will break? These are different and very fascinating questions. The study of dynamics in Part III will help to answer them.

The relation between physics and engineering probably should not be considered too deeply here.

Section 9 - The Speed of Light

PURPOSE To reassure students that the kinematics they have been learning is not wrong just because relativity has been discovered.

CONTENT a. For speeds close to that of light, the rules we have developed for adding velocities no longer work.

b. There is a very useful and extensive domain in which Newtonian kinematics does work.

EMPHASIS Treat very briefly.

COMMENT The theory of relativity is fertile ground for much sterile discussion among high school students. Discussion of these points should be severely limited.

It seems paradoxical to most students that if two rocket ships approach each other, each going 3/4 the speed of light, that a man on one will find that the other is approaching him at something less than (and not 1.5 times) the speed of light. And yet, there is ample evidence that this is true. We cannot here describe the many and varied conclusions of a proper treatment of relativity. Interested students may enjoy reading Barnett's The Universe and Dr. Einstein.[1]

The important thing for students to understand is that even though the kinematics developed here does not hold in all cases, it is still a very useful branch of science. This is because it is almost exactly true for all ordinary motions. It may be worth noting that the results of our kinematics (Newtonian kinematics) differ from the relativistic kinematics by a fraction which is on the order of

$$\left(\frac{\text{speed in question}}{\text{speed of light}}\right)^2.$$

For a speed as great as 10^5 meters per second, this factor is $(10^5/3 \times 10^8)^2$ = approximately 10^{-7} or one part in ten million! Even with the most advanced techniques, very few measurements approach this precision.

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[1] Barnett, Lincoln, The Universe and Dr. Einstein, Sloane, 1957

Chapter 6 - Vectors

For Home, Desk and Lab – Answers to Problems

GENERAL Chapter 6 depends, as did Chapter 5, on considerable problem solving by students. Most of the problems for this chapter should be assigned. Problem 27 is useful to summarize some of the essential characteristics of vectors. It may well be saved for assignment near the close of the work on vectors.

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 6-18 to 6-32.

Section	Easy	Medium	Hard	Class Discussion
1	1, 2*, 3	4*, 5*, 6*, 7		4 and 5 together 2, 6, 7
2	8, 9*, 11*	10, 13*	12, 14*	9, 11, 14*
3	15, 16*, 18*, 19	17, 20*, 27*	21*	16*
4	8	10, 22*	23, 24	10
5, 6		26*	25*	25, 26*

SHORT ANSWERS

1. See discussion on page 6-18.
2. 2 blocks south.
3. (a) See discussion on page 6-19.
   (b) Zero.
4. (a) 3.11 cm, 45° below horizontal.
   (b) 4.4 cm vertically down.
   (c) Zero.
5. (a) 0.7 cm vertically up.
   (b) 2.7 cm, 34° left of vertical.
   (c) 3.7 cm vertically up.
   (d) 3.6 cm, 6.2° left of vertically down.
6. 16.38 cm, 35° up from the horizontal.
7. (a) 2.12 cm, 3.3° west of north.
   (b) 8.48 cm, 3.3° west of north.
   (c) See discussion on page 6-21.
8. See discussion on page 6-21.
9. (a) 448 km/hr.
   (b) 512 km/hr.
10. (a) 4.83 km at 318.5°.
    (b) 0.966 hours.
11. (a) 155.2° clockwise from north.
    (b) 595 mi/hr.
    (c) 149 mi.
12. (a) 7.66° west of north.
    (b) 2.02 hr.
13. (a) 4.3° north of east.
    (b) 280 mph.
14. 18.6° east of north.
15. About 2 miles.
16. (a) 56.3° downstream from straight across.
    (b) 0.05 hours.
    (c) 0.3 miles.
    (d) 0.05 hours.
17. (a) 97.5 m, 280 m, 1300 m.
    (b) See discussion on page 6-26.
    (c) 2850 m.
18. 9.72 m.
19. 20.6 km.

20. (a) 0.30 km.
    (b) SE.
    (c) 2.83 km.
    (d) 2.84 km.

21. (a) 137 km, 344 km.
    (b) 411 km/hr, 149° clockwise from north.
    (c) No.

22. (a) 1 m/sec, 6.4 m/sec.
    (b) 25.5 m, 28.1° with direction of ship.

23. 5° west of north, 340 knots.

24. (a) 1/24″ to the right.
    (b) 1/48″ to the right, 1/4″ down.
    (c) 1/96″ to the right, 1/8″ down, 1/8″ toward reader.

25. (a) 1.035 m/sec toward center of circle from interval mid-point.
    (b) 0.345 m/sec^2 toward center of circle from interval mid-point.

26. (a) 0.21 cm/sec.
    (b) 0.21 cm/sec to right, 0.21 cm/sec vertically down.
    (c) 0.30 cm/sec down and left at 45°.
    (d) 0.02 cm/sec^2 down and left at 45°.

27. See discussion on page 6-32.

COMMENTS AND SOLUTIONS

PROBLEM 1

Draw diagrams to represent the following displacements:

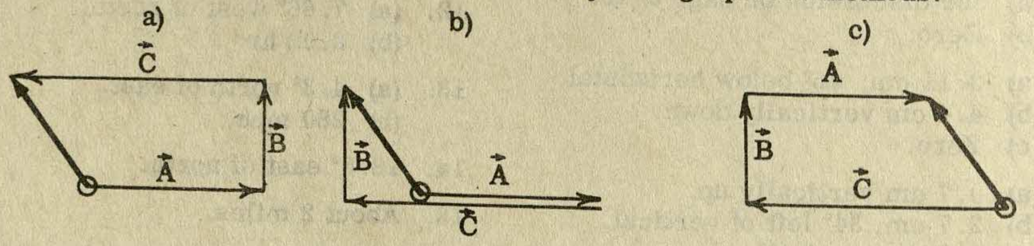
(a) 5.00 cm to the east followed by 3.00 cm north followed by 6.00 cm west.

(b) 5.00 cm to the east followed by 6.00 cm west followed by 3.00 cm north.

(c) 6.00 cm to the west followed by 3.00 cm north followed by 5.00 cm east.

(d) Compare the diagrams of parts (a), (b), and (c) and state a general rule about adding three displacements.

This exercise in drawing vectors, leads to a "proof" that the addition of vectors is commutative. The origin of each sequence of displacements is indicated by a small circle. The displacements are specified to three significant figures to indicate that, with care, students can get better than 1% accuracy with graphical methods.



d) Notice that the sum of these three displacements is independent of the order in which the displacements are added:

$$\vec{A} + \vec{B} + \vec{C} = \vec{A} + \vec{C} + \vec{B} = \vec{C} + \vec{B} + \vec{A}.$$

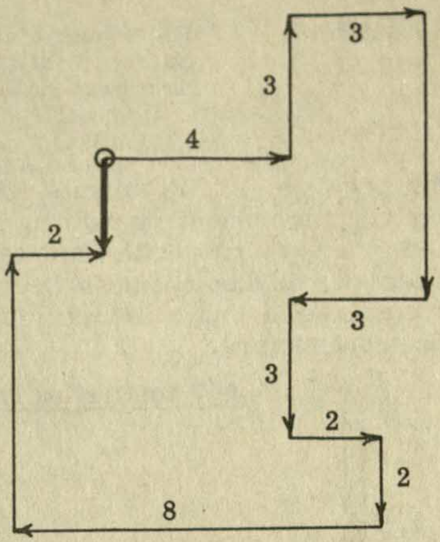
PROBLEM 2

A man follows this route: From his house he travels four blocks east, three blocks north, three blocks east, six blocks south, three blocks west, three blocks south, two blocks east, two blocks south, eight blocks west, six blocks north, and two blocks east. How far and in what direction will he be from home?

Starting to the right (east) from O, the route is as indicated in the diagram.

The man is now two blocks south of his starting point. His total displacement is two blocks south.

Finding the net E-W and N-S displacements is far simpler than adding the successive displacements. The mental addition would go something like this: East = 4 + 3 + 2 + 2 = 11, West = 3 + 8 = 11, so he is neither east or west of his starting point. N = 3 + 6 = 9, S = 6 + 3 + 2 = 11, so he is 2 blocks south of his starting point. This is a fine solution and should be mentioned when you discuss components, since it is essentially the component method of adding vectors.

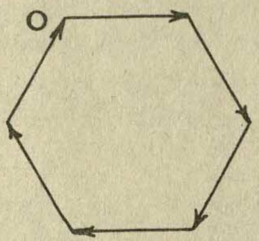


PROBLEM 3 A man walks along a path which is in the shape of a regular hexagon; each side is 10 ft long. Two of the sides run east and west.

(a) Describe his successive displacements.

(b) What is his total displacement?

a) For the hexagon oriented with two sides running east and west, successive displacements starting at O are:



1. 10 feet east
2. 10 feet 30° east of south
3. 10 feet 30° west of south
4. 10 feet west
5. 10 feet 30° west of north
6. 10 feet 30° east of north

b) His total displacement is zero.

PROBLEM 4 The minute hand of a watch is 2.2 cm long.

(a) What is the displacement of the tip of the minute hand between noon and 12:15?

(b) What is the displacement of the tip of the minute hand between noon and 12:30?

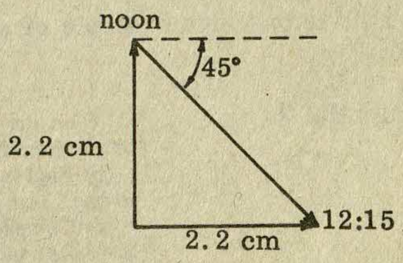
(c) What is the displacement of the tip of the minute hand between noon and 1:00?

Problems 4 and 5 set the stage for problems of circular motion. Both should be assigned.

a) For a watch held vertically, the noon to 12:15 displacement of the tip of the minute hand can be seen from the sketch to be $2.2 \times \sqrt{2} = 3.11$ cm. 45° below the horizontal.

b) 4.4 cm vertically down.

c) Zero.



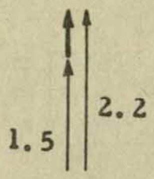
PROBLEM 5

For the watch of Problem 4 the hour hand is 1.5 cm long. What is the displacement of the tip of the minute hand from the tip of the hour hand:

(a) at noon? (c) at 6:00?
(b) at 3:00? (d) at 12:30?

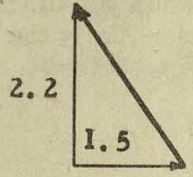
This problem, like Problem 4, gives a simple illustration of vector subtraction and sets the stage for planetary motion. Note the difference between the two problems. Problem 4 is concerned with displacements of a given point at different times. Problem 5 is concerned with displacements between two different points at a single time. This need not be emphasized, but it is probably worthwhile to note the two rather distinct uses of displacement vectors.

a)



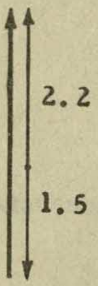
0.7 cm vertically up.

b)



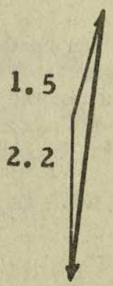
2.7 cm, 34° left of vertical.

c)



3.7 cm vertically up.

d)



This is best done by scale drawing. The answer is 3.6 cm, 6.2° left of vertically down.

PROBLEM 6

A horizontal vector 10.00 cm long is added to another vector 10.00 cm long pointing 70.0° up from the horizontal. Make as accurate a scale drawing as possible to determine the magnitude and direction of their sum. In your answer try for 0.1-mm precision in length and 0.1° precision in angle.

The graphical addition of these vectors shows their sum to be 16.38 cm, at 35° up from horizontal.

If the students try for 0.1 millimeter and 0.1 degree precision, they will probably come within 0.3% of the answer given. There is, of course, nothing wrong with using 20 cm to represent 10 cm. This gives greater precision and illustrates the scaling principle.

Competition between members of the class may produce a precision that will help you show how accurate a graphical solution can be.

This is only one example of many such problems which can be devised if there is time.

PROBLEM 7

(a) Find the result of adding a vector 2 cm east to one 3 cm northwest.

(b) Find the result of adding a vector 8 cm east to one 12 cm northwest.

(c) Compare the results of parts (a) and (b), and state a theorem about adding a pair of vectors which are multiples of another pair. Can you prove the theorem in general?

This problem leads to the idea that if $\vec{A} + \vec{B} = \vec{C}$ then $k\,\vec{A} = k\,\vec{B} = k\,\vec{C}$. It may also help the students understand scaling of vector drawings.

a) 2 cm east + 3 cm northwest = a vector 2.12 cm long, 3.3° west of north.

b) 8 cm east + 12 cm northwest = a vector 8.48 cm long, 3.3° west of north.

c) Notice the vectors of part (b)--including the sum--are four times the vectors of part (a). In general then, if $\vec{A} + \vec{B} = \vec{C}$, $k\,\vec{A} + k\,\vec{B} = k\,\vec{C}$. Both sides of a vector equation may be multiplied by the same constant--or scalar. A geometrical proof is started by noting that the vector sum $\vec{A} + \vec{B} = \vec{C}$ is represented by a triangle, and that the vector sum $k\,\vec{A} = k\,\vec{B} = \vec{D}$ is also represented by a triangle. Two of the sides of one of the triangles are proportional to two sides of the other triangle. The included angles are equal. It then follows that the triangles are similar and that the length of $\vec{D}$ is k times the length of $\vec{C}$ and makes the same angles with $k\,\vec{A}$ and $k\,\vec{B}$ as does $\vec{C}$ with $\vec{A}$ and $\vec{B}$. Hence $\vec{D} = k\,\vec{C}$.

PROBLEM 8 A plane flies southeast at 600 mi/hr. Draw a diagram of its displacement from its starting point after 0.50 hr, 1.00 hr, 1.50 hr, and 2.00 hr.

A velocity of 600 mph southeast means that the plane will fly 600 miles southeast in one hour, 300 miles in 1/2 hour, etc.

PROBLEM 9 The air speed of a plane is 480 km/hr. What is the ground speed of the plane
(a) with a 32 km/hr head wind?
(b) with a 32 km/hr tail wind?

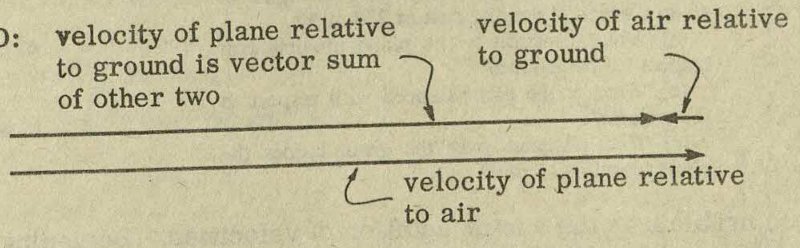
This problem lays the groundwork for more difficult ones on the addition of velocities. The cases of headwinds and tailwinds may be the only ones for which the students have much intuitive feeling, therefore this problem should be assigned and discussed.

a) The key words in the statement of this problem are "air speed". The plane flying 480 km/hr, into a 32 km/hr headwind, travels 480 - 32 = 448 km/hr relative to the ground.

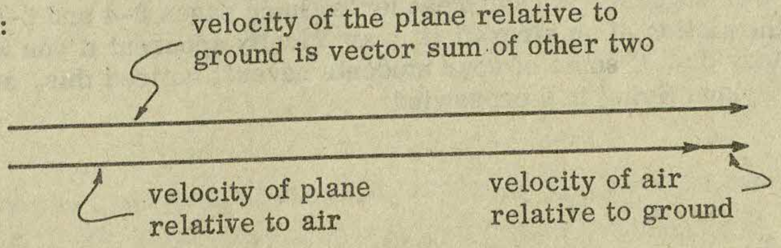
b) If the wind is a tailwind, the plane travels 480 + 32 = 512 km/hr relative to the ground, as shown in the sketches below.

It is essential that students understand this problem before working on problems such as numbers 11 and 12. It may be worthwhile to draw a vector diagram:

HEADWIND:


velocity of plane relative to ground is vector sum of other two
velocity of air relative to ground
velocity of plane relative to air

TAILWIND:


velocity of the plane relative to ground is vector sum of other two
velocity of plane relative to air
velocity of air relative to ground

PROBLEM 10

A group of hikers walking at an average rate of 5.0 km/hr walk the following path:

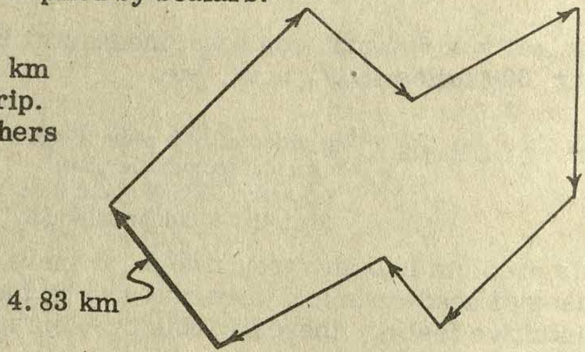
Compass Heading (Clockwise from north)	Time During Which Heading Is Maintained
45°	1.50 hr
130°	0.75 hr
60°	1.00 hr
180°	1.00 hr
225°	1.00 hr
325°	0.50 hr
240°	1.00 hr

(a) In order to return directly to their starting point, in what direction must they walk?

(b) How long will it take them to return?

This problem deals with the addition of displacements, but also subtly introduces the addition of vectors which have been multiplied by scalars.

Walking at a speed of 5.0 km/hr, the hikers walk 1.50 hours × 5.0 km/hr = 7.5 km 45° east of north on the first leg of their trip. This displacement is then plotted. The others are done similarly to give the trip shown:



a) and b) The final result is that they are 4.83 kilometers from home and must therefore walk 0.966 hours at a heading 318.5° clockwise from north in order to get straight home.

A graphical analysis of so many vectors must be done with great care, or cumulative errors are likely to lead to considerable inaccuracy.

PROBLEM 11

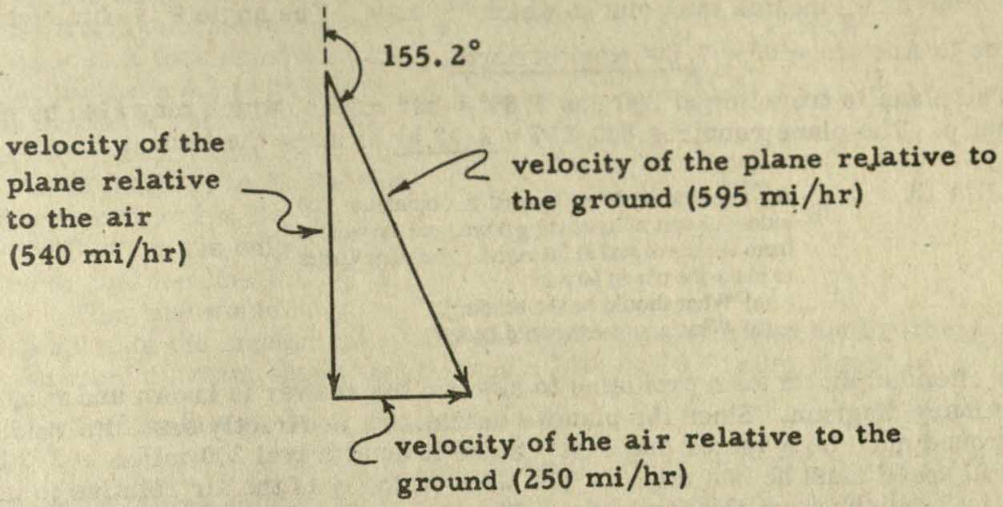
An airplane maintains a heading of due south at an airspeed of 540 mi/hr. It is flying through a jet stream which is moving east at 250 mi/hr.

(a) What direction is the plane moving with respect to the ground?

(b) What is the plane's speed with respect to the ground?

(c) What distance over the ground does the plane travel in 15 min?

This is a basic problem in the vector addition of velocities. Beginning students sometimes have difficulty in visualizing correctly the physical characteristics of this kind of problem. For suggestions, see Teacher's Guide pages 6-4 and 6-5. The effect of wind on the motion of an aircraft is dramatically apparent if you watch a slow plane on a windy day. If some of your students haven't noticed this, suggest they watch a small plane flying in a crosswind.


155.2°
velocity of the plane relative to the air (540 mi/hr)
velocity of the plane relative to the ground (595 mi/hr)
velocity of the air relative to the ground (250 mi/hr)

a) and b) The plane is flying south at 540 mi/hr, but is being blown east at 250 mi/hr. A graphical solution shows that these two motions cause the plane to travel at a heading 155.2° clockwise from north, at 595 mi/hr relative to the ground.

c) At 595 mi/hr the plane will cover 149 miles in 15 minutes.

PROBLEM 12

The pilot of an airplane which flies at an air speed of 300 mi/hr wishes to travel to a city 600 miles due north. There is a 40 mi/hr wind from the west.

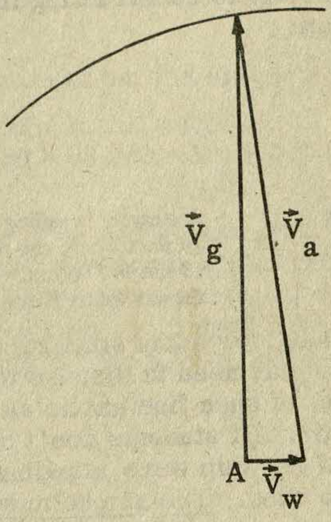
(a) What heading should the plane fly?

(b) How long does it take to make the trip?

A similar problem is analyzed in the boxed material on pages 78 and 79 of the text.

a) The velocity $\vec{V}_g$ of the plane relative to the ground is the vector sum of the velocity $\vec{V}_w$ of the air (wind) relative to the ground and the velocity $\vec{V}_a$ of the plane relative to the air.

From the starting point A, we know that $\vec{V}_g$ must go directly north, so this direction can be laid out from A. The wind velocity vector $\vec{V}_w$ is then drawn east from A with a length representing 40 miles per hour.



To determine the magnitude of the velocity vector $\vec{V}_g$, we must add the velocity vectors $\vec{V}_a$ and $\vec{V}_w$. $\vec{V}_a$ must start at the tip of $\vec{V}_w$ and the length of $\vec{V}_a$ must represent 300 mi/hr. Therefore we construct a circle with center at the tip of $\vec{V}_w$ and with radius representing 300 mi/hr. The intersection of this circle with the line indicating the direction of $\vec{V}_g$ locates the point at which $\vec{V}_a$ ends. The angle θ is obtained graphically, or is Arc sin 4/30 = 7.66° west of north.

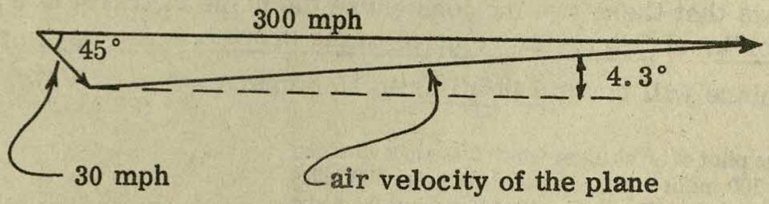
b) The plane is traveling at 300 cos 7.66° = 297 mi/hr, which may also be obtained graphically. The plane requires 600/297 = 2.02 hr to make the trip.

PROBLEM 13

An airplane is flying toward a destination 200 miles due east of its starting point, and the wind is from the northwest at 30 mi/hr. The pilot wishes to make the trip in 40 min.

(a) What should be the heading?
(b) What air speed should he fly?

It is often helpful in such problems to assume the answer is known and roughly sketch a preliminary diagram. Since the plane's destination is directly east, its velocity relative to the ground must be a vector due east. Since it must travel 200 miles in 2/3 hour its ground speed must be 300 mi/hr. The wind (velocity of the air relative to the ground) is 30 mi/hr from the northwest. The plane must obviously head somewhat north of east. The resulting diagram is drawn here:



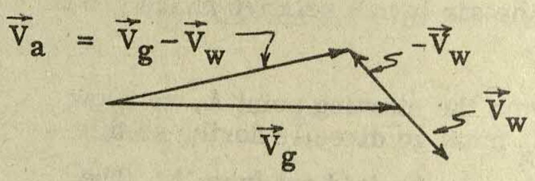
The final step is to make a careful drawing and find that the plane must fly 280 mi/hr, 4.3° north of east.

A more formal alternative:

$$\begin{pmatrix}\text{velocity of a plane}\\ \text{relative to ground}\end{pmatrix} = \begin{pmatrix}\text{velocity of a plane}\\ \text{relative to air}\end{pmatrix} + \begin{pmatrix}\text{velocity of air}\\ \text{relative to ground}\end{pmatrix}$$

or, $\vec{V}_g = \vec{V}_a + \vec{V}_w$. Hence, $\vec{V}_a = \vec{V}_g - \vec{V}_w$.

This leads directly to constructing the vector diagram:

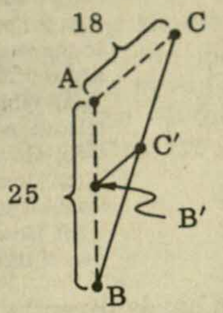


PROBLEM 14

A steamer is sailing directly south at 25 km/hr in an area where the wind is from the southwest at 18 km/hr. What is the angle from true north of the smoke trail from the stack?

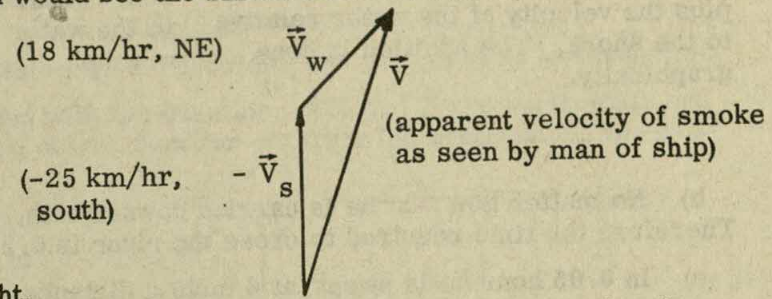
It will be hard for many students to understand what is happening physically in this problem. You may need to discuss what happens to smoke when it leaves the stack. Smoke consists of such fine particles that the smoke from the stack would stop immediately in still air. (If students don't believe it, let them try to throw some smoke!) Conversely, if the ship were standing still and a wind were blowing, the smoke would travel with the wind. The smoke moves with the air and has no motion of its own through the air.

Consider a ship which emits a puff of smoke at point A and then travels south to point B. Suppose it took an hour to go to B. Then the distance AB is 25 kilometers in our problem. Meanwhile the smoke puff is blown 18 kilometers northeast to C. If, halfway to B, the ship had blown off a second puff at B′, that puff would be at C′ when the ship is at B. It should be clear that the line B C′ C is a straight line. This line would be in fact the straight line of the smoke trail if the smoke had been emitted continuously. A carefully constructed diagram shows that the angle ABC is 18.6° east of north.



Another approach to this problem may be helpful.

If there were no wind, the man would see the smoke move backwards with a velocity equal to the negative of the ship's velocity: $-\vec{V}_s$. The effect of the wind is to carry off the smoke particles with a velocity $\vec{V}_w$; the total motion seen by the observer on the ship is thus: $\vec{V} = -\vec{V}_s + \vec{V}_w$. This vector sum is shown in the diagram at the right. Clearly this diagram is equivalent to the previous one, and when drawn to scale, the angle can be read off easily. It is left as an exercise for the reader to say why the apparent velocity seen by a man on the ship gives the direction of the smoke trail as seen in the earth's frame of reference.



PROBLEM 15

The small winged seed of a tall evergreen reaches a constant vertical component of velocity of 1 ft/sec almost immediately after it is blown from the cone. About how far from the base of a 180-ft tree on level ground would seeds from the top of the tree be carried by 40 mi/hr winds?

This problem can be handled as a vector velocity problem but the simplest attack is:

A seed falling 180 feet, at 1 ft/sec, reaches the ground in 180 seconds. A 40 mph wind is a 58 2/3 ft/sec wind. The seed is thus blown 180 × 58 2/3 = 10,560 ft or approximately 2 miles.

No one would expect each seed to leave the top of the tree and go exactly 2 miles-- even if the wind were exactly 40 mi/hr. Winds are gusty, and seeds differ rather widely. However, as an order of magnitude estimate, this may be pretty good. You would certainly guess that under some circumstances the tree could seed out to a one mile radius, and you would be surprised to find it seeding as far as ten miles, except under unusual circumstances involving large updrafts.

PROBLEM 16

A man rows a boat "across" a river at 4.0 mi/hr (i.e., the boat is always kept headed at right angles to the stream). The river is flowing at 6.0 mi/hr and is 0.20 mi across.

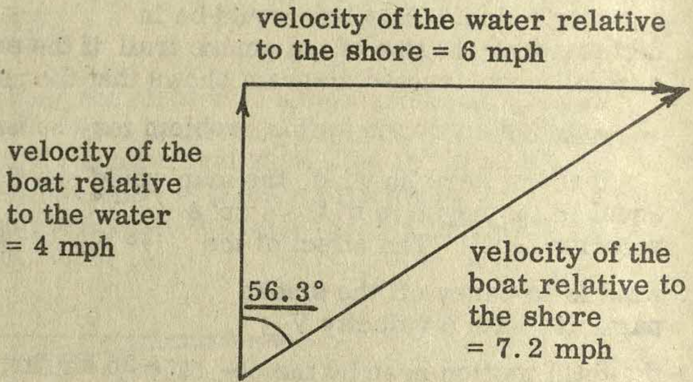
(a) What direction does his boat actually go relative to the shore?

(b) How long does it take him to cross the river?

(c) How far is his landing point downstream from his starting point?

(d) How long would it take him to cross the river if there were no current?

a) The man heads directly across the river, i.e., perpendicular to the banks. At the same time, the river carries him downstream. His velocity relative to the ground (shore) is his velocity relative to the water, plus the velocity of the water relative to the shore. The addition is done graphically.


velocity of the water relative to the shore = 6 mph
velocity of the boat relative to the water = 4 mph
56.3°
velocity of the boat relative to the shore = 7.2 mph

b) No matter how fast he is carried downstream, he goes across the river at 4 mph. Therefore the time required to cross the river is 0.2 miles ÷ 4 mph = 0.05 hours.

c) In 0.05 hour he is swept (at 6 mph) a distance of 0.3 miles downstream.

d) If there were no current he would still cross the river in 0.2 miles ÷ 4 mph = 0.05 hours since his motion across the river is unaffected by the perpendicular motion downstream.

PROBLEM 17

The weather bureau determines the height of cloud layers by measuring the angle of elevation to the point where the light of a vertical beam is reflected by clouds. The angle is measured at an observation station separated from the foot of the light beam by a base line.

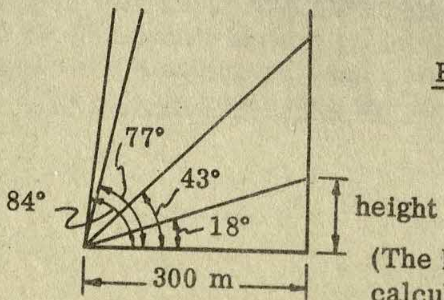
(a) If the base line is 300 meters in length, what is the altitude of the layers which are observed at 18°, 43°, and 77°?

(b) For observations at angles greater than 84°, the heights are classed as estimations. Why should they differ from readings at lesser angles?

(c) What is the height of a layer which reflects light at this 84° angle to the 300-meter base line?

This problem is not directly related to kinematics. It serves to give practice in graphical analysis of triangles. Students knowing trigonometry will solve this one more easily analytically.

a) and c) A scale drawing as shown will give cloud heights:



Angle of Elevation	Cloud Height
18°	97.5 m
43°	280 m
77°	1300 m
84°	2850 m

(The heights shown here were calculated by the formula $h = 300 \tan \theta$.)

b) Students doing this problem trigonometrically may have difficulty seeing why the heights are uncertain if the angle is greater than 84°. But they should see what a large change in height results from a 1° error in measuring the angle. Students solving the problem graphically will see readily that a very small mistake in angular measurement will cause a large error in height for such large angles. The weather bureau cannot make precise measurements of the angle. Their calculations are, therefore, subject to the same inaccuracy.

PROBLEM 18

What is the magnitude of a displacement whose components along the perpendicular X, Y, and Z axes are respectively 4.00 m, 2.50 m, and 8.50 m?

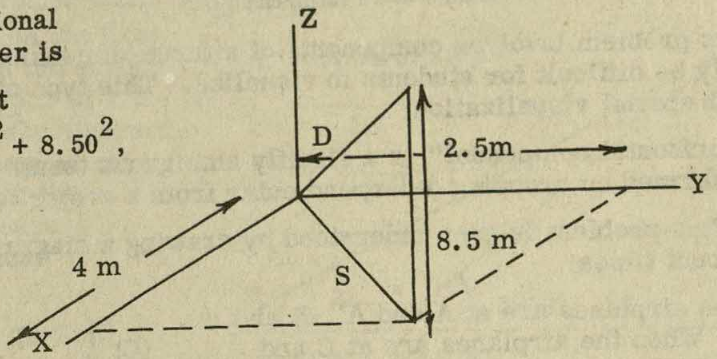
If the students know the three-dimensional Pythagorean theorem the answer is simply:

$$\text{Magnitude of displacement} = \sqrt{4.00^2 + 2.50^2 + 8.50^2}$$
$$= \underline{9.72\ \text{m}}.$$

If this requires proof, class discussion of this problem may include it as shown below.

This problem may be approached with the question: If X and Y are horizontal, and a ball is dropped from the displaced object, how far out from the origin would the ball hit?

By the familiar two-dimensional Pythagorean theorem, the answer is clearly $S = \sqrt{4.00^2 + 2.50^2}$. But $D^2 = S^2 + 8.50^2 = 4.00^2 + 2.50^2 + 8.50^2$, and so to the answer.



PROBLEM 19

At what range will a radar set show a plane flying at 10 km above the ground and at a distance on the map of 18 km from the radar station?

A distance of 18 km out and 10 km up gives a range (magnitude of displacement) of $\sqrt{18^2 + 10^2} = \underline{20.6\ \text{km}}$.

PROBLEM 20

An airplane takes off due south from an airport and maintains a ground speed of 4.0 km/min while climbing at a rate of 0.30 km/min. After 0.50 min, it turns due east, maintaining the same ground speed and the same rate of climb. At the end of 1.0 min after taking off:

(a) How high is the airplane?

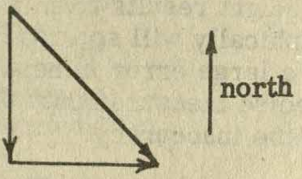
(b) What is its direction from the airport?

(c) How far is the point on the ground directly under the airplane from the point of take-off?

(d) How far is the airplane from the take-off point?

This rather difficult problem in three-dimensional displacements requires a considerable amount of spatial visualization. No formal procedure should be worked out for dealing with this type of problem.

a) If the airplane has climbed at the rate of 0.30 km/min for 1 minute, it is 0.30 km above the ground.



b) The airplane traveled at a constant velocity due south for half a minute and due east for half a minute. It therefore is exactly southeast of the airport.

c) In half a minute the airplane travels 2 km. It is then 2 km south and 2 km east of the airport or $\sqrt{2^2 + 2^2} = 2.83$ km from the airport.

d) The airplane is $\sqrt{2^2 + 2^2 + .3^2} = 2.84$ km from the take-off point.

PROBLEM 21

A plane flying north at 320 km/hr passes directly under another plane flying east at 260 km/hr.

(a) What is the horizontal component of the displacement of the second plane relative to the first 20 minutes after they pass each other? 50 minutes after they pass?

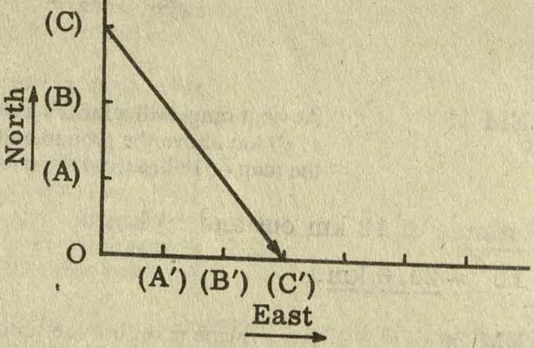
(b) What is the horizontal component of the velocity of the plane flying east relative to the plane flying north?

(c) Does the direction of this velocity vector (relative to the earth) change?

This problem involves components of a three-dimensional velocity vector which will probably be difficult for students to visualize. This type of problem is useful in developing such spatial visualization.

"Horizontal component" is a slightly ambiguous term. It means, in this case, the vector formed by dropping a perpendicular from a vector to a plane rather than a line.

a) This problem is best understood by drawing a diagram which shows the airplanes at different times:



The airplanes are at A and A′, B and B′ etc. When the airplanes are at C and C′, the displacement vector is as shown. Note that this is only the horizontal component. The real vector tilts up because the airplanes are at different heights. After 20 minutes, the distance OC is 320 km/hr × 1/3 hr = 320/3 km; OC′ is 260 × 1/3 = 260/3 km. The horizontal component of the displacement is

$$\sqrt{(320/3)^2 + (260/3)^2} = \sqrt{102400/9 + 67600/9}$$

$$= \sqrt{\frac{1.7 \times 10^5}{9}} = \sqrt{\frac{17 \times 10^4}{9}} = \frac{10^2}{3} \times \sqrt{17} = \frac{4.12 \times 10^2}{3} = 137 \text{ km.}$$

Similarly, after 50 minutes the horizontal component of the displacement is 344 km.

b) The horizontal component of the velocity vector is 137 km ÷ 1/3 hr = 411 km/hr. It is at 149° clockwise from north.

c) In the diagram above, the vectors AA′, BB′, and CC′ are all in the same direction. The relative velocity vector must therefore be in this same direction and does not change.

Notice that in this problem, the actual three-dimensional displacement vector of one airplane from the other is a very complicated thing indeed, changing its magnitude and direction in a very peculiar way. The horizontal component, however, is quite simple.

PROBLEM 22

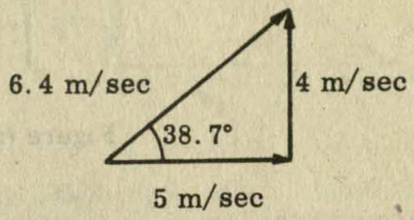
An ocean liner is traveling at 18 km/hr. A passenger on deck walks toward the rear of the ship at a rate of 4.0 m/sec. After walking 30 meters he turns right and walks at the same rate to the rail which is 12 meters from his turning point.

(a) What is his velocity relative to the water surface while walking to the rear? While walking toward the rail?

(b) Draw the displacement vectors relative to the water surface for his stroll. What was the total displacement from his starting point?

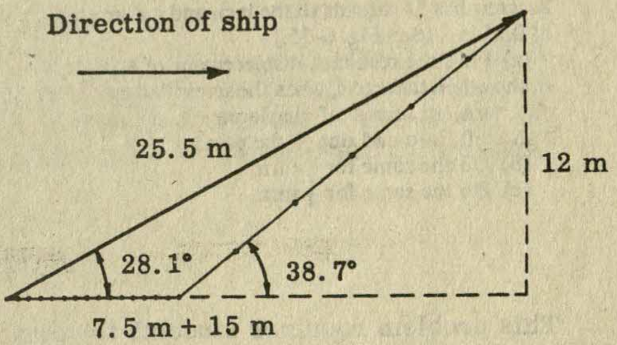
a) For the first part of his walk, the passenger is going back at 4 m/sec while the ship is going forward at 18 km/hr = 5 m/sec. Relative to the water he is traveling in the same direction as the ship at 1 m/sec.

He then walks 4 m/sec toward the side while, at the same time, the ship is carrying him forward 5 m/sec.



He moves $\sqrt{4^2 + 5^2}$ = 6.4 m/sec, 38.7° away from the ship's direction of travel.

b) He walks 7 1/2 seconds on the first part of his stroll, 3 seconds on the second. His positions relative to the water at 1/2 second intervals are shown in the diagram at the right. His total displacement of 25.5 m at an angle of 28.1° with the direction of the ship can be measured from the diagram. An analytical solution is the displacement = $\sqrt{(7\ 1/2\ \text{m} + 15\ \text{m})^2 + 12\ \text{m}^2}$ = 25.5 m at an angle with direction of ship = Arc tan 12 m/(7 1/2 m + 15 m) = 28.1°



PROBLEM 23

A navy pilot, in plotting his trip to rendezvous with his carrier, learns that he is 600 nautical miles due south of the carrier at the time his position is determined. The carrier is on a course 15° east of north and is making 25 knots (nautical miles per hour). The wind at his flight altitude is from 70° west of north at 40 knots. What must be his heading and air speed to make his rendezvous in just two hours' time?

In Figure (a) the plane is at P and the carrier is 600 nautical miles due north at C. During the next two hours, the carrier will move to R, 50 nautical miles 15° east of north from its present position. (The diagram has been distorted to show the small angles and vectors. Students will have to draw the diagram to a much larger scale.) In order to rendezvous with the carrier the plane will have to travel the course from P to R in two hours. The displacement $\overrightarrow{PR}$ will be the vector sum of two simultaneous displacements; a wind displacement $\vec{W}$, 80 nautical miles, 20° south of east and an unknown displacement of the plane through the air mass ($\overrightarrow{PR} = \vec{W} + \vec{X}$). The latter may be found by subtracting $\vec{W}$ from $\overrightarrow{PR}$ as in Figure (b). $\overrightarrow{PE}$ thus represents the required direction and the distance the plane must travel through the air mass in two hours: 680 nautical miles with a heading 5° west of north. Since 680 nautical miles must be covered in two hours, the required velocity is 340 nautical miles per hour.

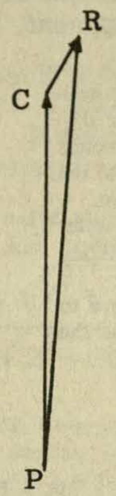


Figure (a)

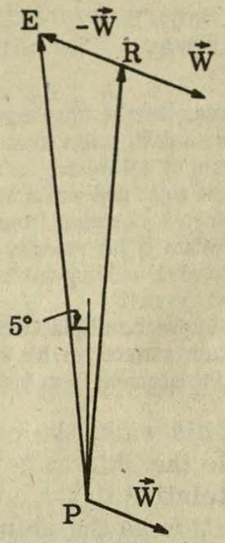


Figure (b)

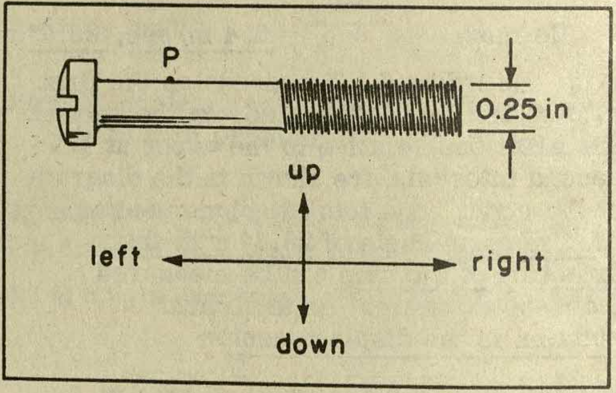
PROBLEM 24

A screw has 24 threads to the inch and a diameter of 0.25 in. (See Fig. 6–35.)

(a) Find the resultant displacement of a point, *P*, shown on the screw, when the screw "advances" one turn, in terms of displacement up, down, right, left, into and out of the paper.

(b) Do the same for ½ turn.

(c) Do the same for ¼ turn.



This problem requires students to visualize three-dimensional displacement vectors. It also illustrates the convenience of the component notation.

a) When the screw advances one turn, the point P is again up. However, it has gone 1/24″ to the right. This is its net displacement.

b) For 1/2 turn, the point P moves 1/48″ to the right and 1/4″ down. Note that these components completely specify the displacement. It is almost superfluous to say that the displacement is also $\sqrt{(1/48)^2 + (1/4)^2} = (1/48) \times \sqrt{145}$ inches, and 4.8° to the right of vertically down.

c) For 1/4 turn (and a right-handed screw), the point P moves 1/96″ to the right, 1/8″ down, and 1/8″ toward the viewer. Here, especially, note how easily the component notation specifies the displacement. It is also easy enough to say that the magnitude of its displacement is $\sqrt{\left(\frac{1}{8}\right)^2 + \left(\frac{1}{8}\right)^2 + \left(\frac{1}{96}\right)^2} = \frac{17}{96}$ inches, but describing the direction is more difficult.

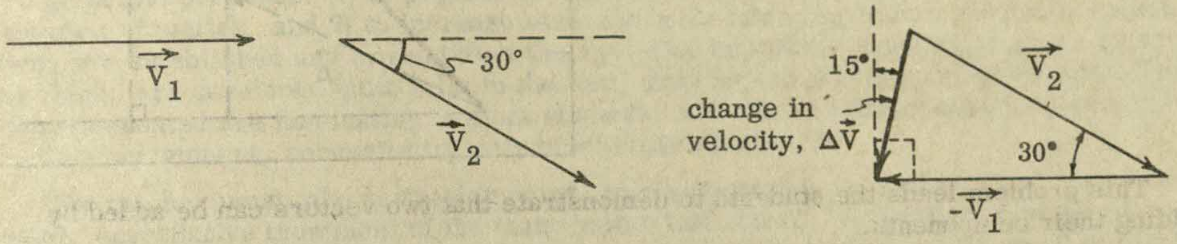
PROBLEM 25

An object moving in a circular path with a constant speed of 2.0 m/sec changes direction by 30° in 3.0 seconds.

(a) What is its change in velocity?

(b) What is its average acceleration during the 3.0 seconds?

a) Suppose that at time t = 0 the object is moving with the velocity shown in diagram (a) as $\vec{V}_1$. Three seconds later, (b), it is moving with the velocity $\vec{V}_2$ (if we assume clockwise motion). The change in velocity is then $\vec{V}_2 - \vec{V}_1$, shown in diagram (c).



(a) (b) (c)

From diagram (c), $1/2\ \Delta\vec{V} = 2 \sin 15° = 2 \times .2588$. $\Delta\vec{V} = 1.035$ m/sec. During the 3 sec interval the change in velocity is 1.035 m/sec. $\Delta\vec{V}$ will always be directed from the position of the object when it is at the mid-point of the interval toward the center of the circular path. Assuming clockwise motion and $\vec{V}_1$ horizontal toward the right, $\Delta\vec{V}$ will be directed 15° clockwise from vertically down.

b) Since the change in velocity occurred in 3 seconds, the velocity was changing at the rate of $1.035 \div 3 =$ 0.345 meters per second per second. This is the average acceleration during the 3 second interval. Like $\Delta\vec{V}$, the acceleration is directed from the position of the object when it is at the mid-point of the interval toward the center of the circular path.

PROBLEM 26

A watch has a second hand 2.0 cm long.

(a) Compute the speed of the tip of the second hand.

(b) What is the velocity of the tip of the second hand at 0.0 seconds? at 15 seconds?

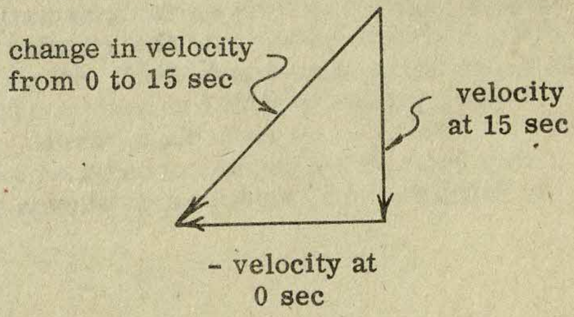
(c) Compute its change in velocity between 0.0 and 15 seconds.

(d) Compute its average vector acceleration between 0.0 and 15 seconds.

a) The tip of the second hand travels $2\pi \times \text{radius} = 2\pi \times 2 = 4\pi$ centimeters in 60 seconds. It therefore has a speed of $\pi/15 =$ 0.21 cm/sec.

b) Considering the watch is held vertically, the velocity at 0.0 sec is a vector 0.21 cm/sec, horizontally to the right; at the 15 second point, it is a vector 0.21 cm/sec vertically down.

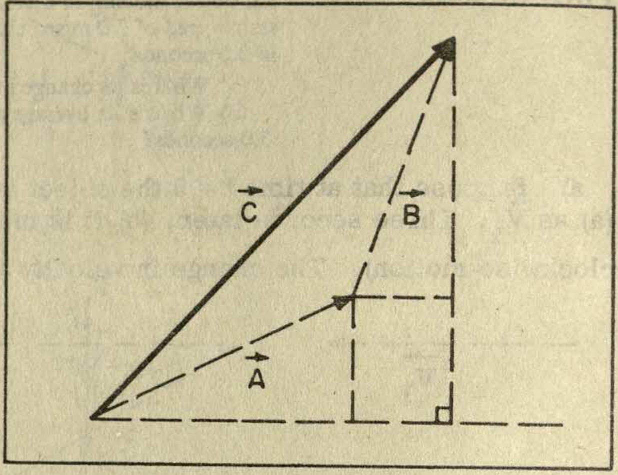
c) The change in velocity is the vector difference between the velocities at 15 and 0 seconds. It is a vector $0.21 \times \sqrt{2} =$ 0.30 cm/sec directed at 45° down and to the left.



d) The average vector acceleration is the change in velocity divided by the time interval in which this change took place, in this example 15 seconds. The average vector acceleration of the tip of the second hand is then $0.30 \div 15 =$ 0.02 cm/sec^2 down, and to the left at 45°.

PROBLEM 27

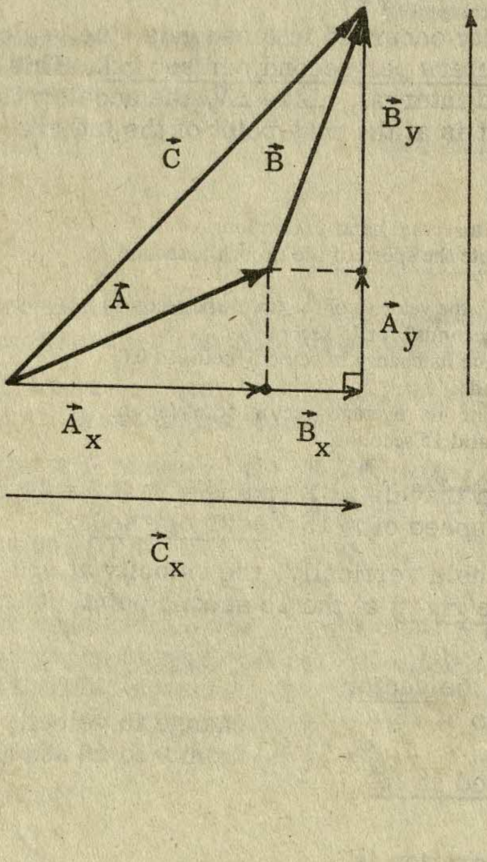
Indicate on Fig. 6-36, the components of *A* and *B* along each axis. Show that the sum of the component of *A* and the component of *B* along each axis is the component of *C* along that axis. Then notice that you can get *C* either by adding *A* and *B* directly or by adding the components of *A* + *B*.



This problem leads the students to demonstrate that two vectors can be added by adding their components.

The solution is almost given in Figure 6-36 of the text. Notice that the component of $\vec{A}$ along the horizontal axis is $\vec{A}_x$ and of $\vec{B}$, $\vec{B}_x$. $\vec{A}_x + \vec{B}_x$ give the vector $\vec{C}_x$ drawn here below the diagram. $\vec{C}_x$ is the horizontal component of $\vec{C}$.

$\vec{C} = \vec{A} + \vec{B}$, but notice also that $\vec{C} = \vec{A}_x + \vec{B}_x + \vec{A}_y + \vec{B}_y$.



Chapter 7 - Mass, Elements, and Atoms

Chapter 7 is the first of three chapters which present a brief but over-all view of some of the most important properties of matter, particularly of its constituent atoms as building blocks, and of the almost limitless variety of structures and properties resulting from combinations of atoms. Students who have studied chemistry will be familiar with many of the conclusions reached in these chapters. Other students may have superficial familiarity with some of the ideas. Very few students, no matter what their background, already will have a clear idea of why the conclusions presented are valid. Thus these chapters serve two principal purposes: 1) to provide an introductory picture of the structure and characteristics of matter, and 2) to increase students' understanding of how scientific conclusions are established and woven into a theory. The important concepts, and the evidence for them, are developed quite fully in the text, drawing from common experience. The ideas presented are fascinating to most students, and can be readily absorbed as a home reading assignment, necessitating only brief class discussion.

The teacher must take particular pains, in Chapters 7-9, to avoid undertaking a detailed, quantitative treatment of the many topics introduced. A detailed view is not warranted at this point. Many of the important concepts will be developed in greater detail in Parts III and IV after quantitative analytical tools have been developed. There are only a few facts to be "learned" in Chapter 7. A glance at Section 15 will provide a quick resume of the salient points which students are expected to appreciate as a result of studying this chapter.

Chapter 7 treats matter first from a macroscopic viewpoint by considering mass, density, and the conservation of mass. Attention is then focussed on the microscopic aspects of matter, first on the chemical and spectroscopic evidence for the elements, and then on various physical measurements which enable determination of the sizes of atoms and their regular arrangement in solids.

CHAPTER SUMMARY.

Part A - Mass (Sections 1-6) Mass as measured in an equal arm balance is shown to be a reasonable measure for comparing the quantity of matter in various bodies, since it is an unchanging property of a body; in contrast, weight varies with the location of a body. The density of an object, the ratio of its mass to its volume, is a particularly useful messure for comparing different materials, since density is independent of the size or shape of an object. The total mass in a given closed system is found to be conserved, despite physical or chemical changes which may occur within the system (for exactness, the mass equivalent of radiant energy must be included, usually as a negligible correction).

Part B - The Chemical Elements (Sections 7-9) Many materials can be decomposed chemically into simpler substances. The elements are those substances which cannot be further decomposed by chemical means. Individual elements exhibit unique spectra when they emit light; different sources of evidence, spectroscopic analysis and chemical methods, agree on the identification of the elements. Spectrum analysis is a simple but powerful tool for determining the atomic composition of various substances.

Part C - The Atom (Sections 10-15) Various kinds of evidence of the sizes of individual atoms and molecules are presented. The electron microscope and field-ion microscope permit precise determinations of the spacings between individual atoms in solids. Other evidence is furnished by the thickness of monolayers. Individual atomic particles which are emitted in radioactive disintegrations may be counted in bubble chambers, photographic emulsions, and scintillation counters. Atoms, while discrete, are seen to be so small that an extraordinary number of them are required to comprise a piece of visible matter, so many, in fact, that matter appears completely continuous upon ordinary examination.

SCHEDULING CHAPTER 7

As noted before, most of the material of this chapter should be treated as a reading assignment, and little time for class discussion need be scheduled. Class time is best devoted to discussion of experiments and solution of a few quantitative problems.

If your class is on a short schedule, this chapter (along with Chapters 8 and 9) should be treated principally as a reading assignment with a minimum of class discussion. In this case, your class discussion should be limited sharply to the central ideas and experiments and to the solution of a few problems. (For a good summary of the main ideas of Chapter 7, refer to Section 15, page 123, of the text.)

Many teachers who have been able to afford the time (something approximating the "long" recommended schedule), have concluded that the understanding of chemistry fundamentals reached by their students was well worth the time spent.

	10-week schedule for Part I			6-week schedule for Part I		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1-6	1	1	I-6	1/2	1	I-6
Secs. 7-9	1	1	I-7	0	0	-
Secs. 10-15	1	1	I-8	1/2	1	I-8

RELATED MATERIALS FOR CHAPTER 7

Laboratory. Experiment I-6 - Small Masses. In this experiment, students assemble and calibrate a "soda-straw" balance which permits accurate measurement of masses as small as that of a single human hair. Students will get insight into some of the design problems of instrumentation for measuring small masses. This experiment can be done after Section 2.

Experiment I-7 - The Spectra of Elements. In this experiment, students use a simple spectroscope to observe the characteristic spectra of a number of elements. This experiment should be done with Section 9.

Experiment I-8 - Molecular Layers. In this experiment, students make a direct measurement of the upper limit on the size of molecules, by determining the thickness of a monomolecular film. The experiment can be done any time after Section 9 is completed.

See the yellow pages for suggestions on these experiments.

Home, Desk and Lab. The following tables classify problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 7-11 ; detailed comments and solutions on page 7-13 to 7-28.

A. Matter and Mass

Section	Easy	Medium	Hard	Class Discussion
1, 2, 3	1, 4	2, 3*		1, 2*, 4
4, 5	5, 6, 8, 10	7, 9*		6, 7, 9
6		11		

B. The Chemical Elements

Section	Easy	Medium	Hard	Class Discussion	Home Project
7		1		1	
8	2, 5	3, 4		2, 3	
9					6

C. The Atom

Section	Easy	Medium	Hard	Class Discussion	Home Project
11	1*	3	6	1*, 3	
12		5	4*	4*	2
13	8*	7	9	7, 9	

Films. A film is planned on "Atoms and Spectra", pertinent to Part B of the chapter. This film is not available at the time this guide is being written. The film "Measuring Small Distances", while not directly pertinent to Chapter 7, shows some near-to-atomic distances as sensed with electron and field-emission microscopes. If you showed this film earlier, you can remind students that they saw a little of this kind of evidence when you are discussing the sizes of atoms.

A. Matter and Mass

Section 1 - Matter and Its Measure
Section 2 - Mass in the Balance
Section 3 - The Meaning of Mass
Section 4 - Density
Section 5 - Indirect Measurements of Mass
Section 6 - Conservation of Mass and Conservation of Matter

PURPOSE To introduce mass as a measure of the quantity of matter, to show how the mass of an object can be determined, and to give the masses of typical objects.

CONTENT a. The equal arm balance can be used to compare masses.

b. Mass is an unchanging property; in contrast, an object's weight depends on its location as well as on its mass.

c. The density of an object is the ratio of its mass to its volume; the concept of density is helpful in estimating masses.

d. Total mass is conserved; in rare cases, such as in nuclear reactions, the mass equivalent of energy must be included in the accounting of mass.

EMPHASIS This chapter is an introductory exploration of concepts that are useful in dealing with matter. We begin to develop the idea of mass. The fact that gravitational mass is defined by balancing should be the central idea. This is a simple refinement of crude notions about the quantity of matter. In Parts III and IV, students' ideas of mass (and energy) will be broadened.

The idea of density requires no great emphasis. It is simply a handy idea for comparing the masses of unit volumes of different samples of matter. Density is one convenient index to an aspect of the distribution of matter in space.

Classwork on these sections should be concerned principally with directing students' attention to the central ideas, and clarifying students' questions rather than extending the concept of mass (and density) beyond the level presented in the text.

DEVELOPMENT Class discussion of problems such as 1, 2, and 4 can be used to consider the meaning of mass. Problems such as 6, 7, and 9 can be used as a base for a discussion of density.

Using the concept of density to estimate masses may also be helpful. Some discussion of Table 2, page 101, should be interesting. The table below gives the densities of a few other substances.

Table of Densities

Light Elements

	Density gm/cm^3
Lithium	0.53
Sodium	1.5
Magnesium	1.7
Beryllium	1.85
Graphite	2.2
Diamond	3.5
Boron	2.65
Aluminum	2.7

Heavy Elements

Iron	7.9
Copper	8.9
Bismuth	9.8
Lead	11.3
Mercury (liquid)	13.6

Very Heavy Elements

	Density gm/cm^3
Gold	19.3
Platinum	21.4
Iridium	22.4
Osmium	22.5

Miscellaneous Substances

Liquid Hydrogen (-255° C)	0.07
Gasoline	0.68
Ethyl Alcohol	0.79
Ice	0.92
Sand	2.3
Rocks in Crust of Earth about	2.7
Woods (Oven Dry)	
Balsa	0.12 - 0.20
Ash	0.52 - 0.64
Beech	0.52 - 0.72
Cedar	0.31 - 0.70
Elm	0.55 - 0.66
Maple	0.50 - 0.68
Oak	0.65 - 0.98
Pine	0.37 - 0.64

Laboratory Experiment I-6 will be helpful in giving students a feeling for how small masses can be sensed.

Estimating Densities and Masses: If you want to extend your discussion of mass and density beyond the minimum, you can have some samples of different materials available, in simple geometric shapes if possible; it helps to have samples of equal volume. Have individual students lift these samples and estimate the densities; a reference sample of water will improve their ability to estimate. Let them check their estimates with a balance.

Density can be approximated by using water to balance the sample. Mark the volume of the sample, twice its volume, etc. on a beaker or graduated cylinder. (The volume of the sample can be determined by its displacement of water.) The level of the water whose mass is equal to that of the sample, is a direct visual indication of the sample density (in gm/cm^3). (Of course, the empty beaker must be balanced before the measurement begins. Using colored water will make the results easily visible.)

Students can become familiar with densities of common substances by trying to estimate the mass of large objects of known size. You might suggest common objects, or perhaps simplify the geometry by asking for the mass carried by a 10 m^3 truck filled with different materials. Interest can be enhanced by having the students suggest the materials. Let one student use a handbook in which to look up the densities of any suggested material.

Start with liquids and have the students find the mass of a given volume of water, alcohol, gasoline, etc. Then, try extremes such as liquid hydrogen or mercury. When you get to solids, you might begin with woods or rocks and then ask the students to suggest heavier materials. The most common suggestion for a heavy substance is lead; it is often a surprise to students that copper and silver are almost as dense, and gold and platinum are far more dense. The most dense metal is osmium.

Archimedes' Principle Many of your students probably will have met Archimedes' Principle, but if you have time, assign Problem 9, and discuss it in class. See the solution to Problem 9 in the green pages, and emphasize floating rather than submerged objects. A demonstration will help the students remember Archimedes' Principle.

If you are forced into a discussion of submerged objects, it would be best to speak of an apparent reduction in mass equal to the mass of the displaced water. Avoid any detailed discussion of forces at this time. If you have discussed submerged objects, you might mention the apparent reduction of mass of an object which is displacing air. (This amounts to 1.2×10^{-3} gm per cm^3; in precise measurements of mass a correction must be made for the buoyancy of air if the sample size is different from the volume of the standard masses, or "weights"). Challenge the better students to find the true mass of a block of aluminum (density 2.7 gm/cm^3) if the apparent mass determined on a balance with standard brass masses (density 8.4 gm/cm^3) is 8.4000 gm. (The aluminum displaces 3.1 cm^3 of air while the brass displaces only 1 cm^3; therefore a net 2.1 cm^3 of air has been displaced by aluminum corresponding to 2.5 mg. The actual mass of the aluminum block is therefore 8.4025 gm.)

COMMENT Mass vs. Weight Try to guard against the confusion between mass and weight without getting into a detailed discussion of forces or units. One way to emphasize this idea is to refer, after doing a problem, to the same problem on the moon, and ask which quantities are the same. Students are usually familiar with the idea that the weight of an object is much less on the moon than on the earth. The principal purpose of introducing mass at this stage is to establish one quantitative measure of matter. Weight is introduced because it is used in the simplest, intuitive determination of mass. (Problem 3 illustrates the distinction between mass and weight, and a brief discussion of this problem usually is enough to make the necessary distinction.)

This is clearly not the time to attempt a rigorous treatment of mass, which would involve forces, inertia, and energy all of which will be covered in detail in Part III. The only concepts which need be emphasized now are that there is a constant associated with each object, its gravitational mass, and that its weight depends on its gravitational mass and its position. These concepts can be covered quickly in a discussion of Problem 3.

Topics to Avoid (1) Newton's Law of Universal Gravitation, which shows the proportionality between weight and mass, should not be covered until it appears in the text in Part III.

(2) Inertial mass, which will be defined in Part III in terms of Newton's law of motion, should not be discussed now. The relation between inertial and gravitational mass is discussed in detail in the Guide to Part III.

(3) The Equivalence of Mass and Energy. Do not get specific about the implications of "total mass" as used in Section 6 to distinguish it from "matter". If some students ask you after class, you might tell them that the total mass is the sum of the mass which an object would have at rest (i.e., the rest mass) and the mass equivalent of the energy; all energy must be included: energy of motion, potential energy, and radiant energy. You can stress that the mass equivalent of energy is small, but it is best to defer quantitative statements or formulas until after energy has been defined in Part III.

Supplement on Mass and Energy If students ask, you can tell them that radiant energy, such as light or heat, does have gravitational mass. For example, light is bent in the presence of a very massive object such as a star.

The now famous formula, $E = mc^2$, expresses the equivalence of mass and energy. The constant of proportionality between them is the square of the speed of light, c^2. Mass is energy and energy is mass. The mass of a pail of water in which the water is at rest is less than the mass of the same pail of water with the water swirling around. However, no balance is sensitive enough to detect this difference.

In many chemical reactions, energy is released as the chemical changes occur. The mass (energy) which is thus carried away is indeed negligible. For example, one kilogram of TNT releases about 10^6 joules of energy when it explodes, but this corresponds to only

$$\Delta m = \frac{10^6 \text{ joules}}{(3 \times 10^8 \text{ m/sec})^2} \approx 10^{-11} \text{ kilograms.}$$

At this point in the course, the energy radiated as light or heat can be thought of as analagous to the gas that might escape from a chemical reaction. If the vessel is closed, no gas will escape; if the walls of the vessel were completely opaque, no radiant energy could escape, and the mass measured on a balance would remain constant. If energy does escape, as pointed out above, its mass is usually negligibly small. Even in fantastically energetic nuclear reactions that occur in the atomic bomb, only about 0.1% of the original mass is released as energy. If we could completely contain the nuclear explosion and all its products in a container on a balance, we would observe no change in mass.

There are some reactions in which there is 100% conversion of what we usually call matter into radiation. For example, a positron (positive electron) which has a mass of about 9×10^{-31} kg can combine with an ordinary (negative) electron whose mass is also 9×10^{-31} kg. The result is that both disappear (annihilate) and in their place appears pure energy equivalent to 18×10^{-31} kg (this is about 1 million electron volts of energy).

From the above discussion you will note that confusion will be avoided if you do not refer to the "conversion of mass to energy". Radiation (energy) has mass.

B. The Chemical Elements

Section 7 - Chemical Analysis and Synthesis
Section 8 - The Elements
Section 9 - Spectrum Analysis

PURPOSE To provide a bridge between the gross measure of matter, mass, and the microscopic building blocks, the atoms.

CONTENT a. Pure substances (i.e., substances with uniform properties) can undergo chemical reactions such as rearrangement, decomposition, and synthesis.

b. Chemical compounds are specific, unvarying combinations of elements; the compounds can be decomposed chemically whereas the elements cannot.

c. The analysis of spectra reinforces the conclusion, originally based on chemical evidence, that all matter is composed of a basic set of about 100 elements.

EMPHASIS This material should be read at home, but will not require any significant class time for development because students have probably been introduced to most of these ideas in earlier science courses. Treat briefly.

COMMENT Avoid Rigorous Definitions Many students will have seen oversimplified or circular definitions of the words "compounds", "elements", and "atoms". (For example, an element is sometimes defined as a pure substance which cannot be decomposed by chemical means; but the term "chemical means" is difficult to define without using the terms "elements" and "atoms".) Discourage the use of these definitions; by the time they have finished the course, the students will have a very clear idea of atoms and elements. At this stage of the course, the ideas as developed in the text are adequate, but they should not be memorized as though they give a complete picture.

The simplest modern definition of an atom is based on atomic structure and the known existence of an atomic nucleus, consisting of protons and neutrons. An atom is an atomic nucleus surrounded by a number of electrons equal to its number of protons. An element is made up of atoms each of which has the same number of protons in its nucleus.

The older definitions are circular because, to be correct, they must exclude nuclear changes. Thus, the terms "decomposable" or "by chemical means" are sufficiently restrictive only insofar as they explicitly exclude nuclear changes. For example, a standard chemical decomposition technique is heating, but extremely high temperatures (as are being sought for thermonuclear reactions) can produce nuclear changes and convert one atomic species into another.

In a similar way, the distinction between mixtures and compounds sometimes becomes fuzzy. Mixtures like salt water can be distinguished from simple chemical compounds such as salt or water which have unvarying chemical composition. But this distinction cannot be applied to very large molecules (such as protein molecules) which might contain millions of atoms. A few "mistakes" in the number or arrangement of the atoms does not necessarily change the chemical behavior of so complex a "compound".

Spectrum Analysis Do not dwell on spectrum analysis or its implications now. Students can observe some spectra and do Experiment I-7, but do not try to go beyond qualitative notions at this stage.

To give students an intuitive idea of a spectrum, the "spectroscopes" described in Section 9 are intentionally crude so that students are more likely to concentrate on the spectrum rather than on wondering about the details of the instrument. There will be ample opportunity in Part II of the course for students to understand how and why spectroscopes work.

In a similar way, the students should not be told now why elements emit discrete spectral lines or why these lines differ for different elements. The spectra can be mentioned qualitatively as a possible important clue to atomic structure. This clue will be picked up again and used in Part IV.

Bands occur in molecular spectra because of the interactions between the atomic nuclei which are superimposed on the basic interactions between the outermost electrons and the nuclei. This effect tends to "smear out" the discreteness of atomic energy levels. In solids, the electrons interact with many nuclei and the spectrum of emitted lines becomes almost continuous, resembling the spectrum of sunlight. The relative intensity of different colors in the spectrum depends more on the temperature of the solid than on the material from which the solid is made.

C. The Atom

Section 10 - Spelling Out Matter – Atoms
Section 11 - Seeing Atoms
Section 12 - More Evidence of Atoms
Section 13 - Counting Atomic Particles
Section 14 - The Crowds of Atoms
Section 15 - A Glance at the Chapter

PURPOSE To point out that atoms are the basic building blocks of matter, and to present a number of direct experimental observations which support the atomic model.

CONTENT a. Any kind of matter is merely a particular arrangement of the approximately 100 types of atoms.

b. Atomic size can be inferred from several types of measurements; the typical linear dimension of an atom is of the order of 2×10^{-10} m.

c. A count of radioactive particles can be used to confirm estimates of atomic spacings.

d. Although the atom forms a natural unit, the huge number of atoms in even small bits of material obscures the individual building blocks and makes matter seem continuous.

EMPHASIS This subject matter should be treated briefly. Students can get the general ideas from the text, and they can learn these better if they are not overwhelmed with more detail about experiments or other evidences pertaining to atoms.

Use the limited time you have to go over the calculations in one or two simple experiments (see DEVELOPMENT below). Supplement these examples with laboratory work or demonstrations.

COMMENT The two most important ideas presented in these sections are that matter is composed of atoms and that one can learn some of the properties of atoms from direct measurements. Most students can extract these ideas from the text, but some students will need help because they mistakenly will feel that they are expected to learn or understand more than is intended.

The best way to help the students is to go over the reasoning and calculations involved in either a monolayer experiment or in a radioactivity counting experiment.

This is definitely not the occasion for trying to convince students that these experiments (or additional facts you provide) necessarily prove the existence of atoms or give unchallengeable values for atomic sizes. Avoid introducing additional arguments and information to justify the deductions given in the text and problems; these invariably raise additional questions, and involve details of physics and chemistry which are too complex to introduce at this point.

It is best to begin a quantitative discussion assuming the existence of atoms (without making any special issue of this assumption) and then calculate atomic properties. Simple experiments and simple reasoning can lead to good estimates of atomic spacing. This spacing can be used to find the number of atoms in a given sample; the number of atoms together with the total mass then gives the mass of an atom. After the students have gone through these steps, they should have a working knowledge of atoms as the building blocks of matter, and a better appreciation for the reality of atoms than they could have acquired by more rigorous arguments.

Some students may be troubled by the idea of "seeing" atoms. They think of the term "see" in the usual sense, and they wonder what an atom looks like; some may even think that atoms look like the cork balls in Figure 7-16. If questions arise, make it clear that, while the evidence given by the electron microscope, the field ion microscope, X-rays, and monolayers is indirect, the fact that it is indirect makes it no less convincing. More direct information about atoms and atomic structure will be given in Part IV. In these sections of Chapter 7, there is no direct evidence about what an atom looks like; there is indirect, but interpretable evidence about the spacing of atoms. This spacing, we now know, is essentially the same as the atomic size, but the relation between spacing and size is not obvious to everyone, and some students may miss it.

DEVELOPMENT Two different types of experiments which give rather direct information about atoms, and which students can readily understand, are measurements of the thickness of monolayers, and counting radioactive decay particles.

A. The Thickness of Films. Students should do Experiment I-8, Molecular Layers. Class discussion of Problem 1 (see green pages for suggestions) can be used as a good prelude for this experiment. Suggestions for introducing the experiment itself will be found in the yellow pages on Experiment I-8. After completing the experiment, class discussion of Problem 4 can help in making sure that students get the full meaning of the experiment.

If you have time, you can show how to obtain information about molecular mass by doing Problem 5 in class. Note that the information given in the statement of Problem 5 implies additional types of experimental data. A film thickness experiment gives only one linear dimension, but the problem also gives the cross sectional area. This type of detail can often be deduced with the aid of X-rays. (If the molecule were assumed symmetric its cross section area would have been estimated as $(1.12 \times 10^{-9} \text{m})^2 = 1.24 \times 10^{-18} \text{m}^2$, which is about 2.7 times the actual cross sectional area. However, an error factor of 3 in the volume would not change the order of magnitude estimates.) Additional experimental evidence is also needed to determine that there are 54 atoms per molecule; this number can be determined using chemical techniques.

B. Atomic Size Deduced by Counting Atoms. Many students can understand Section 13 without help if they study it carefully, but, if you have time, an interesting class discussion can be organized around Problems 7, 8, and 9.

Problem 8 is the easiest and it can be used to motivate the other two. From the statement of 8, one learns the number of particles emitted by a milligram; Problems 7 and 9 go over, rather slowly, the reasoning used to find this number. Problem 7 shows clearly how one can learn what fraction of the emitted particles reach a given detector. Problem 9 shows how one can decide what fraction of the decays occur in a given time.

Note that even after these three problems have been completed, there are still unchecked assumptions used in the calculation. For example, one must assume that exactly one particle is emitted by each atom. There are nuclear radioactive decays in which several particles are emitted; there are others in which one particular type of particle is given off only by some of the atoms while other particles (which might not be detected in a particular experiment) are given off by the other atoms. We also assume that every emitted particle escapes from the source and travels a distance at least equal to the distance from the source to the detector. Further we assume that the detector registers every particle.

Once again, these assumptions can only be justified by other experiments. However, these auxiliary experiments should not be mentioned. The students should understand the assumptions in order to understand what they are calculating, but the assumptions need not be stressed. For example, the assumptions can be summarized by stating that (a) (exactly) one atomic particle is emitted by each atom and (b) each atomic particle reaching the detector is recorded.

Chapter 7 - Mass, Elements and Atoms

For Home, Desk and Lab – Answers to Problems

GENERAL Unless your schedule permits a fair amount of time, you will probably want to assign only a few of the problems in this chapter. Problem 9, while somewhat difficult, may be particularly intriguing to your class, since it involves the "discovery" of Archimedes' Principle, which is not treated elsewhere in the text.

The following tables classify the problems for Parts A, B, and C of this chapter according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the tables. Detailed solutions are given on pages 7-13 to 7-28.

A. Matter and Mass

Section	Easy	Medium	Hard	Class Discussion
1, 2, 3	1, 4	2, 3*		1, 2*, 4
4, 5	5, 6, 8, 10	7, 9*		6, 7, 9
6		11		

B. The Chemical Elements

Section	Easy	Medium	Hard	Class Discussion	Home Project
7		1		1	
8	2, 5	3, 4		2, 3	
9					6

C. The Atom

Section	Easy	Medium	Hard	Class Discussion	Home Project
11	1*	3	6	1*, 3	
12		5	4*	4*	2
13	8*	7	9	7, 9	

SHORT ANSWERS, Part A

1. (a) No.
   (b) Volume changes.
   (c) No.
2. See discussion on page 7-14.
   (a) Yes.
   (b) Use nugget as standard, calibrate rocks or any stable objects to make multiples and submultiples.

(c) Send the standard.

3. (a) Most in Greenland. Least in Canal Zone.
   (b) Canal Zone, 0.25% light; Denver, 0.11% light; Worcester, 0.03% light; Greenland, 0.19% heavy. Weight increases as latitude increases. Weight decreases as altitude increases.
   (c) No.

4. A is wrong; cracking a nut does not depend upon the pull of gravity.
   B could not do weighing because an equal arm balance does not work in the absence of gravity, but he could crack the walnut.
   C is right.

5. (a) 5.50×10^3 kg/m^3.
   (b) Presumably the lighter materials floated to the surface when the earth was molten.

6. 1.3×10^2 kg. Probably not.

7. (mass concrete + mass air) ÷ combined volume < density of water.

8. (a) 2×10^5 boxes.
   (b) 9×10^4 kg.
   (c) 7.0×10^2 kg/m^3.

9. Material	(a) Density, gm/cm^3	(b) Mass displ. water in gm
Cork	0.17	10
Wood	0.75	27
Sponge	0.50	20

(c) Fraction submerged in water = density of body.
    Mass of displaced water = mass of floating body.

10. (a) Compare weights of a number of pieces of paper of equal area.
    (b) Assume uniform thickness of paper.
    (c) Optical micrometer could be used to check thickness at different points.

11. (a) 0.889 kg.
    (b) 0.090 kg.
    (c) 0.270 kg.

SHORT ANSWERS, Part B

1. See discussion on page 7-20.

2. (a) No.
   (b) Possibly.
   (c) No.

3. (a) No.
   (b) No.
   (c) Zn and H_2 are elements. H Cl and Zn Cl_2 are compounds.

4. (a) Hydrogen is the lightest element listed.
   (b) Silicon atom has a mass about 1/2 as great as the mass of the iron atom.
   (c) No, there is no information about volume.

5. Water.

6. The light from different sources has different colors and intensities.

SHORT ANSWERS, Part C

1. Atoms of this size could be seen.
2. See discussion on page 7-23.
3. $V = 1.66 \times 10^{-29}$ m^3.
   $m = 4.48 \times 10^{-26}$ kg.
   $R = 1.27 \times 10^{-10}$ m.
4. (a) 0.45%.
   (b) 4.0×10^{-3} gm.
   (c) 8.0×10^{-5} gm.
   (d) 8.9×10^{-5} cm^3.
   (e) 1.1×10^{-7} cm.
   (f) Yes.
5. (a) 2.1×10^{18} molecules/mg.
   (b) 4.8×10^{-22} gm.
   (c) 8.9×10^{-24} gm.
   (d) 1.9×10^{-24} gm.
6. 8×10^{-24} m^3.
   10^6 atoms.
7. See discussion on page 7-26.
8. (a) 3×10^{-22} gm/atom.
   (b) 3×10^{-29} m^3/atom. Agrees with text computation.
9. (a) See graph on page 7-28.
   (b) 11%.
   (c) 16%.
   (d) 4 days.
   (e) 3×10^{17} particles. This agrees with the polonium count.

COMMENTS AND SOLUTIONS, Part A.

PROBLEM 1

When shelling peas you may find that 12 full pea pods yield 12 empty pods and 91 peas, that is 103 pieces in place of the original 12.
(a) Has the quantity of matter changed?
(b) Has the volume changed?
(c) Has the mass — as determined by weighing on a balance — changed?

This problem shows that mass is more suitable than volume as a measure of quantity of matter.

a) The quantity of matter has not changed, although the number of pieces has changed. The pieces have simply been rearranged in space.

b) The volume changes. How it changes probably depends on how the pods are handled in shelling! If the pods are opened carefully along an edge, and the peas removed, allowing the pods to retain their original shape, the total volume of peas plus pods would increase. If, on the other hand, the pods are ripped apart so that they can pack flat, the total volume of peas plus pods probably would be less.

c) There would be no change in mass as determined by weighing. One must assume that there are no changes such as water evaporation.

PROBLEM 2

Assume you are sent on a South American jungle expedition to analyze ore deposits. On arrival you find the laboratory equipment complete except for the standard masses. You do have a nugget of pure gold of unknown mass that you estimate to be about one kilogram.

(a) Could you use the nugget as a standard mass for the duration of your work?

(b) It is impractical to have masses sent to you and you are determined to complete the work. How would you improvise a set of masses that would give dependable results?

(c) Your quantitative results are to be mailed to the main laboratory in London. Obviously, the data in terms of your own standard mass would have limited meaning to them. How would you solve the problem of making the data completely useful for them?

This exercise can be used as the basis of a brief class discussion. Students (and teachers) may wonder why the intrepid explorer chooses to use a gold nugget rather than water as his temporary mass standard; but using water would make things too easy.

a) The nugget can be used as a standard mass. All "standard masses" are simply calibrated in terms of one another, and the actual "standard" is quite arbitrary. In this case, the explorer would simply define his nugget as his standard, equal perhaps to one nug. (He should probably not trust his guess that this is a kilogram.)

b) A set of working masses can easily be assembled. Search for rocks, etc., which will exactly balance the nugget (mass one nug), then for one to balance the nugget plus a one nug rock (mass 2 nugs), etc. In this way one could build up any desired sequence of masses greater than one nug. A useful arrangement would be 1, 2, 2, 5, 10, 20, 20, 50, 100, etc., which will give any mass between 1 nug and 200 nugs to an accuracy of one nug. If the explorer wishes greater accuracy than this, he will have to assemble a set of fractional nug masses, using a similar procedure (e.g., carefully divide a one nug mass into two equal parts, keeping all the pieces, until the two halves balance—a stone which will balance one half is then a standard 1/2 nug, etc.).

The above order of masses is the one commonly used for laboratory work. However, this is not the only satisfactory set. In fact, it would probably be somewhat easier to assemble a set to give a binary rather than a decimal order, such as 1, 2, 4, 8, 16, 32, 64, 128, etc. In this sequence, each new mass is simply the sum of all the smaller masses minus one unit.

c) The best way to avoid ambiguity in the interpretation of the quantitative results is to send a standard one nug mass to London along with the data. If the explorer were provided with a way of determining the actual mass of his standard (e.g., by precise measurement of the volume of pure water which would balance it), he would presumably have done this at the beginning of his measurements, and provided himself with a standard kilogram rather than a nug.

If the explorer is so stubborn as to refuse to part with one of his precious one nug masses, he could at least measure the mass of his data books in nugs before he shipped them off, and let the home lab use this mass to convert.

PROBLEM 3 The apparent pull of gravity varies with the altitude and the latitude. The relative amount of this pull at various places is indicated in the following table:

A standard station (sea level at 45° latitude)	1.000000
Canal Zone	0.997530
Denver, Colo.	0.998923
Worcester, Mass.	0.999652
Greenland	1.001906

(a) At which place would an object weigh the most? The least?

(b) By what per cent would each of these weights differ from the weight at the standard station? In general, what would you say about the variation in weight from place to place?

(c) Does the mass of the object change from place to place?

The purpose of this problem is to remind the student that mass is an unvarying measure of the quantity of matter, while the apparent pull of gravity on matter (weight) can vary.

a) Since weight is the apparent pull of gravity, inspection of the table shows that the object would weigh the most in Greenland where the pull of gravity is greatest. It would weigh the least in the Canal Zone.

b) The percentage difference in weights at each of the locations is as follows:

Canal Zone: $\frac{1.000000 - 0.997530}{1.000000} \times 100 = 0.2470\%$ (light)

Denver: 0.1077% (light)

Worcester: 0.0348% (light)

Greenland: 0.1906% (heavy)

The weight of an object varies from place to place. It appears to be greater at low altitudes than at high altitudes (Worcester and Denver are at about the same latitude, but Worcester is a few hundred feet above sea level and Denver is "the mile high city") and to be greater at a large degree of latitude than near the equator (compare Canal Zone and Greenland). Students may ask for an explanation of this variation in weight. At this stage it is probably best to say no more than that it is due to the combination of differences in distance from the center of the earth and to the rotational motion of the earth. An understanding of these factors should come through study of Part III (in Chapters 21 and 22) of the course.

c) The mass of the object does not change when it is moved from place to place. Mass can be measured by comparison with a standard on an equal arm balance. An equal arm balance that is balanced at one location will be balanced at all other places because any change in the earth's pull will be the same for both the standard mass and the object.

PROBLEM 4

The crewmen in a spaceship are discussing whether they could crack a walnut with a hammer. Man A says that hitting the walnut with the hammer would be no more effective than hitting it with a feather because the hammer is weightless out in space. B states that he found the mass of the hammer to be about one kilogram by comparing it with known masses on an equal arm balance; therefore, he could break the walnut as easily as back on earth. C maintains that the comparison with other masses proved nothing because out in space they too are weightless.

Comment on the statements of the crewmen. Can they crack the walnut?

This problem points up the distinction between mass and weight. Avoid getting in too deep here—the students do not yet have any firm ideas about energy or momentum. Of course, crewman C is correct, since B could not have "weighed" the hammer in space—it would have balanced as well with a feather as with a standard kilogram mass. However, B is probably right in assuming that he can use the hammer to crack the walnut, since the impact on the walnut depends on the mass (actually on the momentum, mv) of the hammer, not on its weight. Since A is completely wrong, and B is about half right, and C expressed no opinion on whether the walnut could be cracked, B and C should each get a half of the walnut as their reward!

PROBLEM 5

The mass of the earth has been calculated to be 5.98×10^{24} kg. The radius of the earth is 6.38×10^{6} m.

(a) What is the average density of the earth?

(b) Would you expect the average density of the material near the surface to be the same as the average density near the center of the earth? State your reasons.

a) Since density = mass/volume, the average density of the earth is

$$5.98 \times 10^{24}\ \text{kg} \div \frac{4}{3}\pi\,(6.38 \times 10^{6}\ \text{m})^{3} = \underline{5.50 \times 10^{3}\ \text{kg/m}^{3}}.$$

b) The average density of the material near the earth's surface is about half this value while that of the material near the earth's center is about twice this value. Presumably this segregation took place early in the history of the earth when the lighter constituents literally "floated" to the top of the molten sphere and the heavier constituents "sank" to the center.

Students who are interested in the density and structure of the earth will enjoy reading the first chapter of the Science Study Series monograph "How Old is the Earth" by Patrick M. Hurley. In particular, on pages 27 to 29, he states the density of the earth as a whole as 5.515 grams per cubic centimeter. Figure 4 on page 29 shows the densities at various depths.

PROBLEM 6

Cork is only one fourth as dense as water. Could you lift a sphere of cork one meter in diameter?

A simple problem in density determination with a bit of a surprise ending for students who think of cork as "light". A sphere of cork 1 meter in diameter would have a mass of

$$\frac{1}{4} \times \frac{4}{3}\pi\left(\frac{1}{2}\ \text{m}\right)^{3} \times 10^{3}\ \text{kg/m}^{3} = \underline{1.3 \times 10^{2}\ \text{kg}}.$$

In more familiar units, this "light" cork ball weighs some 290 pounds! Most students would have trouble lifting it.

PROBLEM 7

During World War I a number of cargo vessels were made of concrete. It is known that any object which floats must have an average density less than that of water. How can you account for the fact that these vessels floated?

This problem is concerned with density, but the discussion can get a bit sticky if the argument is permitted to run into excessive talk about the forces acting on a floating body. Concrete, of course, is more dense than water, and a block of concrete will literally "sink like a stone" if dropped into water. Concrete cargo vessels did not reach such an untimely end, as a usual occurrence, because they were filled with air, not water, and the average density of the ship (mass of concrete plus mass of air/ volume of concrete plus air) was less than that of water.

Caution - This is no time to discuss the details of buoyant forces or the stability of floating objects. Confine your discussion to the problem at hand or continue with a discussion of Problem 9. A possible variation on the above problem is to inquire why a sponge floats when first tossed into water, but eventually sinks.

PROBLEM 8

A quantity of confectioner's sugar fills a package 16 cm × 8.0 cm × 5.0 cm.

(a) How many packages can be placed in a freight car 22 m × 2.4 m × 2.4 m?

(b) If the mass of sugar in one of the small packages is 450 grams, what is the mass of sugar in the freight car?

(c) What is the density of the sugar?

a) The volume of each pack of sugar is

$$16\text{ cm} \times 8.0\text{ cm} \times 5.0\text{ cm} = 640\text{ cm}^3 = 6.4 \times 10^{-4}\text{ m}^3 .$$

The volume of the freight car is $22\text{ m} \times 2.4\text{ m} \times 2.4\text{ m} = 1.3 \times 10^2\text{ m}^3$.

Therefore, the car will hold $\dfrac{1.3 \times 10^2}{6.4 \times 10^{-4}} \approx \underline{2 \times 10^5\text{ boxes}}$.

b) The mass of sugar in the car is $2 \times 10^5 \times 0.45\text{ kg} = \underline{9 \times 10^4\text{ kg}}$.

c) The density of the sugar is $450\text{ gm}/640\text{ cm}^3 = 0.70\text{ gm/cm}^3 = \underline{7.0 \times 10^2\text{ kg/m}^3}$.

Students may worry excessively about whether or not the sugar boxes exactly "fit" into the freight car. For example, is 22 m/16 cm a whole number? According to the data given, this is not a fair question, since the length of the car is given only to the nearest meter, and may actually be 22.16 meters for all we know. Because of this we are concerned only with roughly how many times one volume is contained in the other.

PROBLEM 9

A student knows that dense objects sink in water and that less dense objects float. He undertakes to find out something about the relations involved. He carefully measures the mass, volume, and submerged volume of several floating objects with the following results:

	Volume	Mass	Submerged fraction
Cork	60 cm^3	10 gm	0.17
Wood	36 cm^3	27 gm	0.75
Sponge rubber	40 cm^3	20 gm	0.50

(a) What is the density of each material?

(b) What is the mass of the water displaced by each object?

(c) What general conclusions can you draw by comparing your computations with the original data? Your conclusions should be the same as those of Archimedes when he leaped from the bath shouting, "Eureka!"

This exercise affords an opportunity for students to "invent" Archimedes' Principle for themselves. It can serve well as a basis for class discussion.

a) and b) Students should be encouraged to construct the following table:

Material	Volume cm^3	Mass gm	Submerged fraction	(a) Density gm/cm^3	(b) Mass displ. water gm
Cork	60	10	0.17	$10/60 = 0.17$	$0.17 \times 60 \times 1 = 10$
Wood	36	27	0.75	$27/36 = 0.75$	$0.75 \times 36 \times 1 = 27$
Sponge	40	20	0.50	$20/40 = 0.50$	$0.50 \times 40 \times 1 = 20$

c) The general conclusions to be drawn are immediately apparent:

(1) A body which floats in water has a fraction of its volume submerged which is numerically equal to the density of the body in gm/cm^3 (i.e., the specific gravity of the body); or

(2) A body which floats in water displaces a mass of water equal to the mass of the body.

The second statement of Archimedes' Principle is probably a little better to emphasize. To show that it is the mass of the displaced water which enters, suggest that students quickly estimate the mass of the displaced fluid if the liquid were of density 2 gm/cm^3, or 10 gm/cm^3.

It is probably best not to get into the extension of this argument to buoyant forces on submerged objects. This will involve Pascal's Law and arguments about forces which would be too diversionary now.

PROBLEM 10

(a) How would you use a sensitive balance to determine whether the *local density* varies in a sheet of notebook paper?

(b) What assumption would you have to make about the paper?

(c) How could you check the correctness of this assumption? Could you use the optical micrometer to check?

a) The simplest method would be to cut a sheet of notebook paper into a number of pieces of equal area and compare the weights of the pieces.

b) This method would be based on the assumption that the paper was of uniform thickness.

c) This assumption could be checked by measuring the thickness at a number of different places with a micrometer caliper. The optical micrometer could be used.

PROBLEM 11

One kg of water is decomposed into the elements hydrogen and oxygen. The mass of hydrogen released is 111 grams.

(a) What mass of oxygen do you expect to be released?

When hydrogen is burned in air, water is formed. When 10 grams of hydrogen are burned, 350 grams of air must be used.

(b) What mass of water is formed? (Water always has the same proportions of hydrogen and oxygen.)

(c) What is the mass of the unburned air?

Note: If you do these experiments, the numbers computed from the conservation of mass agree with the numbers observed.

a) Since mass is conserved in chemical reactions the mass of oxygen released = mass of water - mass of hydrogen released = 1 kg - 0.111 kg = $\underline{0.889\text{ kg}}$.

b) Mass of hydrogen burned or released is proportional to the mass of water formed or decomposed.

$$\frac{10\text{ gm}}{111\text{ gm}} = \frac{\text{mass of water}}{1000\text{ gm}},\quad \text{mass of water} = \underline{90\text{ gm}}.$$

c) Mass of unburned air = mass of air - mass of oxygen used
= 350 gm - (90 gm - 10 gm) = $\underline{270\text{ gm}}$.

COMMENTS AND SOLUTIONS, Part B

PROBLEM 1

Three white crystalline substances — silicon dioxide (quartz), calcium chlorate, and sodium carbonate (washing soda) — are mixed together. It is desired to separate each of these substances from the others. Using the properties listed in the table, describe how you could do this.

Substance	Density (gm/cm^3)	Melting Point	Solubility (gm/liter) Cold Water	Hot Water	Alcohol
Silicon dioxide	2.65	1400°C	Insoluble	Insoluble	Insoluble
Calcium chlorate	2.71	100°C	1777.	Very soluble	Soluble
Sodium carbonate	1.44	32.5°C	215.2	4210	Insoluble

This is a good exercise for class discussion. There are a number of simple ways of separating the three materials, using the given physical properties. The differences in density could be used with liquids of intermediate density which did not dissolve any of the compounds (e.g., only the sodium carbonate would float on a liquid of density 1.5 gm/cm^3). The differences in melting point could also be used. If the whole mixture is heated, the sodium carbonate will melt above 32.5° and run off, while just above 100°C the calcium chlorate could be poured off. The simplest separation procedure involves the differences in solubility of the three compounds. If the mixture is poured into a large enough quantity of hot water to dissolve all of the sodium carbonate, the silicon dioxide can be removed by filtration since it is insoluble. The water can be evaporated and the remainder re-dissolved in alcohol. Since only the calcium chlorate is soluble in alcohol, the sodium carbonate can be removed by filtration. The calcium chlorate is finally recovered by evaporating the alcohol.

PROBLEM 2

Hydrochloric acid vapor — a pure substance — can be decomposed into two different gases each of which acts as a pure substance. On the basis of this evidence alone

(a) Can the original vapor be an element?

(b) Can either of the other two gases be an element?

(c) Can you be sure that any of the pure substances mentioned is an element?

a) The original vapor cannot be an element since it was possible to decompose it forming two other pure substances.

b) Yes, the other two gases could be elements.

c) From the evidence presented, we know that the hydrochloric acid was not an element, but we cannot be sure that the gases formed by decomposition are elements; it might be possible, by other means, to further decompose these gases into simpler substances. By further experiment we might be able to show that the gases produced in the decomposition of hydrochloric acid vapor are the elements hydrogen and chlorine. However, no simple chemical experiments could prove that decomposition is impossible. Our confidence that we now know elements is based on extensive chemical and physical evidence which enables us to order the elements in the periodic table.

PROBLEM 3

When zinc metal is put into hydrochloric acid, the metal is "eaten," a gas bubbles off and a "salt" is left behind.

(a) On the basis of this information alone can you be sure which of the pure substances mentioned are compounds and which are elements?

(b) We then find that the gas acts just like hydrogen gas — it has the same chemical and physical properties. Also we find that hydrochloric acid can be decomposed (see Problem 2) into hydrogen and chlorine. Can the "salt" be an element?

(c) Using the table of known elements (Chapter 32), which substances mentioned are elements and which are compounds?

a) On the basis of the information given in part (a) it is impossible to tell which substances are compounds and which are elements.

b) The salt cannot be a single element. The hydrogen must have come from the hydrochloric acid, and it is still necessary to account for the chlorine and zinc. Since only one substance other than hydrogen was formed, it (the salt) must contain zinc and chlorine and thus be a compound or mixture. (Actually, the salt zinc chloride is formed.)

However, from the given data, it is conceivable that the apparent salt is a mixture (rather than a chemical compound) of two or more elements.

c) Zinc and hydrogen are elements. Hydrochloric acid and the "salt", zinc chloride, are compounds.

PROBLEM 4

> By studying the information given in Table 3 can you decide
>
> (a) Why hydrogen makes up only 0.1 per cent of the total mass of the earth's crust, although in relative abundance it ranks fifth on the list?
>
> (b) Whether the mass of a silicon atom is greater or less than that of an iron atom?
>
> (c) Whether silicon is more or less dense than iron?

This problem is helpful in emphasizing the importance of atomic mass. The following answers are reasonable:

a) Because hydrogen contributes only slightly to the mass of the earth's crust, although of high relative abundance, it must be light. From the table it is clear that hydrogen is the lightest of the listed elements. In fact, it is the lightest of all elements.

b) The mass of the silicon atom is less than that of the iron atom because silicon has relatively more atoms for a given mass.

$$\frac{27.7}{33.9} < \frac{5.0}{3.1}$$

c) Nothing can be concluded about the relative densities of silicon and iron from this table. (Silicon happens to have a lower density and a smaller atomic mass, but density is a matter of the mass per unit volume, not the mass per atom.)

PROBLEM 5

> Table 4 suggests that a particular compound makes up the major portion of the human body. What is this compound?

Table 4 shows that the most numerous atoms in the human body are hydrogen and oxygen and that they are present in about the ratio of 2.5:1. Since the most common compound of hydrogen and oxygen is water, in which the ratio of hydrogen to oxygen atoms is 2:1, one might infer correctly that water makes up the major portion of the human body. Of course, the table alone does not prove that water is the major constituent of the body.

PROBLEM 6

> Project — Construct the three-color transparent screen described in the text, Section 7–9. Use your screen to examine the light from various sources such as an incandescent bulb, a neon sign, a sodium vapor lamp, a mercury vapor lamp, etc. What conclusions do you reach?

The student should not expect to make other than a gross determination of differences since it is difficult, using only the eye, to estimate the relative brightness of light, especially when the light is of different colors. For other than gross results the student could use a light meter or photographic film for the measurement of the intensity of light coming through the various colored filters. He could also use a prism or grating as described in Section 9 and could sketch the relative spacing of the spectral lines. From such a study he would have evidence that substances, when glowing in a gaseous state at low pressure, give combinations of colors and intensities that are distinctive. The incandescent bulb gives a continuous spectrum in which the blue part of the spectrum is of low intensity. This spectrum depends mostly on temperature. From these elementary experiments students cannot be expected to know that each element has a distinctive spectrum which provides a basis for the identification of elements.

COMMENTS AND SOLUTIONS, Part C

PROBLEM 1

Why are you sure there are more than 200 atoms in a spoonful of water? Use the evidence of your senses — not some numbers or sizes someone else tells you.

This is a good problem for introducing and setting the tone of the entire discussion on Part C. Instead of dwelling on proofs about the existence of atoms, the students can immediately direct their attention to a problem with which they can cope using data from their own experience.

You will probably get many different responses. The simplest might be that if there were only 200 particles per spoonful, the particles would be visible. Assume that a spoon has a volume of 5 cm^3; then each atom would occupy about $\frac{5\ cm^3}{200} = 2.5 \times 10^{-2}\ cm^3$ which corresponds to about a 0.3 cm linear dimension. Compare this with the size of droplets in fog. They are certainly much smaller. If anyone is unconvinced, challenge him to count the number of sugar crystals in a spoonful of granulated sugar.

Another argument, which will be helpful in discussing monolayers, depends on the area which can be covered by a spoonful of water. If there were only 200 atoms, the linear dimension of 0.3 cm would imply an atomic cross sectional area of about 0.09 cm^2 so that 200 atoms could only cover 18 cm^2. A spoon of water will wet a much larger area. You might demonstrate this by taking a single drop of water (which may vary from 0.02 to 0.1 cm^3) and pressing it between glass plates. If you move the plates around, the drop will easily wet 100 cm^2. Ask the students what lower limit this evidence sets on the number of atoms in a spoonful. They should realize that if the linear atomic dimension is t cm the volume of water covering 100 cm^2 must be at least 100 t cm^3, and that if the volume of the drop is 0.1 cm^3, t is equal to or less than 10^{-3} cm. This implies a volume per atom of about $10^{-9}\ cm^3$ or at least 5×10^9 atoms in a spoonful.

Of course, there are many, many more, and you can ask the students how they might find an increased lower limit to the number of atoms in a spoonful. You will have the students well on the way toward understanding monolayers if they suggest either surfaces other than glass over which the water might spread more easily or using smaller drops of water.

This problem serves as a good introduction to the monolayer problems, 4 and 5.

PROBLEM 2

A moist project (with a sensitive balance).

After estimating the diameter of several equal-sized soap bubbles, try to catch the material of the bubbles on a sheet of lightweight plastic such as Saran wrap. Weigh it and determine the average amount of material per bubble. Can you determine the order of magnitude of the thickness of the bubble wall? Compare this with the thickness of a monolayer.

Note: You will find the solution easier if you use the fact that the inside and outside areas of a bubble are very nearly equal and therefore the volume of bubble material is approximately given by either area times the thickness.

A laboratory exercise for some of your more ambitious students. Don't try this one with your students unless you have gone through it first, to make certain that your balance is sufficiently accurate to permit a quantitative result. Ordinary bubbles are much more than a monolayer in thickness, usually more of the order of magnitude of a wavelength of light (about 10,000 A, about 1000 monolayers).

PROBLEM 3

A cube of aluminum 4.30×10^{-2} m on an edge has a mass of 2.14×10^{-1} kg and contains 4.78×10^{24} atoms.

(a) What is the volume of the cube occupied by each atom?

(b) If the atoms are spherical and just fit inside the cube, what is the diameter of the atom in angstroms?

(c) What is the mass of each atom?

a) This problem is helpful in giving students more familiarity with typical values of atomic volume and mass. The total volume of the cube is

$$(4.30 \times 10^{-2})^3 = 7.95 \times 10^{-5}\ \mathrm{m}^3.$$

Thus, the volume per atom is

$$7.95 \times 10^{-5}/4.78 \times 10^{24} = \underline{1.66 \times 10^{-29}\ \mathrm{m}^3}.$$

b) The mass per atom is

$$2.14 \times 10^{-1}/4.78 \times 10^{24} = \underline{4.48 \times 10^{-26}\ \mathrm{kg}}.$$

c) The exact "radius" of each atom normally depends on the order of packing of the spheres. While this has implications in crystallography, how the atoms pack together is unimportant in calculating their approximate size, as the problem statement implies. It is sufficient for students to assume that the atom is a sphere which fits inside the calculated atomic volume ($1.66 \times 10^{-29}\ \mathrm{m}^3$), with a diameter equal to the length of an edge of the cube;

thus

$$2R = \sqrt[3]{1.66 \times 10^{-29}} = 2.55 \times 10^{-10}\ \mathrm{m};$$

$$R = \underline{1.27 \times 10^{-10}\ \mathrm{m}}.$$

PROBLEM 4

A student performs an experiment with monolayers. He mixes 5.0 cm³ of oleic acid with 95 cm³ of alcohol. He dilutes 5.0 cm³ of this solution with 50 cm³ more of alcohol.

(a) What is the per cent of oleic acid in the solution?

(b) What is the mass of oleic acid in one cm³ of solution? The density of oleic acid is 0.90 gm/cm³.

He now finds that 1.0 cm³ of solution in his laboratory dropper gives 50 drops. One drop of the solution on a water surface forms a monolayer film with an average diameter of 32 cm after the alcohol has dissolved into the water.

(c) What is the mass of the oleic acid in the film?

(d) What is the volume of the film?

(e) What is the thickness of the film?

(f) Can the thickness be obtained without knowing the density?

This problem can be used either to review or prepare for the monolayer experiment. It provides a good check on whether the students understand the experiment.

a) 5 cm^3 oleic acid in 100 cm^3 of solution = 1/20 oleic acid or 1cc in 20cc. If we take 5cc of this it will contain 5/20 = .25cc oleic acid. Mixing this 5cc with 50cc alcohol gives 55cc of solution which will contain .25cc oleic acid.

$$\frac{.25}{55} = \frac{1}{220} = \underline{0.45\% \text{ oleic acid}}$$

b) mass of oleic acid in 1 cm^3 of solution = 4.5×10^{-3} $cm^3 \times 0.90$ gm/cm^3

$$= \underline{4.0 \times 10^{-3} \text{ gm}}.$$

c) mass of oleic acid in drop = 4.0×10^{-3} gm/50 = $\underline{8.0 \times 10^{-5} \text{ gm}}$.

d) volume of film = mass of oleic acid in drop/density of oleic acid

$$= 8.0 \times 10^{-5} \text{ gm}/0.90 \text{ gm/cm}^3$$

$$= \underline{8.9 \times 10^{-5} \text{ cm}^3}.$$

e) thickness of film = volume of film/area of film

$$= 9.0 \times 10^{-5} \text{ cm}^3/\pi \times (16 \text{ cm})^2$$

$$= \underline{1.1 \times 10^{-7} \text{ cm}}.$$

f) The thickness can be determined directly without knowing the density of oleic acid, since the volume of the oleic acid in the film can be calculated directly:

volume of oleic acid in film = volume of oleic acid in drop of solution

$$= 4.5 \times 10^{-3} \text{ cm}^3/50$$

$$= 9.0 \times 10^{-5} \text{ cm}^3.$$

PROBLEM 5

The natural unit of oleic acid (a molecule of it) has a cross-sectional area of about 4.6×10^{-19} m^2 and a length of about 1.12×10^{-9} m. The density of oleic acid is 0.90 gm/cm^3.

(a) How many molecules are there in one milligram of oleic acid?

(b) What is the mass of one oleic acid molecule?

(c) This molecule is made of 54 atoms of different kinds. What is the average mass of these atoms?

(d) Give an upper limit for the mass of the lightest atom in oleic acid. (Make this limit as small as you can.)

This exercise extends the information obtainable in a monolayer experiment to include the mass per molecule and the mass per atom. It thus gives the students some more order of magnitude estimates; this time, of masses. In order to avoid confusion it is best to defer the consideration of mass until after the students have mastered the arguments about size which are used in Problem 4 and in the monolayer experiment.

a) 1 mg of oleic acid occupies a volume equal to this mass divided by the density 0.90 gm/cm^3.

$$\text{Volume of } 10^{-3} \text{ gm} = \frac{10^{-3} \text{ gm}}{0.90 \text{ gm/cm}^3} = 1.11 \times 10^{-3} \text{ cm}^3$$

But each molecule occupies a volume given by the product of its cross sectional area and its length.

$$\text{Volume occupied by 1 molecule} = (4.6 \times 10^{-19}\ \text{m}^2) \times (1.12 \times 10^{-9}\ \text{m}) = 5.2 \times 10^{-28}\ \text{m}^3$$

$$= 5.2 \times 10^{-28}\ \text{m}^3 \times \left(\frac{10^2\ \text{cm}}{1\ \text{m}}\right)^3 = 5.2 \times 10^{-22}\ \text{cm}^3$$

Therefore, the number of molecules per milligram is:

$$\text{Molecules per milligram} = \frac{\text{volume occupied by 1 mg}}{\text{volume occupied by 1 molecule}}$$

$$= \frac{1.1 \times 10^{-3}\ \text{cm}^3}{5.2 \times 10^{-22}\ \text{cm}^3} = \underline{2.1 \times 10^{18}\ \text{molecules/mg}}.$$

Some students may ask about whether one can always be sure that molecules or atoms always pack in so tightly that the volume of one unit times the number of units always gives the total volume (i.e., the students worry about possible empty spaces). If one were trying to do very precise work, the exact packing of units would be a problem. We do not want to worry about such fine details. This chapter and these problems are designed to give direct, uncomplicated indications of atomic parameters. For this purpose, the differences arising from packing factors are trivial.

b) The mass of the molecule can be determined directly from the answer to part (a). Since there are 2.1×10^{18} molecules in one milligram, each molecule must have a mass of:

$$\text{Mass of molecule} = \frac{1\ \text{mg}}{2.1 \times 10^{18}\ \text{molecules}} = 4.8 \times 10^{-19}\ \text{mg} = \underline{4.8 \times 10^{-22}\ \text{gm}}.$$

(Note that if an extra significant figure were carried from the beginning, there would be 2.14×10^{18} molecules each with a mass of 4.67×10^{-19} mg.)

c) The average mass of an atom is the mass of the molecule divided by the number of atoms:

$$\text{Average mass of atom} = \frac{4.8 \times 10^{-22}\ \text{gm}}{54} = \underline{8.9 \times 10^{-24}\ \text{gm}}.$$

d) The upper limit that can be set on the lightest atom in the oleic acid molecule depends on how much is known about the chemical formula of oleic acid. If nothing is known other than the fact given in the problem statement that there are 54 atoms of different kinds present, the upper limit of the lightest could not be set much below 8.9×10^{-24} gm.

If students assume that oleic acid is mostly carbon and hydrogen and that there are about 2 hydrogen atoms for each carbon, this implies that of the 54 atoms, 36 are hydrogen atoms. Using the fact that carbon has 12 times the mass of hydrogen, it can be argued that 2/(2 + 12) or 1/7th of the mass of the molecule is due to hydrogen. Then $\left(\frac{1}{7}\right) \times (4.8 \times 10^{-22}\ \text{gm}) = 6.9 \times 10^{-23}$ gm of mass are contributed by 36 hydrogen atoms or that the maximum mass of hydrogen would be

$$6.9 \times 10^{-23}\ \text{gm/36 atoms} \approx \underline{1.9 \times 10^{-24}\ \text{gm}}.$$

Note: If the students have studied chemistry and know the formula for oleic acid, they can calculate the mass of the hydrogen atom rather than merely giving an upper limit.

The atomic formula for oleic acid is $C_{18}H_{34}O_2$. The total atomic mass in atomic mass units (C = 12, H = 1, and 0 = 16) is $(18 \times 12) + (34 \times 1) + (2 \times 16) = 282$. Thus the molecular mass determination of this experiment, 4.8×10^{-22} gm, implies that one atomic mass unit is $4.8 \times 10^{-22}/282 = 1.9 \times 10^{-24}$ gm. (If more figures had been carried, this value would be 1.66×10^{-24} gm which is much closer to the actual value.)

PROBLEM 6

Fig. 7–15 shows a crystal of necrosis virus protein magnified about 8×10^4 times. Measure the diameters of several individual protein molecules and find the volume of one molecule. Use the known radius of a carbon atom, about 10^{-10} m, to find the order of magnitude of the number of atoms in the virus molecule.

This is an example of the "direct" measurement of the size of a fairly large molecule.

The last part of the question requires an assumption about the dominance of carbon atoms in a virus molecule. From the figure, the diameter of a molecule is about 2 mm. Taking a magnification of 8×10^4, the diameter of each molecule is about 2×10^{-3} m$/8 \times 10^4 \approx 2 \times 10^{-8}$ m. The volume of each molecule is then about $(2 \times 10^{-8})^3 = \underline{8 \times 10^{-24}\ \text{m}^3}$ (in this approximation it is sufficient to replace each little sphere with a cube of edge equal to the diameter). From the data given, the volume of a carbon atom is about $(2 \times 10^{-10}\ \text{m})^3 = 8 \times 10^{-30}\ \text{m}^3$. Thus, taking the spacing of atoms in a virus molecule as the spacing of close packed carbon atoms, the virus molecules each contain about $8 \times 10^{-24}/8 \times 10^{-30} = \underline{10^6\ \text{atoms}}$.

PROBLEM 7

In the description of scintillation counting it was stated that only one particle in every 10^9 sent out from the source reaches a square screen 10^{-4} meters on an edge at a distance of one meter. Prove that this statement is true if the source radiates equally in all directions.

If the particles are sent out in all directions, they are distributed uniformly over any sphere which is centered at the source. If we consider a sphere whose radius is 1 meter, its total surface area, $4\pi R^2$, is 4π meters^2. Thus the particles are distributed uniformly over 4π m^2, but the screen in this case has an area, A, of only $(10^{-4}\text{m})^2 = 10^{-8}$ m^2. Therefore, the fraction hitting A is $\frac{10^{-8}}{4\pi} = 0.8 \times 10^{-9} \approx \underline{10^{-9}}$.

In general, an area A at a radial distance R intercepts a fraction $\frac{A}{4\pi R^2}$ if the source sends particles in all directions. (Of course, A and R^2 must be measured in the same units.)

PROBLEM 8

From scintillation counting we know that about 3×10^{18} particles are emitted by a milligram of polonium in the course of complete decay.

(a) Assuming that one particle is given off by each atom, what is the mass of a polonium atom?

(b) Now, using the approximate density of polonium, 10 gm/cm³, find the volume occupied by a polonium atom. Check with the number computed in Section 7–13.

a) mass = 10^{-3} gm/3×10^{18} atoms = $\underline{3 \times 10^{-22} \text{ gm/atom}}$.

b) volume = mass/density

$$= \frac{3 \times 10^{-22} \text{ gm/atom}}{10 \text{ gm/cm}^3} = 3 \times 10^{-23} \text{ cm}^3\text{/atom} = \underline{3 \times 10^{-29} \text{ m}^3\text{/atom}}.$$

Within the accuracy of the data this result (3×10^{-29}) agrees with the value (3.5×10^{-29} m^3/atom) computed in Section 13.

PROBLEM 9

The rate of decay of a sample of radon gas is timed by counting its activity. On the first day, 0.1 milligram of radon emits 4.3×10^{16} particles. The following readings of per cent of original activity were found for successive days:

Days Elapsed	Per cent of Original Activity	Days Elapsed	Per cent of Original Activity
0	100	6	34
1	84	7	27
2	70	8	24
3	59	9	20
4	49	10	17
5	41		

(a) Plot the activity as a function of time.

(b) What fraction of the original quantity of gas would remain after twelve days have elapsed?

(c) What fraction of the radon changes in a day?

(d) How many days elapse before only half of the radon is left?

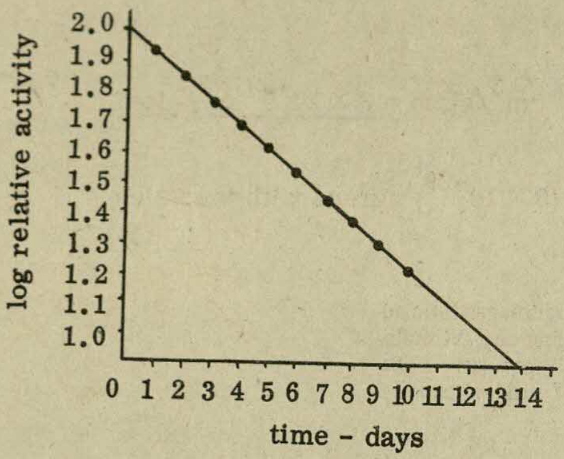
(e) What is the total number of particles that a 0.1-milligram sample of radon gives off? Compare this with the polonium count.

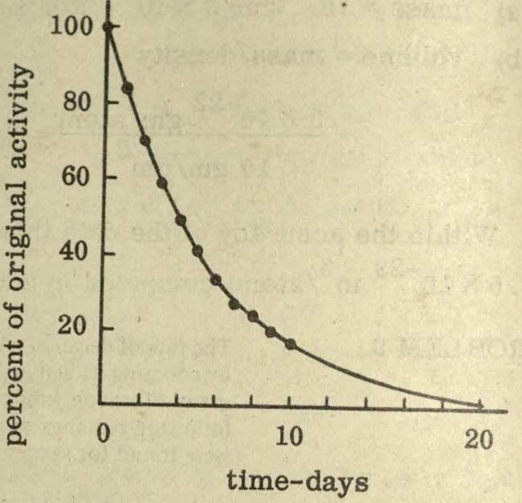
Note: The total number can be obtained by using this geometric series:

$$1 + f + f^2 + f^3 + \cdots = \frac{1}{1-f}$$

Here f is the ratio of the second day's activity to that of the first day. You can also approximate the total by using the area under the curve of activity vs. time.

a) The graph will be approximately linear if plotted on semi-log paper with activity plotted on the logarithmic scale and time on the linear scale (or plot log activity vs. time on linear paper). However, the total number of particles emitted cannot be approximated from the area under the curve on a log plot. A linear plot will be required to show this.





b) Reading from the graph, about 11% of the original gas would be left after 12 days, since the count rate of the remaining sample is about 11% of the initial count rate.

c) About 16% of the radon changes per day (100 - 84)/100; (84 - 70)/84; (70 -59)/70, etc.

d) Half the initial radon is left after about 4 days (4 day half-life).

e) The total number of particles given off in the decay is the sum $4.3 \times 10^{16} + (0.84)\,(4.3)\,10^{16} + (0.7)\,(4.3)\,10^{16} + (0.59)\,(4.3)\,10^{16} + \ldots$ This can be rewritten as

$$4.3 \times 10^{16}\ [1 + 0.84 + 0.7 + 0.59 + \ldots\ldots..]$$

But $0.7 = (0.84)^2$, $0.59 = (0.84)^3$ etc.; that is, every day the counting rate drops by a factor of 0.84. Therefore the total number of counts is

$$4.3 \times 10^{16} \quad \left[1 + 0.84 + (0.84)^2 + (0.84)^3 + \ldots\right]$$

But, from the formula given in the problem statement this is equal to

$$\frac{4.3 \times 10^{16}}{1 - 0.84} \approx \underline{3 \times 10^{17}}.$$

This total count is equal to the polonium count given on page 122 of the text. It is a good idea, particularly if this problem is done in class, to check the statement that the total number can be found by using the area under the curve (remembering that a linear plot is required to show this).

Chapter 8 - Atoms and Molecules

Using chemical and physical evidence, Chapter 8 considers the atomic and molecular nature of matter. Despite its manifold forms and characteristics, all matter is seen to be made up of about a hundred atoms. The chapter begins with a fairly extensive discussion of essentially conventional chemistry, which may have to be treated lightly unless considerable time is available for class discussion.

CHAPTER SUMMARY

The chapter divides naturally into three parts:

Sections 1 through 7 begin with the law of constant chemical composition and the law of multiple proportions. The problem of determining molecular formulas is then attacked with the aid of an actual count (from radioactive decay) of the number of gas atoms per unit volume. The observation that equal volumes of the measured gases contain equal numbers of molecules is then used as the basis of a hypothesis that this fact is true for all gases. Verification of this hypothesis then leads to the law of combining volumes, molecular and atomic masses, and Avogadro's number.

Sections 8 and 9 discuss isotopes and the inner structure of atoms. Attention is called to the fact that all atomic masses are (approximately) integral multiples of the mass of the lightest atom, hydrogen. On the basis of this fact it is presumed that all atoms are built from hydrogenic units.

Sections 10 through 16 discuss the wonderfully complex world of large atomic structures, beginning with relatively simple organic molecules, then to elementary biological entities, and finally to a consideration of the crystalline state of matter.

The development of these topics focuses both on some of our conclusions about the structure of matter and how these conclusions were reached. Through seeing something of how the structure of matter has been unraveled and where the current boundaries are, students should get a deeper understanding of the nature of the scientific quest.

SCHEDULING CHAPTER 8

Unless you are integrating this course with one in chemistry, you will probably want to let the text carry the burden of presenting the material of this chapter, utilizing one or two days of class discussion to highlight a few sections. If you wish to cover Sections 1-7 quantitatively, several days of class discussion should be devoted entirely to these sections, as noted in the 10-week schedule given below. Except for Avogadro's number and atomic dimensions, the material in Chapter 8 will not be needed later in the course; thus a very brief treatment may well suffice. In a short schedule, assign the entire chapter as a home reading assignment prior to class discussion. If at all possible, try to save a half hour for Alan Holden's fascinating movie "Crystals", which is described below.

Subject	10-week schedule for Part I	6-week schedule for Part I
	Class Periods	Class Periods
Secs. 1-7	4	0
Secs. 8, 9	1	1/2 - 1
Secs. 10-12	0	0
Secs. 13-16	1	1/2 - 1

RELATED MATERIALS FOR CHAPTER 8

Laboratory. While there are no laboratory experiments specifically designed for Chapter 8, many students will enjoy growing crystals as home activity, and should be encouraged to try it. "Crystals and Crystal Growing", by Alan Holden and Phylis Singer, is an excellent general description of crystals and their characteristics and supplies a great deal of detailed information on crystal growing. This book is one of the Science Study Series.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 8 - 9; detailed comments and solutions on page 8-10 to 8-17.

Section	Easy	Medium	Hard	Class Discussion
1, 2	2	1, 3*		1, 2, 3*
4, 5		5*	6	
6		4, 7, 8, 9*		7
7	10, 11*	14, 16*		11*
8	13		12	
11	15		17	17
12	18, 19			

Films. "Atoms and Chemistry", by Dr. Robert St. George of The Cambridge School. This film discusses and demonstrates the law of constant composition and the law of multiple proportions, by experiments centering about the electrolysis of water, the recombination of hydrogen and oxygen, and the quantitative decomposition of three nitrogen-oxygen compounds. Running time: 21 minutes.

"Crystals", by Dr. Alan Holden of the Bell Telephone Laboratories. This is a fascinating account of the nature of crystals, describing how crystals are formed and why they are shaped as they are. Actual growth of crystals is recorded under a microscope and recipes are given for growing crystals. Crystal phenomena are directly related to the concept of atoms. Running time: 25 minutes.

A film on "Molecules" is in preparation.

Section 1 - The Laws of Chemical Composition
Section 2 - The Problem of Determining Molecular Formulas
Section 3 - The Number of Particles in Gases
Section 4 - The Determination of Molecular Formulas
Section 5 - The Law of Combining Volumes
Section 6 - Molecular and Atomic Masses
Section 7 - Moles and Avogadro's Number

PURPOSE To summarize the chemical evidence for atoms and the determination of atomic masses.

CONTENT a. The law of constant chemical composition: Every pure sample of a chemical compound contains the same proportions by mass of all the elements into which it can be analyzed.

b. Law of multiple proportions: Whenever two elements combine in more than one way into different molecules, the ratio of the different masses of one element which combine with a given mass of the other is in the ratio of small whole numbers.

c. By counting atomic particles, one can find that, at room temperature and normal pressure, gases which can be measured by this technique have 2.5×10^{25} particles per m^3.

d. Avogadro's hypothesis states that at the same temperature and pressure, the number of molecules in equal volumes of gases is the same.

c. By comparing the masses of a particular element formed from the decomposition of equal volumes of different gaseous molecules containing that element, one can determine the number of atoms of the element in each of the molecules.

f. Law of combining volumes: When compounds are decomposed, the volumes of different gases produced under the same conditions of temperature and pressure bear whole number relations to each other. When compounds are produced by a chemical reaction between gases, the volumes of gases consumed also exhibit simple whole-number relations.

g. Our knowledge of the mass ratio of atoms together with our knowledge of the mass of the hydrogen atom (1.67×10^{-24} gm) enables us to find the mass of any atom.

h. The chemical atomic mass scale defines the mass of oxygen as exactly 16.0000; all other masses are given their values relative to this standard. The physical mass scale is defined by assigning a particular kind of oxygen (O^{16}) a value of exactly 16.0000.

i. Avogadro's number is defined as the number of O^{16} atoms in exactly 16 gms; it has the value of 6.025×10^{23}.

j. A mole of any identical object is 6.025×10^{23} of these objects. The mass in grams of one mole of any atom is numerically equal to the mass of the atom expressed in atomic mass units. A mole of any gas at atmospheric pressure and 0°C occupies 22.4 liters, and has a mass equal to the gram molecular mass of the gas molecule (equal to the gram atomic mass only for monovalent noble gases).

EMPHASIS Sections 1 through 7 briefly sumarize material which usually is taught in chemistry courses. If your students have had chemistry, you may want to go quickly over a few of the exercises for review; these students should find the exercises easy. If your students have not had chemistry, you cannot expect them to learn all of this material in the short time available. If you are rushed for time, you can treat all of these sections simply as a reading assignment. If your time is limited, spend it on Table 3, molecular formulas, and the meaning of a mole.

COMMENT Compounds and Mixtures. In most cases, the distinction between compounds and mixtures is clear. However, a precise determination of whether a given substance is a "compound" or a "mixture" may be far from simple. For one thing, it is very difficult to get an absolutely pure sample of anything. Impurities of one part per million or less are often not extractable.

Most of the metals in common use are mixtures rather than elements or compounds. There are a variety of steels each of which has, mixed with iron, a definite amount of carbon together with other elements such as manganese, cobalt, nickel, etc. Brass is copper and zinc, dural is aluminum and copper, "14 carat gold" contains gold and silver, bronze is copper and tin, and solder is tin and lead. The compositions of many of these may vary without changing the name. The physical properties of alloys vary in a fairly regular way with composition.

Compounds of Nitrogen and Oxygen

<table>
<tr><th>Chemical Name</th><th>Name</th><th>State at 20°C</th><th>Characteristics of Gas</th><th>Boiling Point °C</th><th>Characteristics of Liquid</th><th>Melting Point °C</th><th>Characteristics of Solid</th></tr>
<tr><td>N_2O</td><td>Nitrous oxide</td><td>gas</td><td>colorless
1.98 gm/liter</td><td>-89.5</td><td>colorless
density 1.23</td><td>-102.4</td><td>colorless
cubic crystal</td></tr>
<tr><td colspan="8">Produced by heating ammonium nitrate, $NH_4NO_3 \rightarrow 2\ H_2O + N_2O$
Dissolves in cream under pressure; when released it forms bubbles, like whipped cream. This is "laughing gas", used as general anesthetic.</td></tr>
<tr><td>NO</td><td>Nitric oxide</td><td>gas</td><td>colorless
1.34 gm/liter</td><td>-151.8</td><td>blue
density 1.27</td><td>-163.6</td><td>blue</td></tr>
<tr><td colspan="8">Produced by reduction of dilute nitric acid (6 Normal) with Cu or Hg. $3\ Cu + 8\ H^+ + 2\ NO_3^- \rightarrow 3\ Cu^{++} + 4\ H_2O + 2\ NO$
N_2 and NO_2 are produced in the same reaction.</td></tr>
<tr><td>N_2O_3</td><td>Dinitrogen trioxide</td><td>gas</td><td>red-brown</td><td>3.5</td><td>blue-liquid</td><td>-102</td><td>blue</td></tr>
<tr><td colspan="8">Can be formed by cooling a mixture of NO and NO_2. When dissolved in H_2O, forms nitrous acid: $N_2O_3 + H_2O \rightarrow 2\ HNO_2$</td></tr>
<tr><td>NO_2</td><td>Nitrogen dioxide</td><td>liquid</td><td>red</td><td rowspan="2">21.3</td><td rowspan="2"></td><td rowspan="2">-9.3</td><td rowspan="2"></td></tr>
<tr><td>N_2O_4</td><td>Dinitrogen tetroxide</td><td>liquid</td><td>colorless</td></tr>
<tr><td colspan="8">NO_2 and N_2O_4 always exist in equilibrium; they can be formed from concentrated HNO_3 and Cu</td></tr>
<tr><td>N_2O_5</td><td>Dinitrogen pentoxide</td><td>solid</td><td>decomposes</td><td>47</td><td></td><td>30</td><td>white
crystal</td></tr>
<tr><td colspan="8">Formed by dehydrating HNO_3 with P_2O_5; or oxidize NO_2 with ozone.</td></tr>
<tr><td>NO_3</td><td>Nitrogen trioxide</td><td>blue gas</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="8">Decomposes at ordinary temperature.</td></tr>
</table>

We mention alloys mainly because the students rarely encounter a pure metallic element in everyday life; the one common exception is copper, which is often used in pure form for electrical wiring. The main reason that pure metallic elements are not used is that they are usually quite weak mechanically. Adding other elements to form alloys increases the mechanical strength. However, adding other elements also increases the electrical resistivity, which explains why pure copper is used widely. (This increase in resistivity is so great that if the combination of mechanical strength and low electrical resistance is needed, strands of steel are generally woven together with strands of copper in making the wire, rather than using an alloy.)

The Compounds of Nitrogen and Oxygen. The classic example for the law of multiple proportions are the compounds of nitrogen and oxygen mentioned on page 127 and described in the table on page 8-4. (Some of these compounds are used in some of the demonstrations in the film “Atoms and Chemistry”.) These oxides can also be used to illustrate the variety of properties which can result from different simple compounds made of the same material. The properties of the oxides are given in the table.

Section 8 - Atomic Masses and Integers; Isotopes
Section 9 - The Innermost Structure of Matter

PURPOSE To suggest that atoms themselves are made of building blocks – protons and electrons (and neutrons).

CONTENT a. Atoms of any one element have the same chemistry and (nearly) the same optical spectrum; they may have different masses; in this case they are called different isotopes.

b. Any matter can be broken down to the element hydrogen. Hydrogen itself consists of a pair of natural units – a proton and an electron. All matter has one proton for each electron.

c. Mass is a count of the number of hydrogen units (proton plus electron) into which any piece of matter can ultimately be resolved.

EMPHASIS Students will meet the ideas of these sections again in Part IV after they have learned about electrical charge. If you are rushed, you can defer class discussion. However, this material is the logical climax of the atomic picture at this level. Atoms are the building blocks of matter and “hydrogen units” are the building blocks of atoms. Further, students will have many questions they would like to ask (only some of which you will want to answer). Thus, you should give some class time to this material.

COMMENT *A Simplified Picture of Atoms and Elements*. The discussion of hydrogen units as building blocks opens the way for a simple accurate description of atoms and elements, although these matters will be covered in Part IV.

An atom is an atomic nucleus surrounded by a number of electrons exactly equal to the number of protons in the nucleus. Chemical reactions involve sharing and rearrangement of the outermost electrons. For this reason atoms which have the same number of electrons (and therefore protons) have the same chemical properties, and therefore belong to the same element.

Isotopes of the same element differ only by the number of neutrons in the nucleus. (The neutron is similar to a tightly coupled proton and electron.)

The mass of any single isotope, expressed in atomic mass units, is almost exactly equal to a whole number. This number is the sum of neutrons and protons in the nucleus. For simplicity, the text assigns one hydrogen unit to either the neutron or the proton. Because the number of hydrogen units is (to within 1%) equal to the atomic mass, a count of the hydrogen units is a measure of matter which could be used instead of mass. In fact, since these units are the building blocks, they are a basic measure of matter.

(When the atomic mass of an element differs by more than 1%, from a whole number, this discrepancy is always due to a mixture of isotopes.)

Supplementary Notes on Nuclear Physics. Although this is certainly not the time to discuss any details of nuclear physics, the material in Appendix 2, at the back of this volume, may help you answer questions. None of the information in the Appendix is intended for class presentation. Rather than risk confusing many students, questions relating to this material should be answered outside of class.

Section 10 - Molecules — Structures and Properties
Section 11 - The Study of Organic Molecules
Section 12 - The Chemistry of Life

PURPOSE To show that physical and chemical properties of molecules are related to the atomic characteristics of the molecules; to indicate that organic molecules, despite their complexity, are composed of the fundamental building blocks of matter — atoms; and to extend the discussion of atoms and molecules to the structure of living things.

CONTENT a. The physical and chemical characteristics of molecules are related not only to the identities and numbers of atoms present, but to how the atoms are physically arranged.

b. Organic molecules, especially those encountered in living systems, are frequently of great complexity. However they, like simpler molecules, are built of atoms, and their structure is the key to their behavior.

EMPHASIS While this material is of great interest, it should not be carried much beyond the level presented in the text. Treat as a reading assignment.

COMMENT You may want to have The Physics and Chemistry of Life (A Scientific American Book, published by Simon and Schuster, New York 1955) in your class library. It has many reference works in its bibliography. Additional interesting articles appear constantly, in the Scientific American and other magazines. "The Physical Solution of Biological Problems" is one of the planned titles in the Science Study Series.

Section 13 - Molecules and Solids
Section 14 - Crystals: The Physics of the Solid State
Section 15 - The Geometry of Crystals
Section 16 - Order and Disorder

PURPOSE To introduce, briefly, the role of atoms in forming crystals and how crystalline structure affects properties of solid matter.

CONTENT a. When atoms or molecules take up their places in a solid crystal, the molecules and even atoms sometimes begin to lose their distinctiveness.

b. Most solids have very definite crystalline structures.

c. The elegant shapes of crystals, and the particular geometry of crystals depends on the way the individual atoms or molecules of a crystal arrange themselves; the local stacking (and not the shape of the individual atoms) determines the macroscopic crystalline shape.

d. Many physical properties are related to the crystalline structure (or to imperfections in it).

EMPHASIS The text will carry the subject matter well and little class time will be needed to explain the ideas. It will be far better to observe crystals, cleave them, and grow them than to talk about them.

If you possibly can, show your students the movie "Crystals", by Alan Holden (25 minutes). Suggest that students try growing crystals as suggested in the Science Study Series monograph "Crystals and Crystal Growing", by Alan Holden and Phylis Singer.

Chapter 8 - Atoms and Molecules

For Home, Desk and Lab – Answers to Problems

GENERAL Unless your class schedule permits a fairly extensive treatment of the material of this chapter, you will probably wish to assign only a few of the problems. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 8-10 to 8-17.

Section	Easy	Medium	Hard	Class Discussion
1, 2	2	1, 3*		1, 2, 3*
4, 5		5*	6	
6		4, 7, 8, 9*		7
7	10, 11*	14, 16*		11*
8	13		12	
11	15		17	17
12	18, 19			

SHORT ANSWERS

1. (a) No.
   (b) See discussion on page 8-10.
2. 1 carbon atom to 2 hydrogen atoms.
3. (a) Related by simple integers.
   (b) All except 2nd pair.
4. (a) 3×10^{-7} kg.
   (b) 6×10^{-9} kg.
5.

Sample No.	Mass of Oxygen (grams)	(a) No. of Oxygen atoms/molecule	Mass of Nitrogen (grams)	(b) No. of Nitrogen atoms/molecule	(c) Chemical Formula
(1)	1.6	1	2.8	2	N_2O
(2)	1.6	1	1.4	1	NO
(3)	4.8	3	2.8	2	N_2O_3
(4)	3.2	2	1.4	1	NO_2
(5)	4.8	3	1.4	1	NO_3

6. (a) $2\,NO + O_2 \rightarrow 2\,NO_2$.
   (b) 1/2 volume of oxygen.
   (c) One volume of NO_2.

7. (a) 18.0 gm.
   (b) 16.0 gm.
   (c) 35.5 gm.
   (d) 23.0 gm.
   (e) 1.0 gm.

8. (a) Carbon 1/4.
   Hydrogen 2/4.
   Oxygen 1/4.
   (b) $C_2H_4O_2$.

9. Cu_1Cl_2 and Cu_1Cl_1.

10. (a) 284 amu.
    (b) 4.71×10^{-22} gm.

11. 6.0×10^{23} molecules/mole.

12. (a) No.
    (b) 238.1025 amu.
    (c) No.

13. 63 amu.

14. 3×10^{-23} cm^3.

15. (a) 10^3 cm^2.
    (b) 10^7 cm^2.

16. (a) 18 gm.
    (b) 18 cm^3.
    (c) 3.0×10^{-23} cm^3.
    (d) 3.1×10^{-8} cm.

17. See discussion on page 8-16.

18. (a) 2.5×10^{13}.
    (b) 4×10^8.

19. 3×10^{16} atoms.

COMMENTS AND SOLUTIONS

PROBLEM 1

In sodium hydroxide, the combining masses are 23 grams of sodium to 16 grams of oxygen and 1 gram of hydrogen. In water, as you know, there are 8 grams of oxygen to 1 gram of hydrogen.

(a) If water were HO, as Dalton believed, could sodium hydroxide be NaOH?

(b) What possible pairs of chemical formulas for water and sodium hydroxide are suggested by these combining masses?

a) If water were HO, with a ratio 8:1, then sodium hydroxide, or any substance showing an oxygen to hydrogen ratio of 16:1 would have to contain twice as much oxygen per unit of hydrogen as water—e. g., HO_2, H_2O_4, etc. Thus, if water were HO, sodium hydroxide would be NaO_2H.

b) Possible pairs of formulas:

water	sodium hydroxide
HO	NaO_2H
HO_2	NaO_4H
H_2O	$NaOH$
H_3O	$Na_2O_2H_3$
H_2O_2	NaO_2H
H_2O_3	Na_2O_3H

Note that HO and H_2O_2 (or any simple multiples of a given proportion) could result in the same formula for sodium hydroxide. The above list is only fragmentary—obviously an infinite number of combinations are possible.

PROBLEM 2

Dalton determined the ratios of the masses of carbon and oxygen in carbon monoxide to be as 3/4, of hydrogen and oxygen in water as 1/8, and of carbon and hydrogen in the compound methane as 3/1. Without firm knowledge to the contrary, he assumed water molecules to contain only one hydrogen atom for each atom of oxygen and he asserted correctly the 1/1 ratio of carbon and oxygen atoms in carbon monoxide. What is the ratio of numbers of carbon to hydrogen atoms in methane as determined by these ideas?

If, following Dalton, water were HO, and carbon monoxide CO, methane would be $\underline{CH_2}$, since $2 = \frac{3}{4} \times \frac{8}{1} \times \frac{1}{3}$.

PROBLEM 3

Copper and chlorine combine in different proportions to form two compounds, No. 1 and No. 2, with relative masses of copper to chlorine 0.895 to 1 and 1.79 to 1 respectively.

(a) How does this example illustrate the law of multiple proportions?

(b) Which of the following pairs of formulas may be correct on the basis of the information given?

Compound 1		Compound 2
Cu_1Cl_1	and	Cu_2Cl_1
Cu_1Cl_1	and	Cu_1Cl_2
Cu_1Cl_2	and	Cu_1Cl_1
Cu_1Cl_3	and	Cu_2Cl_3

a) The masses of copper which combine with a unit quantity of chlorine in the two compounds have the simple integer ratio 2/1, i.e., 1.79/0.895 = 2/1.

b) The second pair ($CuCl$ and $CuCl_2$) is not possible. All others contain twice as much copper to chlorine in compound 2 as in compound 1 and are possible.

PROBLEM 4

In an experiment with polonium, over a two-year period a total of 9×10^{17} disintegrations occur, as measured by counting the flashes.

(a) What was the original mass of polonium?

(b) What mass of helium will be formed?

a) Section 3 states that when the activity of 10^{-7} kg of polonium is monitored for two years, 3×10^{17} particles are observed to be ejected. For 9×10^{17} disintegrations to occur in two years there must have been $\underline{3 \times 10^{-7}\text{ kg}}$ of polonium originally present.

b) 9×10^{17} helium particles are formed, each of mass $\frac{4\text{ gm}}{6.025 \times 10^{23}} = 6.6 \times 10^{-24}$ gm.

Mass of helium $= 9 \times 10^{17}$ particles $\times\, 6.6 \times 10^{-24}$ gm/particle $= 6 \times 10^{-6}$ gm $= \underline{6 \times 10^{-9}\text{ kg}}$.

PROBLEM 5

In a certain experiment you have samples of different gaseous compounds of oxygen and nitrogen; all have the same volume and the same temperature and pressure. The gases are decomposed and yield different masses of oxygen as shown in the following table:

	Mass of oxygen (grams)		Mass of oxygen (grams)
(1)	1.6	(4)	3.2
(2)	1.6	(5)	4.8
(3)	4.8		

(a) Give the most probable number of oxygen atoms in the molecule of each substance listed above.

To try to determine the complete formulas, you now measure the masses of nitrogen that result when equivalent samples are decomposed.

	Mass of nitrogen (grams)		Mass of nitrogen (grams)
(1)	2.8	(4)	1.4
(2)	1.4	(5)	1.4
(3)	2.8		

(b) What is the most likely number of nitrogen atoms in each molecule?

(c) Give the complete formula for each substance.

Since there were equal volumes of the various gases at the same temperature and pressure, each volume of gas has the same number of molecules. The number of atoms of oxygen per molecule for these gases is proportional to the masses of oxygen obtained by decomposition. The same is true for nitrogen.

		(a)		(b)	(c)
	Mass of oxygen (grams)	Probable no. of oxygen atoms per molecule	Mass of nitrogen (grams)	Probable no. of nitrogen atoms per molecule	Chemical formula
(1)	1.6	1	2.8	2	N_2O
(2)	1.6	1	1.4	1	NO
(3)	4.8	3	2.8	2	N_2O_3
(4)	3.2	2	1.4	1	NO_2
(5)	4.8	3	1.4	1	NO_3

PROBLEM 6

Nitric oxide (NO) will combine with oxygen from the air to form nitrogen dioxide (NO_2).

(a) Write the equation for this reaction.

(b) If one volume of NO is consumed, what volume of O_2 is consumed?

(c) What volume of NO_2 is obtained at the same pressure and temperature?

Students who have not studied chemistry may need help with this problem.

a) $2\,NO + O_2 \rightarrow 2NO_2$.

b) If one volume of NO is consumed, 1/2 volume of O_2 is consumed, since the volumes have the same ratios as the number of molecules in the formula.

c) If one volume of NO is consumed, one volume of NO_2 is obtained at the same temperature and pressure.

PROBLEM 7

Forty grams of sodium hydroxide (NaOH) react to completion with 36.5 grams of hydrochloric acid (HCl) producing water (H_2O) and common salt (NaCl). After evaporating the water, we find 58.5 grams of salt are left.

(a) Using the law of conservation of mass, compute the number of grams of water that was formed.

(b) Whenever we decompose water we find that the ratio of the masses of hydrogen and oxygen in it is as 1 to 8. Compute the mass of oxygen that was contained in our original materials.

(c) We decompose some hydrochloric acid and find that the masses of hydrogen and chlorine are in the ratio of 2 to 71. How much chlorine was there in the hydrogen chloride that we used to make salt?

(d) How many grams of sodium were there in the sodium hydroxide we started with?

(c) How much hydrogen was contained in the sodium hydroxide?

a) Since initial mass = final mass,
40 gm (NaOH) + 36.5 gm (HCl) = X gm (H_2O) + 58.5 gm (NaCl), X = 18.0 gm.

b) The initial material must have contained the same amount of oxygen as 18.0 gm of H_2O (all of the oxygen is contained in the H_2O after the reaction is complete). Since the ratio of the mass of oxygen to that of hydrogen in water is 8:1, we must have

$$\frac{8}{8+1} \times 18.0 \text{ gm} = \underline{16.0 \text{ gm}} \text{ oxygen.}$$

c) Since the ratio of H to Cl in HCl is 2:71, the initial mass of Cl must be

$$\frac{71}{71+2} \times 36.5 \text{ gm} = \underline{35.5 \text{ gm}} \text{ Cl.}$$

d) Mass Na = mass NaCl - mass Cl = 58.5 gm - 35.5 gm = 23.0 gm Na.

e) Mass H in NaOH = mass NaOH - mass Na - mass O
= 40.0 gm - 23.0 gm - 16.0 gm = 1.0 gm H.

PROBLEM 8

A student carefully analyzes an organic acid and finds the following combining weights of the constituent elements: Carbon 6, Hydrogen 1, Oxygen 8.

(a) Knowing the atomic masses of these elements, indicate the relative numbers of atoms of each that occur in a molecule of the compound.

(b) Organic acids usually contain the carboxyl group COOH. Assuming that there is one such group per molecule, what is the formula of a molecule of this acid?

a)

atom	atomic mass	rel. no. of atoms
carbon	12	1/4
hydrogen	1	2/4
oxygen	16	1/4

b) Since the COOH group contains 2 O atoms, the acid must contain 2 C atoms and 4 H atoms; thus the formula would be CH_3COOH, or $C_2H_4O_2$.

PROBLEM 9

The relative atomic masses of copper and chlorine are 63.6 amu and 35.5 amu. Which of the pairs of formulas in problem 3 (b) are correct in the light of this information?

The ratio of 0.895 to 1 (relative mass of copper to chlorine) is the same as 63.6 to (2 × 35.5) and the ratio of 1.79 to 1 is the same as 63.6 to 35.5. Therefore the third pair of formulas (Cu_1Cl_2 and Cu_1Cl_1) is possible but the other pairs are not possible.

PROBLEM 10

(a) Find the approximate mass in atomic mass units of a molecule of stearic acid ($C_{17}H_{35}COOH$).

(b) Using Avogadro's number find the mass of the molecule in grams.

a) The molecular weight = $(18 \times 12) + (36 \times 1) + (16 \times 2) = \underline{284 \text{ amu}}$.

b) The mass of a molecule = $284 \text{ amu} \div 6.025 \times 10^{23}$ amu/gm

$$= \underline{4.71 \times 10^{-22} \text{ gm}}.$$

PROBLEM 11

The results of a monolayer experiment show that there are 2.1×10^{21} particles per gram of stearic acid (molecular mass 284 amu). Calculate the value of Avogadro's number from this data.

This problem is a good follow-up for the laboratory experiment with monolayers. If there are 2.1×10^{21} particles of molecular weight 284 amu per gm, then in 1 mole (284 gm), there would be

$$284 \text{ gm/mole} \times 2.1 \times 10^{21} \text{ molecules/gm} = \underline{6.0 \times 10^{23} \text{ molecule/mole}}.$$

PROBLEM 12

Uranium undergoes radioactive disintegration forming an entirely different element which may be separated from the parent element by chemical means. This new element, thorium, has different chemical properties, and would exhibit a much lower density if sufficiently large amounts of it could be obtained. Its atomic mass is less by approximately 4 amu, the amount contributed to the other product of each disintegration — one helium atom.

Within a short time, and at a very predictable rate, this thorium undergoes two more disintegration reactions. No perceptible mass change accompanies these reactions, but the eventual product is a material having chemical properties which are indistinguishable from our first material, uranium. This newest element likewise exhibits the general physical properties of uranium, but we know it has lost 4 units of mass and has not, in either of the last two reactions, been able to recover any mass.

(a) In the light of this evidence, can uranium in nature consist of a single isotope?

(b) What is the mass, in amu, of natural uranium which consists of 0.72 per cent of U^{235} whose mass is 235.1156 amu, and 99.28 per cent of U^{238} whose mass is 238.1242 amu?

(c) Can these be the two isotopes involved in the radioactive process described above?

This problem emphasizes the meaning of isotopes. If you assign this problem use only the given information. This is not the time to get into a discussion of radioactive series.

a) Natural uranium must consist of at least two isotopes, the original atom and the new atom described which is 4 mass units smaller.

b) Mass U = $0.0072 \times 235.1156 + 0.9928 \times 238.1242 = \underline{238.1025\text{ amu}}$.

c) No. U^{235} is lighter than U^{238} by 3 amu, while the isotope described is lighter by 4 amu and hence must be U^{234}.

PROBLEM 13

The atomic mass of copper is 63.54, and it is known to consist of a mixture of two stable isotopes. If the one of these which constitutes approximately 30 per cent of copper has a mass of approximately 65 amu, what is the atomic mass of the other isotope to the nearest whole number of amu?

Since $0.30 \times 65 + 0.70\, X = 63.54$, $X \approx 63$. The other isotope must be Cu^{63}, i.e., its atomic mass is 63 amu.

PROBLEM 14

The density of lead is 11.34 gm/cm³ and the mass of a lead atom is about 207 amu. Calculate the effective volume of each atom in this solid state.

Since 207 gm of lead contain 6.025×10^{23} atoms, 1 cm³ (11.34 gm) must contain $\left(\frac{11.34}{207}\right) \times 6.025 \times 10^{23} = 3.3 \times 10^{22}$ atoms. Therefore, the volume per atom is $1\text{ cm}^3/3.3 \times 10^{22} \approx \underline{3 \times 10^{-23}\text{ cm}^3}$.

PROBLEM 15

When an oil film on water forms colored patterns, we know it must be at least 3 to 5 × 10^{-4} cm thick. The patterns of color are indicative of varying thicknesses.

(a) Assuming an average thickness throughout of 10^{-3} cm, what area will 1 cm³ of oil on the troubled waters cover?

(b) Oleic acid molecules are of the order of magnitude of 10^{-7} cm in length. What area of water could be covered by 1 cm³ of this material if it spreads to a monolayer?

a) Since volume = area × thickness, 1 cm³ of oil of thickness 10^{-3} cm will cover an area of $\underline{10^3\text{ cm}^2}$.

b) Similarly, a monolayer of oleic acid of volume 1 cm³ would cover an area of $\underline{10^7\text{ cm}^2}$.

PROBLEM 16 ____ (a) What is the mass in grams of a mole of water?
(b) What is the volume of this mass of water?
(c) Find the volume occupied by one water molecule.
(d) The atom of oxygen in this molecule occupies about half of the volume. What is the approximate diameter of an oxygen atom?

a) The chemical formula of water is H_2O and thus its total atomic mass from Table 3, page 133, is 18. The mass in grams of a mole of water is numerically equal to the mass in amu or 18 gm. $(2 \times 1 \text{ gm}) + (1 \times 16 \text{ gm}) = \underline{18 \text{ gm}}$.

b) If the density of water is assumed to be one gram per cm^3, the volume of a mole of water is $\underline{18 \text{ cm}^3}$.

c) Since 6.025×10^{23} molecules occupy 18 cm^3, the volume of a water molecule $= 18 \text{ cm}^3/6.025 \times 10^{23}$ molecules $= \underline{3.0 \times 10^{-23} \text{ cm}^3}$.

d) The oxygen atom would occupy $1.5 \times 10^{-23} \text{ cm}^3$. Volume of a sphere $= \frac{4}{3} \pi R^3$ and $R = \frac{1}{2} D$. Hence, $V = \frac{\pi D^3}{6}$ and $D^3 = \frac{6 V}{\pi}$.

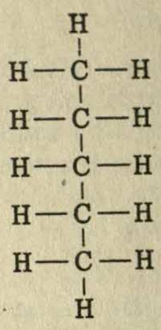
$$D^3 = \frac{6 \times 1.5 \times 10^{-23} \text{ cm}^3}{\pi}$$

$$= 2.9 \times 10^{-23} \text{ cm}^3$$

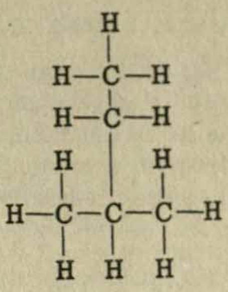
$$D = \underline{3.1 \times 10^{-8} \text{ cm}}.$$

PROBLEM 17 Pentane, which has the formula C_5H_{12}, consists of a chain of carbon atoms, with hydrogen atoms arranged in a manner similar to those of butane in Fig. 8-4. In this molecule each carbon is bonded with four atoms and each hydrogen with one atom.
(a) Show that there can be three compounds with the molecular formula C_5H_{12} in which these bonding relations exist. (The carbon atoms need not be extended in this chain order.) Diagram the three compounds. Do you think more than three compounds can exist?
(b) Do you expect these compounds to have the same properties?

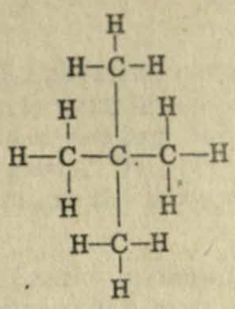
a) This is a difficult exercise in topology as well as in organic chemistry if the student really tries to prove that there can be no more than three possible compounds of formula C_5H_{12}. However, with a little imagination (and with the help of the text), the student should be able to demonstrate that he can draw at least three, as follows:



(1)



(2)



(3)

b) These compounds should not be expected to have the same properties.

PROBLEM 18

Red corpuscles of the human blood stream are known to be flattened discs approximately 7×10^{-6} m in diameter and 10^{-6} m thick. Blood count shows on the order of 5×10^6 of these in each cubic millimeter of blood.

(a) If the adult body contains 5 liters of blood, what is the total number of red corpuscles it contains?

(b) The mass of a hemoglobin molecule is believed to be 6.8×10^4 amu. How many such molecules would there be in a red corpuscle if hemoglobin has a density of one gm/cm^3, and if we assume a red corpuscle is made entirely of hemoglobin?

a) Since there are 5×10^6 red corpuscles per mm^3, and 10^3 cm^3/liter = 10^6 mm^3 per liter, there are about 5×10^6 corpuscles/mm^3 $\times$ 5 liter $\times\, 10^6 \frac{\text{mm}^3}{\text{liter}} = \underline{2.5 \times 10^{13}}$ corpuscles in the adult body.

b) Since the density is one gm/cm^3 the molecular volume of hemoglobin is about 6.8×10^4 cm^3/(6.025×10^{23}) = 1.1×10^{-19} cm^3 = 1.1×10^{-25} m^3. The volume of the red cell is about $\frac{1}{4}\pi\,(7 \times 10^{-6}\text{ m})^2 \times 10^{-6}\text{ m} \approx 4 \times 10^{-17}\text{ m}^3$. Thus, the red cell would contain about 4×10^{-17} m^3/1.1×10^{-25} m^3/molecule $\approx \underline{4 \times 10^8}$ hemoglobin molecules.

PROBLEM 19

A single human nerve cell may extend 100 cm from the spinal column to the foot, and be as much as 10^{-4} cm in average diameter. If we assume that each of the constituent atoms of this structure occupies a volume of 3×10^{-23} cm^3, what total number of atoms is required to build it?

This problem provides an interesting side light on the variability in physical dimension of a single cell. The volume of this long nerve cell is about

$$100\text{ cm} \times \frac{\pi}{4}\,(10^{-4}\text{ cm})^2 \approx 8 \times 10^{-7}\text{ cm}^3$$

Thus, the total number of atoms required to build it is about

$$8 \times 10^{-7}\text{ cm}^3/3 \times 10^{-23}\text{ cm}^3 \approx \underline{3 \times 10^{16}}$$

Chapter 9 - The Nature of a Gas

Chapter 9 is chiefly concerned with introducing the idea of a physical model as a means for analyzing and extrapolating observations on relatively complicated systems. The discussion centers about the molecular model of a gas and shows how this picture enables us to understand many of the important properties of gases. The treatment here is only semi-quantitative, since a full-fledged analysis must follow the study of dynamics. (This material is treated quantitatively in Chapter 26.)

In covering Chapter 9, the instructor should be careful to handle the material at the level presented, and not attempt rigorous dynamical justification of the results. Exact definitions of words like "pressure" should be avoided, since students do not yet have a precise definition of a force. (The text deliberately uses words like "push" instead of force here.) A quantitative discussion of the kinetic theory (as presented in Chapter 26) requires familiarity with the concept of forces and of the laws of conservation of momentum and energy. In the present chapter, it is important only that students develop some feeling for the gas law as a useful description of the properties of gases, and of the way a scientist uses a model, not as just a simplified picture of some rather complicated physical phenomenon, but rather as a tool for extending knowledge and giving direction to experiment.

CHAPTER SUMMARY

Chapter 9 divides naturally into the following three parts:

Sections 1 and 2 Here the text introduces the concept of a physical model, as contrasted to the familiar usage of the word "model". Physical models are seen not as something constructed of balsa wood and cement, but rather of ideas and assumptions which permit us to analyze complicated situations. A model must be tested by experiment, and care must be exercised in extrapolating from the model into a range where it is untested. We take as a model for a gas an assembly of widely-spaced molecules in constant motion, colliding with one another and with the walls of the container. This model appears to be consistent with many of the obvious properties of gases, their low density, high mobility, etc.

Sections 3 through 5 The molecular model is now examined in more detail. We note that the model predicts that the pressure of a gas should be proportional to the number of molecules per unit volume (Boyle's law). This relation can be verified by experiment, if the pressure is not too large; under large pressure, significant deviations are found. When the experiment is repeated at different temperatures, it appears that the pressure also depends on the temperature. This relation may be used to construct a gas thermometer for temperature measurement, and defines an absolute temperature scale. From the molecular model, temperature is seen to be identifiable with the mass and speed of the molecules (actually mv^2, although this fact is not proved at this point.)

Sections 6 and 7 The molecular model predicts that at sufficiently low densities, the random motion of individual molecules should be observable. This is found in observations of Brownian motion, which sets a natural limitation on the precision of any physical measurement. In closing, we note that large masses of gas (e.g., the earth's atmosphere, stellar clouds) may be confined without walls, by gravitational attraction.

SCHEDULING CHAPTER 9

This chapter makes fairly easy reading, and much of the material can be conveyed by the text, alloting only a small amount of time to class discussion. Class periods are probably best confined to discussion of some of the quantitative problems selected from the HDL material appropriate to Sections 3 through 5. A thorough treatment of all the material of Chapter 9 would take far too much time. The major topics which can be handled quantitatively by high school students will be re-examined in detail in Chapter 26.

	10-week schedule for Part I			6-week schedule for Part I		
Subject	Class Period	Lab Period	Exp't	Class Period	Lab Period	Exp't
Secs. 1, 2	0	0	-	0	0	-
Secs. 3, 4, 5	3	1	I-9	2	0	-
Secs. 6, 7	0	0	-	0	0	-

RELATED MATERIALS FOR CHAPTER 9

Laboratory. Experiment I-9 - Natural Temperature Scale. In this experiment, students calibrate a simple gas thermometer at several temperatures. See the yellow pages on Experiment I-9 for suggestions.

Home, Desk and Lab. The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion are indicated. Problems which are particularly recommended are marked with an asterisk (*). Answers to problems are given in the green pages: short answers on page 9 - 9; detailed comments and solutions on page 9-10 to 9-21.

Section	Easy	Medium	Hard	Class Discussion	Home Project
1				1	
2	3, 4, 5 6, 16	2		2, 4	
3	7, 8, 9 10	11, 12	13	9*, 11	
4	14, 17	15, 18		14	
5	20	19, 21			
6	22			22	22

Film. "Behavior of Gases", by Dr. A. V. Baez of Physical Science Study Committee. Brownian motion of smoke particles is shown with the aid of a microscope. The motion is compared with that of a mechanical model consisting of steel balls in rapid motion. A Boyle's law experiment is performed, and explained by means of the molecular model with the aid of animation. Running Time: 15 minutes.

Demonstrations. Go easy with demonstrations for this chapter. This is not the occasion for a full-fledged treatment of heat or of gases. Most of the properties of gases discussed are familiar enough without demonstrations. A demonstration of Boyle's law would be appropriate if time permits.

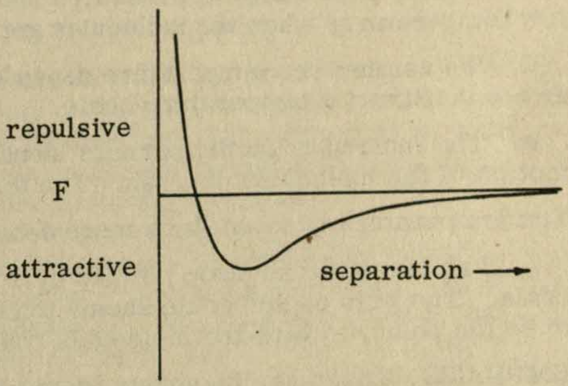
Section 1 - Physical Models
Section 2 - The Molecular Model of a Gas

PURPOSE To explain the role of a model in scientific investigation, and to introduce the familiar molecular model of a gas.

EMPHASIS These sections should be handled entirely as a reading assignment. Extensive class discussion of what is and what is not a "model" is apt to be overly philosophical and fruitless. This material is not intended to be profound, and many students will have partial (though sometimes inaccurate) familiarity with some of it.

COMMENT The "good vacuum" described on page 153, with molecules spaced about 5×10^{-6} m apart, corresponds to a pressure of about 10^{-9} atmospheres (about 10^{-6} mm Hg). This pressure is about 1000 times smaller than that which can be obtained with a mechanical vacuum pump, and can be achieved only with the addition of an auxiliary pump. With new vacuum techniques (usually designated "ultra-high vacuum") it is now possible to achieve pressures as low as 10^{-13} atmospheres in small volumes.

The text notes briefly (page 153) that one of the basic assumptions of the molecular model is that molecules "do not push or pull on one another across long distances". This statement is not precise, but is about the best we can do at this point without getting in deep. Molecules do, of course, interact with one another. Otherwise, we would be quite unable to explain the existence of matter in the liquid or solid phases. The interaction forces are not exactly zero at average intermolecular distances. A rough graph of the force field presented by one atom to another appears as shown at the right.



When molecules are as near each other as an atomic diameter (of the order of 10^{-8} cm) or less, the repulsive and attractive forces are appreciable. At large separations, however, there is very little force between the molecules. In a gas the molecules move mostly in the region of little force and can be thought of as moving freely.

The situation is roughly analogous to a few sticky ping-pong balls being blown about by a blast of air. (The air blast supplies the kinetic energy which, in the atomic case, results from thermal agitation.) If the air stream is strong enough (the temperature high enough), the balls (atoms) will move about wildly, bouncing off each other and the walls of their container. The stickiness (attractive force) will have no noticeable effect. However, if the air stream is weak (temperature low), the balls (atoms) will tend to stick together when they meet and eventually coalesce into a "solid".

Note that the idea of motion is fundamental to the model of a gas. A collection of widely spaced molecules at rest could not exert a pressure on the walls of the container, and, because of their mass, the molecules would eventually simply fall to the bottom of the vessel.

The text notes that the speed of motion of widely spaced molecules is "a bit faster" than the speed of sound (page 153). If questions arise about this point, it is sufficient to explain that sound is propagated by collisions between molecules (don't introduce waves here), and that it takes a small but finite time for collisions to be completed. Furthermore, molecules do not move uniformly in the direction of propagation of the sound. They move in all directions, so the speed of propagation of a sound pulse would be expected to be somewhat less than the mean molecular speed.

Section 3 - Boyle's Law
Section 4 - Temperature and Gas Thermometers
Section 5 - Temperature and the Gas Model

PURPOSE To extend the molecular model, in a semi-quantitative manner, to the detailed prediction and verification of the pressure-volume relationships of gases, and to show that the temperature of a gas is a measure of the molecular speed.

CONTENT a. The pressure required to confine a gas is a measure of the average perpendicular push per unit area exerted by molecules striking the wall of the vessel. Pressure differences can be measured by determining the difference in height of a liquid on two sides of a U-tube.

b. The pressure is directly proportional to the number of molecules per unit volume. The proportionality constant is the same for any gas. Experimentally, at the melting point of ice, P (cms of mercury) = $2.81 \times 10^{-24} \frac{N}{V}$, for N molecules contained in a volume V m^3.

c. The proportionality constant depends on the temperature at which the measurement is made. At the boiling point of water it is 3.84×10^{-24}. In general we can write $P = \theta N/V$, where θ is a function of the temperature. This relation (Boyle's law) holds over a fairly wide range of pressures and temperatures, but fails at high pressures and low temperatures when the molecules get too close together.

d. We can use the temperature dependence of the pressure of a constant volume of gas to construct a temperature scale.

e. The molecular model permits identification of the temperature with the speed of motion of the molecules of a gas. The temperature also depends on the molecular mass. The temperature is found (by a more detailed study) to be proportional to the quantity mv^2.

f. The temperature scale defined by a gas thermometer is the "absolute" temperature scale. The zero of this scale seems to correspond to zero molecular speed. (Now thought to be the point at which the molecules reach a least amount of speed.)

EMPHASIS These are important ideas, but cannot be developed quantitatively at this point. All of this material will be treated in detail in Chapter 26. Defer any dynamical treatment until then. Do not try to introduce the concepts of force, momentum, or kinetic energy at this point. Concentrate your class time here on the solution of problems, so students will know what kinds of problems may be treated by the gas law, even if they must accept it pretty much as an empirical relationship for the moment. Do not concentrate on the definitions of different temperature scales or means for converting between various scales.

COMMENTS Questions may arise about the details of the "Boyle's law" experiment discussed in Section 3. The apparatus shown in Figures 9-5 and 9-6 is highly schematic. We require some provision for placing the known volume of gas (V_o) in an ice bath or in boiling water, and this is hardly provided by the simple setup shown. There are a number of small points which students may bring up, but which otherwise probably should not be mentioned:

1. We assume that the volume V_o is known by prior calibration.

2. We assume that the volume V_o does not change when we add gas to the vessel — this is equivalent to assuming that the volume of the mercury displaced is negligible compared to V_o.

3. We assume that V_o is unchanged by a change in temperature of the vessel — thus we ignore thermal expansion of the glass or other material enclosing V_o.

4. We assume that the pressure calibration does not depend explicitly on temperature — hence we are assuming that the mercury does not change temperature, or, if it does, that we can ignore the change in its density due to thermal expansion.

With these assumptions, the experiment proves empirically that the pressure depends only on the number of molecules per unit volume, at constant temperature. From the form of the equation ($P = \theta N/V$), we see that this is equivalent to the statement that if the number of molecules, N, is kept constant, the product PV will also be a constant at constant temperature. Thus, as a practical matter, when we use our device as a gas thermometer, it makes no difference whether we arrange to measure the change in pressure at constant volume or the change in volume at constant pressure. As long as N is fixed and known, the temperature is determinable from either measurement.

* * *

Some instructors may be disturbed by the slightly strange units used in developing the gas law. Note that this relationship is introduced into this chapter as a purely empirical law, and that pressure differences are defined in centimeters of mercury in a natural way. For now, we cannot define “proper” units for pressure (dynes/cm^2 or newtons/m^2) or for the “gas constant” (joules or calories per degree Kelvin per mole). The present definitions suffice for the purposes at hand.

* * *

The plausibility arguments given here in “deriving” Boyle's law from the molecular model are not intended to be rigorous. Students may realize that one of the implicit assumptions involved is that the average speed of the molecules is not changed when additional gas is forced into a closed vessel. (As we see, this is tantamount to assuming that the temperature is unchanged.) There are other assumptions which are concerned with the way in which we calculate the average push on the wall and relate it to the average speed of the molecules and the number of collisions per second in such a way that the pressure, at constant speed, turns out to be incependent of the mass and speed. This matter is dealt with in Chapter 26. Thoughtful students may wonder about the fact that the molecular mass and speed do not enter explicitly in the pressure, but only in the temperature. This fact is not intuitively obvious, but cannot be verified until later.

* * *

In an idealized definition of absolute temperature in terms of the motion of gas molecules, the absolute zero is defined as that state where molecular motion ceases. This idea is now thought to be incorrect, regardless of whether gases do or do not actually condense into solid or liquid phases at low temperatures. (At this point, this idea will have to be passed as a statement without more explanation than is in the text.) The absolute zero is defined as the temperature at which all systems are in their lowest possible state of energy, but this state need not correspond to a state of zero motion. In fact, at the absolute zero, even in solids, molecules would still be in motion (with the so-called “zero-point” energy). The absolute zero is not something like the North Pole, which we can arrive at or pass, but rather something which we can only approach asymptotically as a limit in any physical measurement.

Here we are defining the absolute temperature scale in terms of a gas thermometer. Students may wonder why we choose this yardstick. We can, of course, define a temperature scale in terms of any temperature-dependent property of any substance. For instance, we might define a scale in terms of the change of length of an iron bar with temperature. We could write $T = kL$. Then any measured value of L would define a corresponding temperature. However, we now have a different temperature defined, for instance, by two bars of different lengths. We could rectify this by rewriting our equation

$$T = \frac{k}{L_o} L$$

where L_o is the length of the bar in some standard environment (ice bath). We now have defined a temperature which would be the same when measured by any iron bar, but suppose we used a brass bar instead of iron? Measured temperatures would not be the same. A different value of k would have to be used for brass, and no values of the two constants, k_{iron} and k_{brass}, would give exact agreement all along the scale. Furthermore, if we attempt to reach the ''absolute zero'' of this temperature scale, corresponding to $L = 0$, we cannot come close to approaching it. It has no real physical significance.

The temperature scale which is defined by the behavior of a gas has much more generality and significance. In the first place, all gases behave in the same way when their densities are sufficiently low. That is, the same proportionality constant gives the same temperature determinations regardless of which gas we use. Secondly, the absolute zero of the gas thermometer scale has empirical significance in that this temperature *is* the limit which even the most ingenious refrigeration systems are just not quite able to reach. Thus while we may use the length of solid or liquid columns (e. g., in thermometers) as secondary standards to measure temperature over limited ranges, we are much better off to use gas thermometers as our primary standards to define the whole temperature scale.

* * *

Some students may wish to know how low-temperature refrigerators work. This is not the occasion to go into a discourse on the first and second laws of thermodynamics. However, the following sort of simple explanation may be offered if questions arise. Most people are familiar with the fact that a gas heats if it is compressed and cools if it is allowed to expand. In a refrigerator the gas is first compressed, thereby raising its temperature. The heated gas then flows through a ''heat exchanger'' where its temperature is reduced back to that of the surroundings. The gas is then allowed to expand, thereby lowering its temperature below that of the surroundings. In this way a large volume of cold ''working gas'' can be accumulated. A small portion of the cold gas can then be compressed again and allowed to cool before expansion to the temperature of the rest of the cold gas. In this way another smaller reservior of still colder gas can be built up. By successive steps of this kind, any gas can eventually be brought to a sufficiently low temperature that it will become liquid. Air will liquify at about 70° K. Helium, which liquifies at about 4° K has the lowest liquefaction temperature of any substance known. The temperature of the liquid helium can be further reduced by ''boiling'' the helium under reduced pressure, and by various other means. In this way, temperatures as low as 0.001° K can be reached rather routinely. By very special techniques, temperatures as much as a factor of 100 or 1000 times smaller sometimes have been achieved. However, no matter how close scientists may eventually approach the absolute zero, it is a certainty that this temperature will never be reached.

Section 6 - Brownian Motion and Noise
Section 7 - Gases without Walls

PURPOSE To point out that molecular motion can be observed (and that gas molecules [like other matter] are subject to the pull of gravity).

CONTENT a. Observation of the erratic motion of small particles (Brownian motion) is evidence of molecular motion.

b. Brownian motion sets a natural upper limit on the sensitivity of any physical measurement.

c. A gas can be ''contained'' without walls, due to gravitational attraction.

EMPHASIS This material can quickly become excessively complicated if an attempt is made to extend the level of discussion beyond that presented in the text. It is best handled principally as a reading assignment. Further references to limitations on measurement are made in the next chapter.

COMMENT Brownian motion is easy to observe with a simple microscope, and students may be interested in doing this in the laboratory or at home. Any suspension of sufficiently small particles will do, although the effect increases as smaller particles are used.

Chapter 9 - The Nature of a Gas

For Home, Desk and Lab — Answers to Problems

The following table classifies problems according to their estimated level of difficulty and the sections to which they relate. Those which are especially suited to class discussion and those which are home projects are indicated. Problems which are particularly recommended are marked with an asterisk (*).

Answers to all problems which call for a numerical or short answer are given following the table. Detailed solutions are given on pages 9-10 to 9-21.

Section	Easy	Medium	Hard	Class Discussion	Home Project
1				1	
2	3, 4, 5 6, 16	2		2, 4	
3	7, 8, 9 10	11, 12	13	9*, 11	
4	14, 17	15, 18		14	
5	20	19, 21			
6	22			22	22

SHORT ANSWERS

1. See detailed discussion on page 9-10.
2. See detailed discussion on page 9-10.
3. See detailed discussion on page 9-10.
4. See detailed discussion on page 9-11.
5. See detailed discussion on page 9-11.
6. 1.24×10^{-3} gm/cm^3.
7. 3 times atmospheric density.
8. 0.83 times sea level density.
9. (a) 76 cm.
   (b) 114 cm.
   (c) See discussion on page 9-12.
10. (a) 76 cm.
    (b) 38 cm.
11. (a) 2.73 m.
    (b) 11 atmospheres
12. Rise.
13. (a) 1.86×10^{-4} gm/cm^3.
    (b) 1.13
14. (a) 1.92
    (b) 1.92
    (c) No change.
15. (a) 3.57×10^{21} molecule/m^3.
    3.57×10^{18} molecules.
    (b) $7.53 \times 10^{-25} \dfrac{\text{cm of mercury}}{(\text{molecules/m}^3)}$.
    73° Kelvin.
16. Less.
17. 1.1
18. (a) 9.10×10^{24} molecules.
    (b) 3.2 atmospheres.
    (c) 5.7×10^{23} molecules.
19. (a) Hydrogen.
    (b) 4 to 1.
20. (a)

Gas	mv^2 in amu m^2/sec^2
He	3.76×10^6
Ne	3.82×10^6
A	3.80×10^6

    (b) See detailed discussion on page 9-18.
21. See detailed discussion on page 9-19.
22. See detailed discussion on page 9-20.

COMMENTS AND SOLUTIONS

PROBLEM 1

Consider the idea that all matter is made up of atoms. Does the idea that matter is constructed from 100 different types of atoms lead to a physical model? Why?

It is not intended that there be a single well-defined answer for this problem. The problem might well be used as the basis for a brief class discussion in which students have a chance to theorize from what they have learned.

In our search for simplicity, we have found that all matter is constituted of only 100 basically different kinds of particles, called atoms. There are certainly many more than this number of basically different substances in the world. Thus we must consider a model in which atoms can combine in various different groupings, compounds, in order to form each of these myriad forms of matter. The smallest particle in which one of these groupings is completely represented could be called a molecule.

PROBLEM 2

(a) Outline the atomic model of solids that was discussed briefly in the last chapter.

(b) How is the volume of a solid related to the volumes of the individual atoms? For instance, is the volume of a solid about the same as the total volume of the individual atoms? Much greater? Much less?

(c) On an atomic model how would you expect the compressibilities of solids and gases to differ?

a) The important features of the atomic model of solids are: Atoms are bonded together to form molecules. The mass of the molecule is equal to the sum of the masses of the atoms making up the molecule. In a gram mole of substance there are 6.025×10^{23} molecules. In chemical reactions there is an interchange of the atoms of one molecule with those of another molecule. The atomic structure of molecules ranges from the simple to the complex. In some substances, such as table salt, the atoms are grouped in such a way that it is hard to define individual molecules. The atomic structure of crystals is reflected in a geometrical pattern of atoms that is always the same for the same substance. Some solids have no particular structure (amorphous solids).

b) The volume of a solid is close to the sum of the volumes of the individual atoms that make up the solid. (In the solution of some of the problems in Chapter 8, the volume of an atom was computed by dividing the total volume by the number of atoms in that volume.)

c) Because atoms in solids are very close together with very little space between them, one should expect solids to be nearly incompressible (as they are). Gases are highly compressible because the average distance between atoms or molecules is large (as compared with the atoms in solids).

PROBLEM 3

In Section 9–2 we pointed out that motion is an essential feature of the molecular model of gases. Use the idea (discussed in Chapter 4) that gravitational effects become less important at small scale to explain that "molecules" of a gas may be in motion, although wooden molecules like those in Fig. 8–6 would just sit still on a table.

With small masses the gravitational forces become negligibly small. The forces are so small that the thermal motion or velocity of a molecule is great enough for it to escape from the attractive effect of the other molecules, the table, or the earth and thus move freely about. The wooden molecules (made on a much larger scale) experience proportionally much greater gravitational forces, and thus are strongly attracted to the earth and table.

While a good student will be able to grasp the general idea in this question, some of the work in Part III will bring it home.

PROBLEM 4

(a) What will happen when you turn a glass upside down and shove it down into water?

(b) Can you use this effect to reconstruct the reasoning that Hero of Alexandria used to show that air is a material substance?

a) As an inverted glass is shoved down into water the water rises inside the glass, but is always below the level of the water outside the tumbler.

b) If air were a void, or lacked material substance, then we would expect the water inside and outside the tumbler to be at the same level. This is not the case. The observed fact can be understood under the assumption that air consists of particles that are moving about and therefore offer resistance to the rising of the water inside the glass.

PROBLEM 5

What evidence can you think of that shows the mobility, interpenetrability, and mixing of gases? As a start, answer the questions, "How do most odors reach you?" and "What happens to the air when you move around?"

The transmission of odors can be explained by assuming that the particles which convey the odor are moving about in rapid motion, that due to this motion they intermingle with the other air particles (interpenetrate), and diffuse rather uniformly (mixing of gases) throughout the room. The time it takes to do this is much longer than one might expect by straight line propagation. This is caused by the collision of the odor particles with the other air particles and thus a change in direction of their motion.

As you move about, the air particles that hit you from the front rebound with a greater velocity and thus exert a greater pressure on you. You feel this especially when you move rapidly. Nevertheless, you can move around in air with relative ease, demonstrating the mobility of air molecules. It is much more difficult to move through water. This would indicate that air molecules have a greater mobility than water.

PROBLEM 6

Two students set out to determine the density of air. First they weigh an empty container and find its mass to be 20 gm. Next they inflate a limp plastic balloon to a diameter of 21 cm and squeeze its contents into the container. The container with the air from the balloon now is found to have a mass of 26 gm. What is the density of air from this measurement?

$$\text{Density of air} = \frac{\text{mass of air}}{\text{volume of air}} = \frac{26\text{ gm} - 20\text{ gm}}{4/3 \times \pi \times (21/2\text{ cm})^3} = \frac{6\text{ gm}}{1/6 \times \pi \times 21^3\text{ cm}^3}$$

$$= \underline{1.24 \times 10^{-3}\text{ gm/cm}^3}.$$

PROBLEM 7

A tire is inflated to three times atmospheric pressure. What is the density of the air in the tire?

We must assume that the temperature of the air in the tire does not change when the tire is inflated. Then, if the pressure is increased by a factor of 3, the density must also be greater by a factor of 3. Since both pressure and density are proportional to the number of molecules per unit volume, changing one by a factor of 3 changes the other by the same factor. Notice that the actual volume of the tire does not enter the argument, so we do not much care whether the tire is made of thin rubber or tough steel.

PROBLEM 8

At sea level a mercury barometer reads a pressure of 76 cm of mercury, and at an elevation of 1500 meters it reads 63 cm. What is the relative density of the air at this altitude compared to that at sea level? Assume that the temperature is the same at both levels. (Note: 1500 meters is about 4500 feet.)

Since at constant temperature the pressure is directly proportional to the density, the pressure at 4,500 ft must be 63/76 = 0.83 times that at sea level. Students may be a bit disturbed about using a centimeter of mercury as a unit of pressure, but it is a perfectly acceptable, if arbitrary unit. In this case, since we are only concerned with a ratio of pressures, it would make no difference what our choice of units was, as long as it was the same at both places. (To drive this point home, you might ask – "If we measured the pressure with a water barometer at sea level, and found that the pressure supported 32 ft of water, what would the height of the water column be at 4,500 ft altitude?")

PROBLEM 9

To the apparatus shown in Fig. 9–5 we attach a plastic bag of volume V_0 full of gas. We squeeze this gas into the gas space V_0 on the right side of the apparatus. The height of the mercury on the left rises about 76 cm. We close the valve on the right; then we attach another bag of gas of volume V_0 at atmospheric pressure. Finally we open the valve and squeeze the gas out of this bag into the volume V_0.

(a) By what additional height does the column of mercury rise?

(b) With the plastic bag still attached we open the valve. By what height does the column of mercury fall?

(c) Why does the column fall farther than 76 cm?

a) As discussed in Section 3, the second extra batch of molecules in V_o will produce the same increment of pressure as the first. Thus the level will rise another 76 cm.

b) In the volume V_o in the apparatus there are now 3 times as many molecules as originally occupied this volume. (The number present initially plus those added in the two extra batches.) When the valve is opened and the gas permitted to enter the empty plastic bag, which has volume V_o when inflated, the molecules of gas will be equally shared between the two equal volumes so that in the apparatus there will be 1 1/2 batches of molecules or 1/2 batch of extra molecules. The excess pressure will be 1/2 that exerted by one batch of molecules; thus the mercury column will now be 38 cm higher than it was initially, bringing it to $h_o + 38$. Before opening the valve, it stood at $h_o + 76 + 76 = h_o + 152$ cm. Thus it must have dropped 152 - 38 = 114 cm.

c) If air corresponding to a volume V_o at atmospheric pressure had escaped from the apparatus, the mercury level would have fallen 76 cm (see part a). Instead, air corresponding to 1.5 V_o at atmospheric pressure (see part b) escaped. Thus the column dropped 1.5 × 76 = 114 cm.

PROBLEM 10

In a mercury barometer (Fig. 9-16), at sea level the normal pressure of the air (one atmosphere), acting on the mercury in the dish, supports a column of mercury in a closed tube.

(a) What height *h* would you expect for the column of mercury in the barometer?

(b) If you go up in the air until the density has fallen to half its sea-level value, what height *h* would you now expect?

a) Standard air pressure at sea level is taken as 76 cm of mercury which is the height h in Figure 9-16.

b) The density of a gas varies directly with the pressure exerted by the gas. At half the sea-level density there must be half sea-level pressure. The pressure would be 38 cm of mercury.

PROBLEM 11

Every column of water 10 m high exerts a pressure approximately equal to that of the atmosphere. A diving bell (a cylinder open at the bottom) with a diameter of 2 meters and a length of 3 meters is sunk in 100 meters of water.

(a) Assuming that the temperature of the water is the same at all depths, how far in the diving bell will the water rise?

(b) What must be the pressure of the compressed air pumped into the diving bell to keep it entirely free of water?

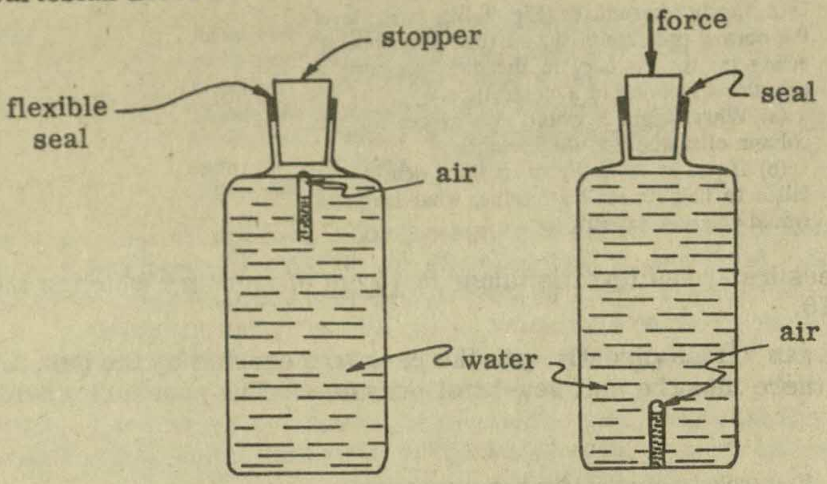
(c) Have you ever made a Cartesian diver? (Your teacher is not to grade you on your answer to this question!)

a) Assume that the initial pressure in the bell, before it is lowered, is 1 atmosphere. Then, at a depth of 100 meters in water, the pressure will have increased by 100 meters ÷ 10 m/atmosphere = 10 atmospheres. The total final pressure will then be 1 + 10 = 11 atmospheres (we must add the pressure of the air on top of the water, which is 1 atmosphere). Then since the initial and final values of PV must be the same (Boyle's law), the final volume must be 1/11th of the initial volume. The column of air in the diving bell will change in length only, not in diameter, as the water enters, so that the final length of the air column must be 3 m/11 = 0.27 m. Thus the water must rise 3.00 m - 0.27 m = 2.73 m into the bell.

b) To keep the bell free of water, we should have to supply a pressure of at least 11 atmospheres of compressed air (equal to the pressure of the water and the air above the water.)

c. Rather than take class time, students may be told how to make a Cartesian diver on their own time. (The best way to do this is to have a working one available for observation and experimentation.) As shown in the sketches, invert a small closed tube (a small cylindrical vial of the sort pharmacists use for pills will work well) in a bottle of water which has a flexible seal and a loose-fitting cork as a piston. Put enough water into the tube so that it just floats when no pressure is applied to the stopper. When pressure is applied, the tube will sink, since the increased pressure will be transmitted through the water, compressing the air in the tube and decreasing the volume of the air which is trapped in the tube. When the pressure is released, the "diver" rises. Commercial versions of this device are available with a tube shaped in the form of a human figure—usually called "Cartesian Imps".

Cartesian Diver



PROBLEM 12

A log or boat will float in water if its mass is less than the mass of water it displaces; a balloon will rise as long as the mass of the container and the enclosed gas is less than that of an equal volume of the air around it. If a soap bubble 20 cm in diameter with walls 10^{-3} cm thick is filled with neon gas, will it rise or sink through air (density 1.2×10^{-3} g/cm^3) of the same temperature? Consider the density of the bubble material to be 1.1 gm/cm^3, and the density of neon gas to be 8.4×10^{-4} g/cm^3.

We must calculate the relative masses of the neon-filled soap bubble and an equal volume of air. The mass of the soap bubble is

bubble mass = bubble material vol. × bubble material density + neon vol. × neon density,

$$= 4\pi (10 \text{ cm})^2 (10^{-3} \text{ cm}) \times 1.1 \text{ gm/cm}^3 + \frac{4}{3}\pi (10 \text{ cm})^3 \times 8.4 \times 10^{-4} \text{ gm/cm}^3$$

$$= 1.4 \text{ gm} + 3.5 \text{ gm} = 4.9 \text{ gm}.$$

$$\text{Mass of displaced air} = \frac{4}{3}\pi (10 \text{ cm})^3 \times 1.2 \times 10^{-3} \text{ gm/cm}^3 = 5.0 \text{ gm}.$$

Since 5.0 gm is greater than 4.9 gm, the bubble will rise.

PROBLEM 13

A balloon with a mass of 10.3 gm is filled with helium to a diameter of 32.0 cm. It rises in the air and lifts, in addition to the balloon, 9.2 meters of string before it comes to equilibrium. The string is found to have a mass of 0.78 gm/m.

(a) What is the density of the helium in the balloon?

(b) What is the ratio of the pressures inside and outside the balloon?

Note: The density of air is 1.20 gm/liter; the density of helium at room temperature is 0.165 gm/liter.

a) A good, straightforward, if somewhat difficult, exercise dealing with the density of gases and the meaning of equilibrium. Try this one only on your better students.

If the balloon is in equilibrium, we know that total mass of balloon = total mass of displaced air. The mass of the displaced air is $\frac{4}{3}\pi (16 \text{ cm})^3 \times 1.2 \times 10^{-3} \text{ gm/cm}^3 =$ 20.7 gm. The mass of the balloon is mass of material + mass of helium + mass of 9.2 m string = 10.3 gm + mass of helium + 9.2 m × 0.78 gm/m = 17.5 gm + mass of helium. Therefore, the mass of the helium must be 20.7 m - 17.5 gm = 3.2 gm. Since the balloon has a volume of $1.72 \times 10^4\ \mathrm{cm}^3$ (calculated before in determining the mass of displaced air), the density of the helium is

$$\text{density of helium} = \frac{3.2\ \mathrm{g}}{1.72 \times 10^4\ \mathrm{cm}^3} = \underline{1.86 \times 10^{-4}\ \mathrm{gm/cm}^3}.$$

b) Since the normal density of helium is $1.65 \times 10^{-4}\ \mathrm{gm/cm}^3$, the pressure inside the balloon must be 186/165 = 1.13 times greater than the pressure outside.

PROBLEM 14

(a) A gas thermometer containing helium is at the temperature of melting ammonia in which it has been immersed for some time. The volume of He at standard atmospheric pressure is read. Then the thermometer is moved into a bath of boiling water. The pressure on the He gas volume is still atmospheric pressure. By what factor does the volume of He change? (Use Fig. 9–10 to estimate the temperature of melting ammonia.)

(b) If the thermometer had a constant volume, by what factor would the pressure of the He gas change?

(c) If we used oxygen gas instead of He in the thermometer, how would that affect the answers to (a) and (b)?

a) For the same number of molecules and pressure the volume of a gas is proportional to the basic or Kelvin temperature scale. The factor by which the volume changes is the same as the factor by which the temperature changes. The temperature of melting ammonia is $2.00 \times 10^{-24}\ \frac{\text{cm of mercury}}{(\text{molecules/m}^3)}$ or 195° Kelvin, and for boiling water is $3.84 \times 10^{-24}\ \frac{\text{cm of mercury}}{(\text{molecules/m}^3)}$ or 373° Kelvin. The volume changes by the factor

$$\frac{3.84 \times 10^{-24}}{2.00 \times 10^{-24}} = \frac{373^\circ}{195^\circ} = \underline{1.92}.$$

b) For constant volume the factor by which the pressure changes is the same as the factor by which the temperature changes $\left(\frac{PV}{N} = \theta\right)$ and thus is the same factor as above, 1.92.

c) A gas thermometer filled with oxygen instead of helium would give the same results. Over a wide range of temperature, simple gases behave almost exactly alike with reference to pressure and volume expansion. For this reason, temperature can be measured in a more fundamental way using gas thermometers than by using the expansion of other substances. A gas thermometer will depart from the ideal gas behavior when the temperature approaches the temperature of liquefaction of the gas used. For oxygen, this would be about 80° K.

PROBLEM 15 The pressure in a tube of volume 1.00×10^{-3} m^3 containing helium has been reduced to 1.00×10^{-2} cm height of mercury at 0°C.

(a) What is the number of molecules per unit volume? What is the total number of molecules in the tube?

(b) The tube is immersed in liquid nitrogen. After waiting for a while, the pressure gauge connected to it settles down to 2.68×10^{-3} cm of mercury. What is the temperature on the basic gas scale (θ)? What is the temperature on the Kelvin scale?

(c) If the helium were replaced by air, how would your answers to (a) and (b) be affected? (Look at Fig. 9–10.)

a) From Boyle's law $PV/N = \theta$, the number of molecules per unit volume is

$$N/V = P/\theta$$

$$= \frac{1.00 \times 10^{-2} \text{ cm of Hg}}{2.81 \times 10^{-24} \dfrac{\text{cm of Hg}}{(\text{molecules/m}^3)}} \qquad \text{(See page 156 of the text.)}$$

$$= \underline{3.57 \times 10^{21} \text{ molecules/m}^3}$$

The total number of molecules = $N = 3.57 \times 10^{21}$ molecules/m^3 $\times 1.00 \times 10^{-3}$ m^3

$$= \underline{3.57 \times 10^{18} \text{ molecules.}}$$

b) Since the temperature is proportional to the pressure of the helium gas the temperature of boiling nitrogen would be found from

$$\frac{P_N}{P_i} = \frac{\theta_N}{\theta_i}, \text{ or } \theta_N = \frac{P_N}{P_i}\,\theta_i$$

$$\theta_N = \frac{2.68 \times 10^{-3} \text{ cm}}{1.00 \times 10^{-2} \text{ cm}}\,\theta_i$$

$$= 0.268 \times 2.81 \times 10^{-24} \frac{\text{cm of mercury}}{(\text{molecules/m}^3)}, \text{ or}$$

$$0.268 \times 273° \text{ Kelvin}$$

$$= \underline{7.53 \times 10^{-25} \frac{\text{cm of mercury}}{(\text{molecules/m}^3} \text{ or } 73° \text{ Kelvin.}}$$

c) All the gas components of the air would liquify at this temperature or at higher temperatures. Thus the air would not behave in accordance with Boyle's law and could not be used in a gas thermometer to measure the temperature of boiling nitrogen. The gas in the thermometer, changing to a liquid or even a solid, would exert less pressure than it would if it were a gas, giving too low a temperature reading for the boiling point of nitrogen.

PROBLEM 16 Is air containing water vapor denser or less dense than dry air at the same temperature and pressure?

Air containing water vapor is less dense than dry air at the same temperature and pressure. At a given temperature and pressure, the number of molecules per unit volume is a constant, independent of the masses of the molecules. Thus a water molecule must replace a nitrogen or oxygen molecule if the pressure is to remain fixed. But the molecular masses involved are:

N_2: $2 \times 14 = 28$

O_2: $2 \times 16 = 32$

H_2O: $(2 \times 1) + 16 = 18$

Thus the gaseous mixture with H_2O molecules present will be less dense than the dry air.

PROBLEM 17

When a car is driven for an extended period, particularly in summer, the tires and their inflating air are heated both from the flexing and friction on the tires and from the warmth of the road surface. When the daytime temperature is 27°C, by what factor would you expect the pressure to increase with a 30°C temperature rise in the tire and air?

Since the volume of the tire structure does not change appreciably with pressure and temperature, the pressure will be proportional to the temperature, and the factor by which it changes is the same as the factor by which the temperature changes on the Kelvin scale. $\frac{P_2}{P_1} = \frac{\theta_2}{\theta_1} = \frac{30 + 27 + 273}{27 + 273} = \frac{330}{300} = \underline{1.1}$. So, if P_1 is, say, 28 psi, the pressure would increase to 28 psi × 1.1 = 31 psi. Since the tire will surely expand somewhat, the factor will be smaller than 1.1.

PROBLEM 18

Note: In this problem you will probably find it an advantage to use ratios to find the answers at different temperatures and pressures. For this purpose, recall that at 273°K and one atmosphere pressure there are ~ 2.70×10^{25} molecules/m³ (Section 9–3).

A certain tire contains (10/81) m³ of air. It is normally inflated to a total pressure of 3 atmospheres (3 times the standard atmospheric pressure).

(a) If this pressure was measured at a temperature of 27°C, how many molecules of air are in the tire?

(b) After a few hours driving the temperature within the tire rises to 47°C. What is the pressure now?

(c) How many air molecules would you have to release from the heated tire to reduce the pressure back to 3 atmospheres at 47°C?

a) $\frac{P_1V_1/N_1}{P_2V_2/N_2} = \frac{\theta_1}{\theta_2}$, or $N_2 = N_1 \frac{P_2V_2\theta_1}{P_1V_1\theta_2}$

$N_2 = 2.70 \times 10^{25}$ molecules $\times \frac{3}{1} \times \frac{10/81}{1} \times \frac{273}{300} = \underline{9.10 \times 10^{24}\ \text{molecules}}$.

b) $P_3 = P_2 \frac{\theta_3}{\theta_2}$ = 3 atmospheres $\frac{47 + 273}{300} = 3 \times \frac{320}{300}$ atmospheres = $\underline{3.2\ \text{atmospheres}}$.

c) The number of molecules that would give 3 atmospheres pressure at 47°C is

$$N_4 = N_2 \frac{P_4}{P_2} = 9.10 \times 10^{24} \times \frac{3}{3.2} = 8.53 \times 10^{24} \text{ molecules.}$$

The number of molecules that would have to be released to reduce the pressure back to three atmospheres would be simply the difference

$$N = N_2 - N_4 = 9.10 \times 10^{24} - 8.53 \times 10^{24} = 0.57 \times 10^{24} = \underline{5.7 \times 10^{23} \text{ molecules}}.$$

(It might be well to note that it is not considered good practice to "bleed" the tires under these conditions. Increased flexing of the tires causes a temperature rise which will bring the pressure up again.)

PROBLEM 19

A gas is made up of molecules of hydrogen and oxygen.
(a) Which molecules move faster?
(b) About what is the ratio of the speeds?

a) Since the temperature of a gas is proportional to mv^2 and hydrogen molecules have a lesser mass than oxygen molecules, the hydrogen molecules will move with higher speed than the oxygen molecules (when the two gases are at the same temperature).

b) For the same temperature, we know that $m_H v_H^2 = m_O v_O^2$, thus

$$\frac{v_H}{v_O} = \sqrt{\frac{m_O}{m_H}} = \sqrt{\frac{2 \times 16}{2 \times 1}} = 4.$$

The hydrogen molecules have a speed about 4 times that of the oxygen molecules.

PROBLEM 20

There is good reason to believe that for monatomic gases, the speed of sound is always a definite fraction of the speed of molecular motion. (Crudely, it is molecular motion that carries sound; consequently the two speeds must be related.)
(a) Using the following table compute the products of mass times the square of the speed of sound for each of the monatomic gases listed.

Gas	Mass (amu)	Speed of Sound (m/sec)
He	4.00	970
Ne	20.2	435
A	39.9	308.5

(b) Explain the relationship of your results to the statements about thermal speeds of molecules near the end of Section 9-5.

a) Gas	mv^2 (in amu m^2/sec^2)
He	3.76×10^6
Ne	3.82×10^6
A	3.80×10^6

b) The speed of sound and the speed of molecular motion for monatomic gases are proportional. Thus the ratios of the above values are the same as the ratios of mv^2 for the different molecules. Since the values of mv^2 are the same for all cases, this is an indication that the quoted speeds of sound were all determined at the same temperature.

PROBLEM 21

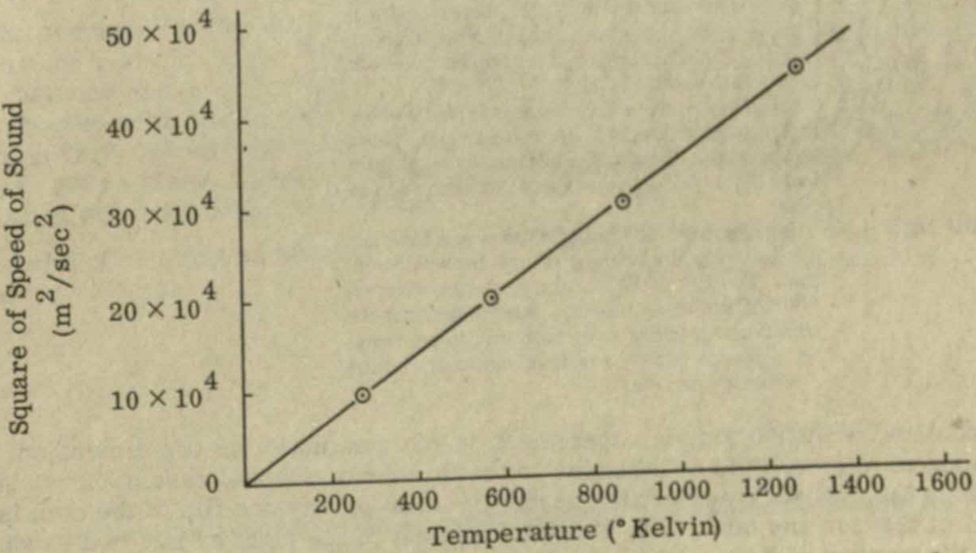
Temp. (°K)	Speed of Sound v_s (m/sec)	v_s^2 (m²/sec²)
273	308.5	9.55×10^4
573	446.5	19.9
873	551.1	30.0
1273	665.5	44.3

This table for argon gas shows the measured speed of sound and the square of the speed of sound at different temperatures on the Kelvin scale.

(a) Plot the square of the speed of sound vertically against absolute temperature horizontally.

(b) Assuming that the speed of sound is always the same fraction of the molecular speed, check your result against the statement near the end of Section 9-5 that temperature and the square of the molecular speed are proportional.

a) Square of Speed of Sound versus Temperature.



b) Assuming that the speed of sound is always the same fraction of the molecular speed, the fact that the points on the graph of $(\text{speed})^2$ versus temperature very nearly fall on a straight line again indicates that temperature is directly proportional to the square of the velocity of the molecules. This adds evidence for the gas model.

The graph of the given function is a straight line through the origin. Thus the relationship represented can be expressed by an equation of the form: $v_s^2 = AT$. (On our graph, A has the approximate value 3.4×10^2 meters²/sec² per degree Kelvin.)

If v_s^2 is proportional to T, mv_s^2 must also be proportional to T.

PROBLEM 22

* Brownian motion occurs because in a short interval of time a particle is hit by more molecules from one side than the other. In this way the particle moves at random to the right or the left.

(a) As a rough model of this Brownian motion, flip a coin and move one pace to the right if it comes up heads, or one pace to the left if it comes up tails. Flip the coin 10 times over, following the directions it gives you each time. Then write down how far away you are from your starting point and whether you are to the right or to the left. Go through this whole procedure several times. After 10 flips do you always come out at the same place?

(b) Pool your results with those of your classmates and make a plot showing (vertically) the total number of times that any of you came to a given final position against (horizontally) the final position measured from the starting point. For example, if in all the various tries you or any one of your classmates landed 3 paces to the left 18 different times, you will plot a point on the graph 3 units to the left of center and 18 units high. If you and your classmates landed back at the start 27 times, you will plot a point at the center of the horizontal axis 27 units high, etc.

(c) Examine this plot to see where you (or a particle in Brownian motion) are most likely to end up. Should the curve be symmetric about the starting point? Is it?

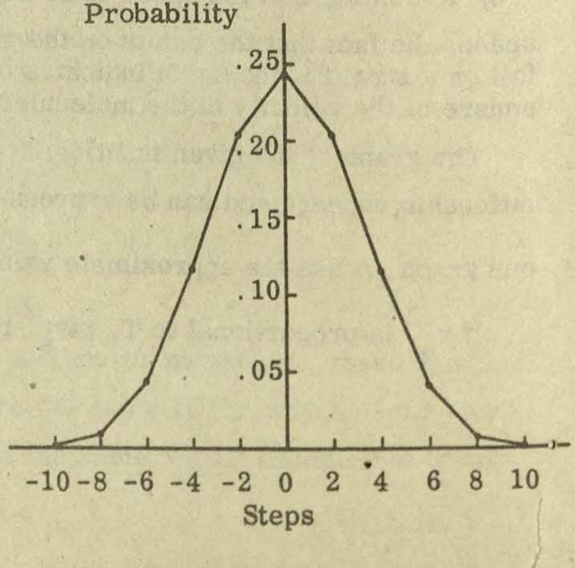
(d) If you measure the distances moved without regard to direction away from the starting point, about what is the average distance in paces gone in 10 flips? Would you expect this average to be zero?

(e) Compare the kind of motion you have just studied with observations of real Brownian motion. See Fig. 9-17 or take your own observations of Brownian motion. Remember that the coin flipping, telling you which way to go, represents chance bombardment of molecules hitting a Brownian particle.

This problem involves a simple experiment on random motion in one dimension. The student is asked to extend his conclusions to the three-dimensional case of Brownian motion. Since the probability of getting heads (or tails) on any one flip of the coin is independent of that for any other flip, the displacement to the right or left is a random function of the number of flips. This is called a random walk.

a) If, in 10 flips, a student comes out at the same place 10 times, you had better ask to examine his coin! The probability of ending up 10 steps to the right (10 heads in a row), for example, is 1 in $2^{10} = 1024$.

b) The slope of the plot of your class' results should approximate that shown at the right. Since we do not know the number of students in your class, the vertical axis of our plot is a probability scale rather than the number of cases at each place. If you wish your class to see how the class distribution compares with the theoretical distribution on the plot for the actual



distribution. If you have N students each of whom contributed 10 trials, the theoretical number of trials arriving at each step is:

Step	Expected number of cases at each step
0	10 N × 0.246
-2 or +2	10 N × 0.205
-4 or +4	10 N × 0.117
-6 or +6	10 N × 0.044
-8 or +8	10 N × 0.00975
-10 or +10	10 N × 0.00098

c) The most likely ending position is at the origin, or starting point. Unless the coin is loaded, there are equal probabilities for being a given distance either to the right or left of the starting point. The class' plot should make this clear (unless your class is very small).

d) Students should not expect the magnitude of the average displacement to be zero. All of the trials would have to result in zero displacement for the average magnitude to be zero.

The average magnitude of displacement is found by multiplying each of the displacements (including zero displacement) by the number of such displacements, summing these products, and dividing the sum of the products by the number of tirals.

The average displacement found by your class can be compared to that which would be expected theoretically. We are not interested in differentiating distance to the right from distance to the left. Thus, we find from the curve (in part b) that the probability for ending up zero steps from the starting point is 0.246; the probability for ending up ± 2 steps from the starting point will be .205 + .205 = .410; for 4 steps we have .234; 6 steps .088; 8 steps .0195; and 10 steps .00196. If we made 1000 trials we would expect to get about the following results:

Total distance moved after taking 10 steps	Number of times in 1000 that this result is likely to be obtained
0	246
2	410
4	234
6	88
8	20
10	2

To calculate the average total distance covered, we follow the standard averaging procedure

$$\frac{0 \times 246 + 2 \times 410 + 4 \times 234 + 6 \times 88 + 8 \times 20 + 10 \times 2}{1000} = \frac{820 + 936 + 528 + 160 + 20}{1000} = \frac{2464}{1000}$$

$$= \underline{2.46 \text{ steps}}.$$

e) If the motion in a random walk of one dimension is extended to three dimensions, it would compare with Brownian motion of small suspended particles, where, instead of flipping coins to decide which way to move, the random unbalanced collisions cause the motion. The two differ in that the viscosity of the fluid results in retarding forces that vary with the velocity. In our simple example of random motion no consideration was given to the velocity. Also the time between impacts in Brownian motion affects the time at which we will find a particle in any particular position. No consideration was given to this in the random walk.

If more information is desired on random functions and the random walk, one good source is Introduction to Mathematical Physics by William Band, Van Nostrand, 1959, pp. 163-169.

Chapter 10 - Measurement

CHAPTER SUMMARY

The intent of this chapter is to examine in a preliminary way the fundamental nature and limitations of measurement; to consider measurement itself as one of the branches of physics; and to show the place of measurement in physical investigation.

Because of its relation to many current discussions of information theory you or some of your students may be interested in the chapter. It is not essential to the further development of this course. You can easily use it as a reading assignment. You can demand a study of it or cut it completely according to your schedule or taste, without danger that essential material will be missed. There are six sections in the chapter:

Sections 1 and 2 Human judgement is required in any physical measurement. Even though complex instruments are used for amplification and display, a human observer must make "yes-no" (binary) decisions about the value of the measured quantity. The number of "yes-no" decisions which must be made is a quantitative measure of the precision of the measurement. In the case of an instrument which seems to perform a measurement and give a direct "answer", such decisions are often taken in designing the instrument.

Section 3 There are natural uncertainties (noise, exactly where the object "ends", etc.) that limit the precision of any measurement.

Section 4 Instruments may be used for measurement without a detailed knowledge of "what makes them tick". Used in this way, they must be calibrated by using them to measure quantities we already know.

Section 5 and 6 The act of measurement inherently disturbs the system being measured. Useful measurement requires that the disturbance be inconsequential, or that the disturbance be taken account of through allowing for it. Section 6 serves as an introduction to Part II.

SCHEDULING CHAPTER 10

Chapter 10 deals implicitly with that branch of science known as information theory. It is difficult to treat this subject more than superficially without first introducing considerable material on statistics. This would be entirely too difficult for students at this level. However, the general results of such studies can be presented qualitatively (as they are here). Such matters form an important part of students' understanding of the nature and problems of scientific investigation.

This chapter is intended to be read. It should not require much class time unless you wish to develop a few of the topics more fully than they are covered in the text. The exercises are intended largely as "thought" questions for students, and need not be assigned or discussed in class. If time is short, the chapter can be handled entirely as a reading assignment, or it can be skipped without causing difficulty later.

SUPPLEMENTARY MATERIALS FOR THE CHAPTER

Home, Desk and Lab. The first three exercises in the end-of-chapter material are discussed in the green pages (page 10-7). These are quantitative exercises, but should not be assigned unless you plan to spend time on class discussion of the material of the chapter. The other three exercises are "thought questions", or home "experiments".

Films. "Measurements", by William Siebert of Massachusetts Institute of Technology. Measurement of the speed of a rifle bullet is used as the basis for a discussion of measurement; problems discussed include noise, bias, "black boxes", and the element of decision inherent in all measurements. Running time: 22 minutes.

This film can be used to summarize the ideas of the chapter or to introduce a general class discussion of measurement. The film, although originally intended to fit in with the study of Chapter 10, also can be used quite effectively near the end of Chapter 3.

Section 1 - Decision: The Unit of Measurement

PURPOSE To indicate the role of the observer in a measurement, and to point out the ultimate binary nature of decision.

EMPHASIS Treat largely as a reading assignment. Some of your students may enjoy a brief further discussion such as that below.

CLASS DISCUSSION Students will not think that six or seven yes-no decisions are needed to measure the length of a piece of paper as 28 cm. They are overlooking the fact that such decisions are implicit in the process. Some of them were taken in building a ruler so as to avoid having to repeat them each time we wish to measure.

The text refers to the fact that there is a minimum number of decisions which must be made to determine a quantity to a specified degree of precision. Problems 1 and 2 deal with this relationship. The proof of this theorem is somewhat difficult, but many of your students should be able to see that there are efficient and inefficient ways of establishing the value of a quantity through a series of binary decisions.

Suppose, for example, we wish to use a series of yes-no decisions to determine the length of a given table to the nearest centimeter with a one-meter rule. The basic problem is to decide how many 1/100th parts of a meter most closely approximate the length of the table. Assuming the table to be not more than a meter long, and making only binary decisions, we could take as many as 100 decisions in measuring the table. For example, we could construct a list of all possible lengths to the nearest centimeter, and successively ask whether the table corresponded to each length:

Length	1 cm	2 cm	3 cm	4 cm	5 cm...	65 cm...	98 cm	99 cm	100 cm
Equal to table?	no	no	no	no	no ...	yes.....	no	no	no

Again assuming the table to be not more than a meter long and using a meter stick as our rule, we might make the following sequence of binary decisions:

Question: Is the table-	Answer	Conclusion
1. longer than 1/2 rule (50 cm)?	Yes	$50\text{ cm} < L < 100\text{ cm}$
2. longer than 50 cm + 1/4 = 3/4 rule (75 cm)?	No	$50\text{ cm} < L < 75\text{ cm}$
3. longer than 50 cm + 1/8 = 5/8 rule (62.5 cm)?	Yes	$62.5\text{ cm} < L < 75\text{ cm}$
4. longer than 62.5 cm + 1/6 = 11/16 rule (68.8 cm)?	No	$62.5\text{ cm} < L < 68.8\text{ cm}$
5. longer than 62.5 cm + 1/32 = 21/32 rule (65.6 cm)?	No	$62.5\text{ cm} < L < 65.6\text{ cm}$
6. longer than 62.5 cm + 1/64 = 41/64 rule (64.1 cm)?	Yes	$64.1\text{ cm} < L < 65.6\text{ cm}$
7. longer than 64.1 cm + 1/128 = 83/128 rule (64.8 cm)?	Yes	$64.8\text{ cm} < L < 65.6\text{ cm}$
8. longer than 64.8 cm + 1/256 = 167/256 rule (65.2 cm)?	No	$64.8\text{ cm} < L < 65.2\text{ cm}$ = 65 cm QED

It is apparent that this process consist of finding which half-part of a given range includes the quantity sought, then finding which half of this part includes the quantity, and so on. It is to be noted that step number 7 narrowed the range within which the desired quantity fell to less than 1 cm. This step specified the quantity to within 1/100th of a meter.* It is no accident that the smallest power of 2 which gives a number greater than

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*At this point we could say that $L = 65.2 \pm 0.4$ cm. This specifies the length within an interval of less than 1 cm. However, the specified range, 64.8 cm-65.6 cm straddles 65 cm in such a way that we cannot be sure that the quantity is not nearer to 66 cm than 65 cm. We could call it either 65 cm or 66 cm and be correct to within 1/100th of a meter. If we wish to specify the length to the nearest whole number of centimeters, one further decision is required in this particular case. This further decision specifies the quantity to about 1/250th of a meter. It follows that when one calls for a measurement "to the nearest whole centimeter", he is sometimes asking for a finer distinction than "to 1/100th of a meter".

100 is 2^7. Seven decisions is the least number which can be taken to specify a quantity to 1/100th (actually $\frac{1}{2^7} = \frac{1}{128}$) of a given standard.

Thus we can determine the least number of decisions required to make any measurement to a specified precision. For example, in Problem 1, we are asked to determine the minimum number of decisions necessary to specify the time to the nearest hour, minute, and second. It does not matter what particular time we are asked to specify. It does not matter whether we are dealing with a clock with one hand (hour), two hands (hour and minute), or three hands (hour, minute and second). Fundamentally, we are being asked to specify an angle to $\frac{1}{12 \times 60 \times 60} = \frac{1}{43,200}$ th of 360° (12 hours). The smallest power of 2 greater than 43,200 is $2^{16} = 56,472$. Therefore a minimum of 16 decisions is necessary to determine the time to within one second. ** We could have arrived at the same result by noting that $2^4 = 16 > 12$; $2^6 = 64 > 60$; $2^6 = 64 > 60$. Adding the exponents, $4 + 6 + 6 = 16$.

Section 2 - Amplification and Display

PURPOSE To point out that the use of instruments for amplification and display does not relieve the human observer from the necessity of making fundamental decisions about the result of a measurement.

EMPHASIS Treat as part of a reading assignment, simply making sure that students see that even the most elaborate devices used in measurement do not take the place of human judgement. They are simply aids to help an observer "see better". Even in simple counting, someone has to decide what is countable and what is not. See COMMENTS under Section 3.

Section 3 - Signals and Noise

PURPOSE To point out that there are inherent limitations on the accuracy of any physical measurement.

EMPHASIS Treat primarily as part of a reading assignment, simply making sure that students understand some of the limitations on precision of measurement.

COMMENTS The examples in the text--the limiting effect of precisely where the end of the table is; the width of the marks on the measuring scale; and the effect, in the realm of atomic size, of Brownian motion; can be used as illustrations of natural limitations on precision of measurement.

* * *

Students may be inclined to think of counting as a form of measurement that can be accomplished with a high degree of precision. Sometimes this is so. However, as the

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**As was the case in our example dealing with measuring a table to the nearest centimeter, if we want to define the time to the nearest whole number of seconds, a further decision may be needed. It will be required if the interval of less than one second stipulated by the 16th decision does not "straddle" a specific second in such a way that all values within the interval are closer to that second than to the preceding or following second.

text points out in Section 1 of this chapter, it is often difficult to decide what to count and what not to count. Perhaps an illustration such as the following could be used to make this point.

If, for some reason, we sent several people to count all the marbles in town, it is unlikely that their count would agree. This would not be due simply to the fact that some would find marbles that others would not find. Sometimes it would be hard to determine what was and what was not a marble. How would we define a marble? By its appearance? By its size? By whether it had been used to play marbles? How big or how small does a glass ball have to be before it is something other than a marble? Could we set a range of sizes and say with confidence that all glass balls within this range are marbles, and all bigger and smaller glass balls are not marbles?

Some boys use steel ball bearings as marbles. How would we distinguish between "steelies" that are marbles and "steelies" that are ball bearings? By the fact that they are in a boy's pocket instead of an automotive parts store? Some of those in the store will find their way to a boy's pocket. Which ones shall we count?

Clearly our count will not be precise unless our counters can agree on a test which will enable them to make a clear-cut binary decision: "It is a marble" or "It is not a marble". While it might be hard to find a useful purpose for such a count as the above, often the purposes for which we want a count will suggest the nature of the test or definition. For example, if we wanted to know how many marbles were swallowed by infants last year, it might be somewhat easier to devise a test for what will be counted as marbles.

Your students can probably think of some of the natural limitations on precision in counting stars.

* * *

Students may be interested to know that the only way to reduce the effects of thermal noise on measurement is to reduce the temperature. There is no degree of sophistication of instruments which can be used to reduce the ambiguity of a single measurement beyond this limit. However, if a sequence of measurements can be made, and if there is some a priori knowledge available about the state of the system (e.g., that it is actually stationary), then various correlation techniques can be used to increase the precision. For example, the average value of the position of a stationary object can be found to arbitrarily high precision: for a sequence of n observations of uncertainty X, the uncertainty in the average (assuming a random set of observations) is $\frac{X}{\sqrt{n}}$. (e.g., in 100 observations, the error is reduced by a factor of 10.)

Section 4 - Black Boxes and Calibration

PURPOSE This section is an elementary discussion of the philosophy of measurement, intended to let students realize that, if they are careful in calibrating their instruments, they need not know everything about the inner workings in order to obtain useful data.

EMPHASIS Treat as a reading assignment.

COMMENT While "black boxes" are often useful, the text is careful to point out that must of the growth of physics depends upon opening black boxes. It is only by detailed knowledge of the "works" of a piece of equipment that we can hope to understand the magnitude of the systematic errors (as contrasted to random errors) which enter into physical measurements. Calibration enables us to determine the relative precision of an instrument, which is a measure of the random errors inherent in its use, but frequently random errors are only part of the story. We may calibrate a thermocouple against a thermometer, but if the thermometer itself is inaccurate, our real precision in the measurement of temperature with a potentiometer reading of the thermocouple emf may be much poorer than we think.

Section 5 - Interaction

PURPOSE To point out that the act of measurement inherently alters the system being measured, and places a further fundamental limit on the precision of the measurement.

EMPHASIS This section alludes to such ideas as the famous Uncertainty Principle of Heisenberg, but a detailed class discussion of such matters would lead to chaos at this point. Let students understand interaction at the level presented without trying to penetrate deeper. Treat as a reading assignment.

COMMENT The innocent-seeming statements of this section stem from deep and controversial philosophical issues. We must answer questions such as ''Is the system real if we cannot measure it?'' (What do we mean by saying that something is there? Is it there even if we do not determine this fact, or is it where we determine it to be. . . there plus or minus some uncertainty due to the measurement?) These are not matters for class discussion, but you may want to stimulate some of your better students outside of class.

Section 6 - Light

PURPOSE Introduction to Part II.

EMPHASIS Treat entirely as a reading assignment.

Chapter 10 - Measurement

For Home, Desk and Lab – Answers to Problems

COMMENT None of the problems for this chapter need be assigned or discussed in class. They may be considered as "thought questions" for students to consolidate their reading. A solution of Problems 1 and 2 cannot be expected unless class time is devoted to a discussion of the material of Section 1 (see previous comments). Problems 4-6 do not require solutions, and are not discussed below.

PROBLEM 1

Analyze the series of decisions you make in discovering that your watch reads 37 minutes and 23 seconds past eleven o'clock. How will the set of decisions change if the watch is turned 90 degrees from its normal position? If you read the face in a mirror what decisions would you make?

This is a problem in information theory. As discussed in the guide for Section 1, 16 decisions are required to determine the time to a precision of 1 second (sometimes 17 decisions are required to determine the time to the nearest whole second). In principle, rotation or reflection of the watch makes no difference since we are still concerned with measuring an angle to a specified precision. However, we may require further decisions to deduce the meaning of the measured angle (an even simpler case – is it AM or PM?).

PROBLEM 2

Which measurement is more complex and why: measuring a table approximately 2.5 feet long to the nearest sixteenth inch, or measuring the diameter of a hair to the nearest hundredth of a millimeter under the microscope? Discuss the decisions required.

This is another problem in information theory. In the case of the table, we are asking for a precision of at least 1 part in $2.5 \times 12 \times 16 = 1:480$. In the case of the hair, we are asking for a precision of 1 part in, perhaps, from 2 to 10 (probably most hair is between .01 and .1 mm in diameter). The first measurement is far more complex. The first measurement requires 9 to 10 decisions ($2^9 = 512$), while the second requires only 1 to 4 decisions ($2^1 = 2$, $2^4 = 16$), although the second measurement may take much more time (setting up a microscope, etc.).

PROBLEM 3

Make a table showing all possible combinations of on-off conditions of a set of five lights. Use these to make an on-off code representing the alphabet.

An exercise in binary arithmetic. Since $2^5 = 32$, there are 32 possible combinations of "on-off" for five lights. These are 00000, 00001, 00010, 00011, 00100, 00101, 00110, 00111, 01000, 01001, 01010, 01011, 01100, 01101, 01110, 01111, 10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111, 11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111.

APPENDIX

Appendix 1. Acceleration in Circular Motion

(Supplement to Teacher's Guide for Chapter 6, Section 6*)

Traditionally one of the hardest subjects to put across in elementary physics is the acceleration of a body moving at constant speed in a circle. Developing a clear understanding is important for several reasons:

1. The blocks to understanding are of a fundamental nature and worth clearing up in their own right.

2. Uniform circular motion is one of the two cases which can be treated relatively simply in which the vector nature of acceleration is important. (The other case is that of a constant acceleration vector).

3. The text uses circular motion to help verify the vector nature of $\vec{F} = m\vec{a}$ (Chapter 21). We must therefore be able to deduce $\vec{a}$ (given $\vec{r}$ and $\vec{v}$) independently of statements such as: "The acceleration points inward along a radius because the force points inwards along a radius".

4. Alternatively, for planetary motion where there is no "string" visible pulling inwards, we need knowledge of $\vec{a}$ to deduce the presence of the invisible gravitational force.

In the following pages we suggest some steps which should help to isolate the difficulties so that they are met one at a time and to bring them out into the open where they may be attacked.

The Objective

Our aim is to find the instantaneous acceleration of a body moving in uniform circular motion. From Chapter 6, we know that the average acceleration, $\vec{a}_{av}$, between times t_1 and t_2 (corresponding velocities $\vec{v}_1$ and $\vec{v}_2$) is

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

The instantaneous acceleration $\vec{a}$ at any time, t, between t_1 and t_2 is the limit of the average acceleration when the times t_1 and t_2 approach each other and approach t.

There are four major difficulties which generally trouble students in calculating the instantaneous acceleration:

1. Given two vectors $\vec{v}_1$ and $\vec{v}_2$, how do we take the difference? (This problem is especially hard when both vectors have the same length and we have no help from our incorrect natural desire to subtract the shorter from the longer.)

2. Which vector do we subtract from which? (Do we want $\vec{v}_2 - \vec{v}_1$ or $\vec{v}_1 - \vec{v}_2$? It does not matter as long as we divide by $t_2 - t_1$ or $t_1 - t_2$ respectively, noting that dividing a vector by a negative scalar reverses the direction of the resultant vector.)

3. How do we choose a proper scale? An arbitrary scale for drawing an acceleration vector may be chosen independently from the scale chosen for the velocity vector. In

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*This supplement was originally prepared for Chapter 21 of Part III. Because of its relation to the work in Chapter 6, it is reproduced here. You may wish to refer to it again when you are teaching Chapter 21.

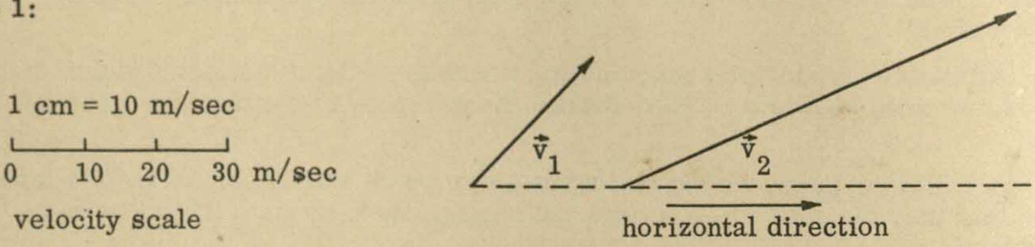
particular, we can not draw any vector without specifying the scale we are using. General haziness about scales probably explains why many students can find $\vec{v}_2 - \vec{v}_1$, but do not know the next step to take to draw the acceleration.

4. What happens when we take the limit of the average acceleration? Both the limiting magnitude and the limiting direction must be found. Students have difficulty seeing that although the vector $\vec{v}_1 - \vec{v}_2$ grows shorter when t_2 and t_1 get closer, the ratio $\frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$ approaches a limiting value.

Sample Sequence of Problems

It is essential that students draw some velocity and acceleration vectors themselves. Many sample problems can be devised. One sequence which illustrates what can be done is given below. If you use such problems in class, it would be best to have them duplicated by some process such as ditto or mimeograph.

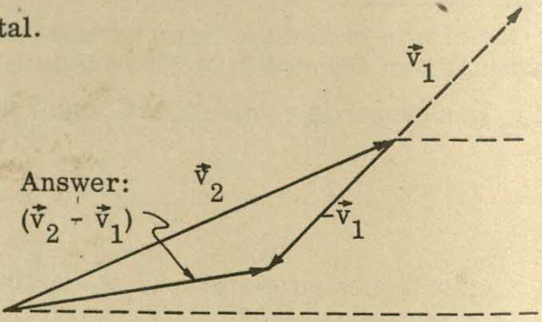
Example 1:



Problem: Find $\vec{v}_2 - \vec{v}_1$ graphically. Find its magnitude in meters/sec, and give the direction in terms of an angle from the horizontal.

Solution: magnitude of $(\vec{v}_2 - \vec{v}_1) = 38$ m/sec.

direction of $(\vec{v}_2 - \vec{v}_1)$:
9° above horizontal and to the right



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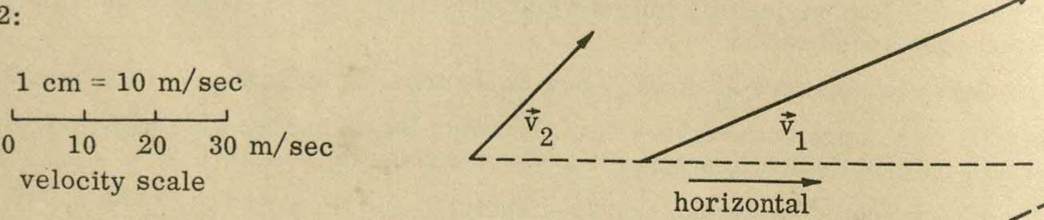
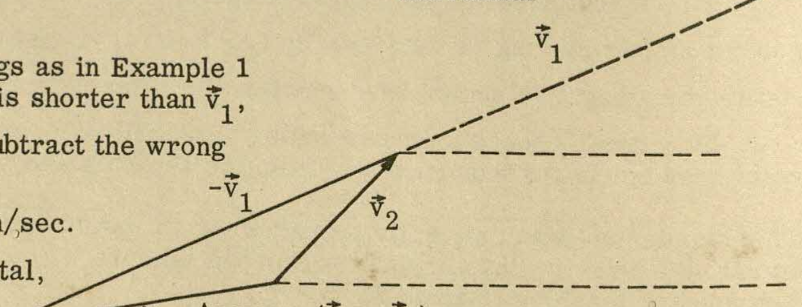
Example 2:



Problem: Find the same things as in Example 1 (find $\vec{v}_2 - \vec{v}_1$). (Here since $\vec{v}_2$ is shorter than $\vec{v}_1$, students may be tempted to subtract the wrong way.)

magnitude of $\vec{v}_2 - \vec{v}_1 = 38$ m/sec.

direction: 9° below horizontal,
to the left.



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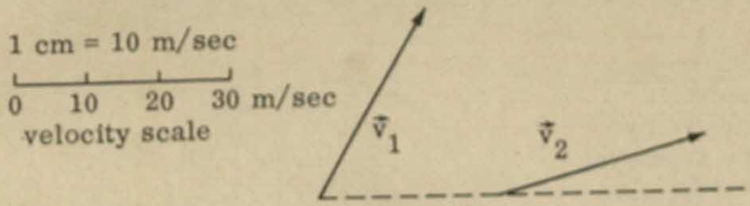
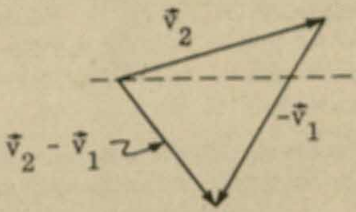
Example 3: Same as Examples 1 or 2, (find $\vec{v}_2 - \vec{v}_1$), except we use two vectors of the same length.





magnitude of $\vec{v}_2 - \vec{v}_1 = 23$ m/sec
direction: 52° below horizontal to right

The examples so far merely stress the subtraction of vectors. The next element to be added is computation of the acceleration.

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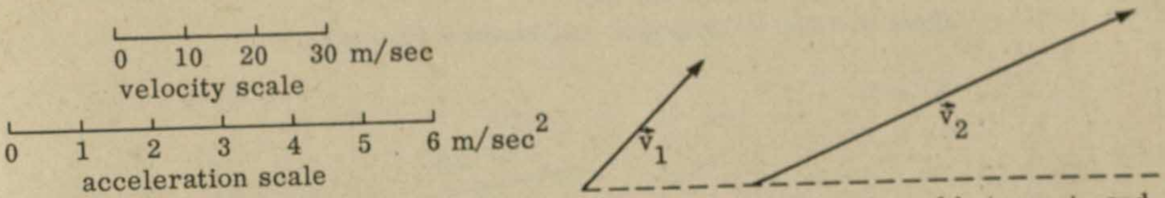
Example 4: Given $\vec{v}_1$ and $\vec{v}_2$ the same as in Example 1, $t_1 = 2$ seconds, $t_2 = 6$ seconds.

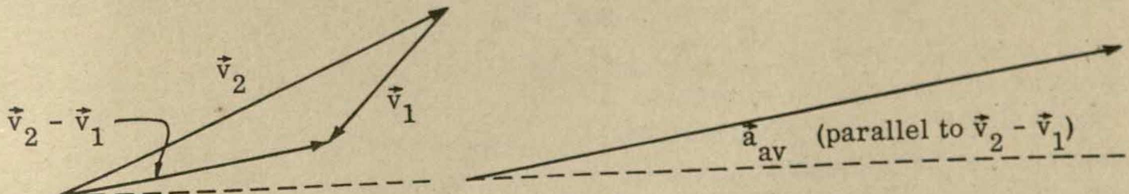


Problem: Find the magnitude of the average acceleration in the interval between t_1 and t_2. Draw a vector of the acceleration to scale showing its magnitude and direction.

Solution: The average acceleration, $\vec{a}_{av}$, is given by $\vec{a}_{av} = \dfrac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$.

Its direction is parallel to $\vec{v}_2 - \vec{v}_1$, and its magnitude is found by dividing the magnitude of the vector $(\vec{v}_2 - \vec{v}_1)$ by the time interval $t_2 - t_1$.

From Example 1 the magnitude of $(\vec{v}_2 - \vec{v}_1)$ is 38 m/sec, and $t_2 - t_1 = 6 - 2 = 4$ seconds. Therefore, the magnitude of the acceleration is $\frac{38}{4} = 9.5$ m/sec^2. Using the scale of acceleration we plot the vector of proper length, parallel to $\vec{v}_2 - \vec{v}_1$.



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It may be necessary to give several other examples to drive home the steps of

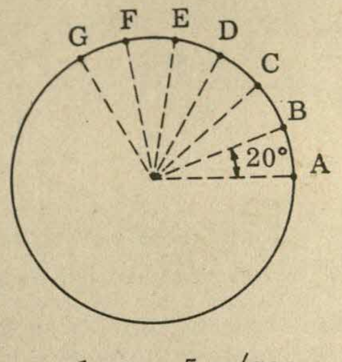
(i) Find $\vec{v}_2 - \vec{v}_1$.

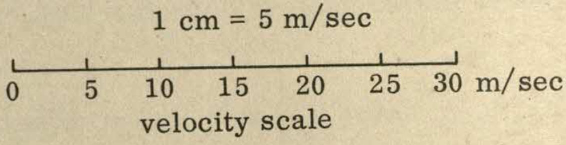
(ii) Measure the length of $(\vec{v}_2 - \vec{v}_1)$ and, from the scale, determine the magnitude of $(\vec{v}_2 - \vec{v}_1)$ in m/sec.

(iii) Divide the magnitude of $(\vec{v}_2 - \vec{v}_1)$ by the time interval $(t_2 - t_1)$ to get the magnitude of the average acceleration.

(iv) Using the acceleration scale to get the length, plot the vector $\vec{a}_{av}$ as parallel to $\vec{v}_2 - \vec{v}_1$.

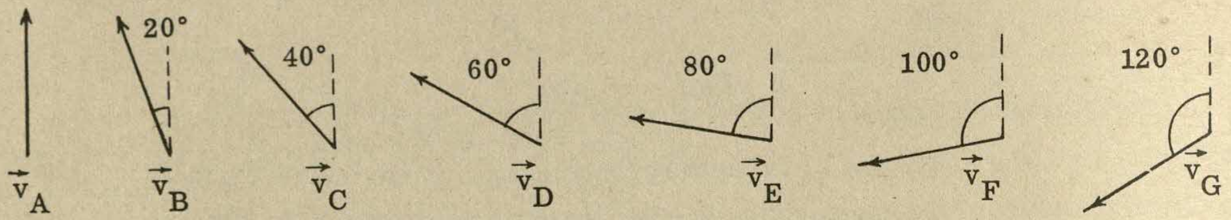
Example 5: We now turn to circular motion. We show a circular path with consecutive points A, B, C, D, E, F each separated by 20° of arc. We assume the particle is moving at 10 m/sec and that it takes 5 seconds to go from A to B, B to C, etc.



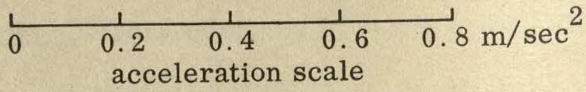


Problem: Draw a series of separate diagrams showing the instantaneous velocities $\vec{v}_A$, $\vec{v}_B$, $\vec{v}_C$, etc., at the points A, B, C, etc. Draw all velocity vectors to the scale shown and indicate the direction of each by a clearly specified angle.

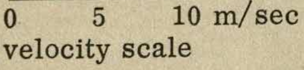
Solution: The velocity vectors are tangent to the path, point in the direction of motion, and have a length corresponding to 10 m/sec on our scale. Thus we have:

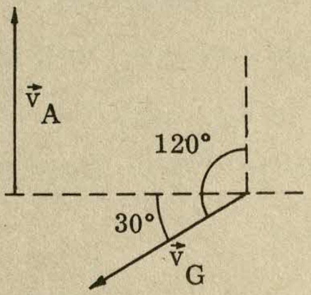


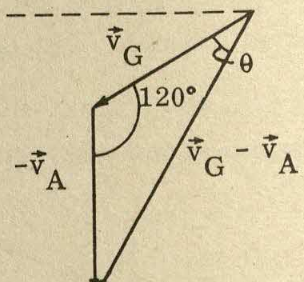
Problem: Compute the magnitude and direction of the average acceleration for the following intervals, and draw them on a clearly labeled figure using the scale indicated: Intervals AG, AE, AC, AB.



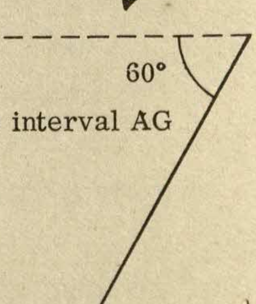
Solution: First we compute $\vec{a}_{av}$ for the interval AG







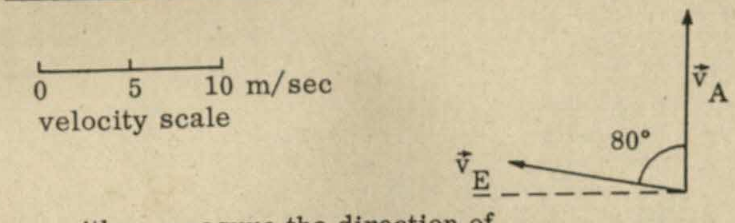
Measurements show that the magnitude of $\vec{v}_G - \vec{v}_A$ is 17.5 m/sec. By measurement or geometry (using the fact that we have an isosceles triangle) we find the angle θ is 30°, and since $\vec{v}_G$ points 30° below the horizontal, we see that $\vec{v}_G - \vec{v}_A$ is to the left at 60° below horizontal. The time interval $t_G - t_A$ is 30 seconds. Therefore, the average accelera-

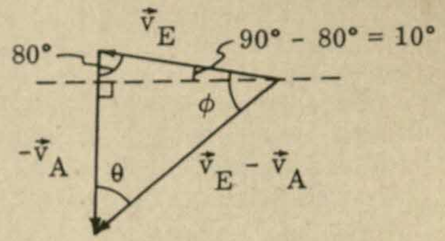


$\vec{a}_{av}$ for the interval AG

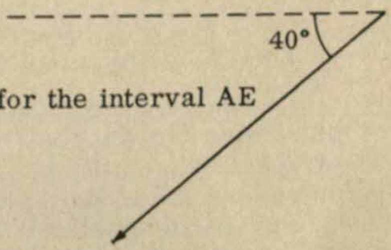
tion between A and G is 0.58 m/sec^2 in the direction 60° below the horizontal toward the left.

For the interval AE, we construct the figure below:



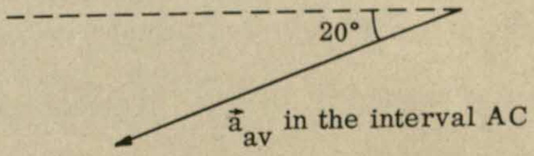


We can either measure the direction of $\vec{v}_E - \vec{v}_A$, or else use geometry. By the fact that we get an isosceles triangle, we see that $\theta = \phi$. Therefore, $2\phi = 180° - 80° = 100°$; $\phi = 50°$ so $\vec{v}_E - \vec{v}_A$ lies at 40° below the horizontal and to the left.

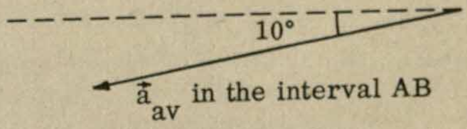


Measuring the length of $\vec{v}_E - \vec{v}_A$ shows its magnitude is 13 m/sec. The time interval is 20 seconds. The average acceleration has a magnitude $\frac{13}{20} = 0.64$ m/sec^2.

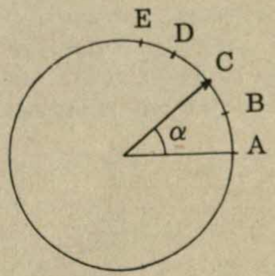
For AC the answer is $\vec{a}_{av}$ has a magnitude of 0.68 m/sec^2 and a direction 20° below horizontal and to the left.



For AB, $\vec{a}_{av}$ has a magnitude of 0.70 m/sec^2 and points at 10° below horizontal to the left.



Studying the results we note two things:



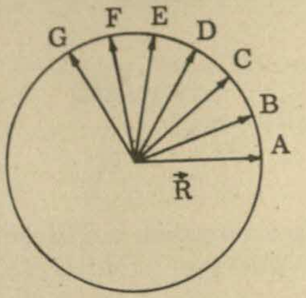
(i) The magnitude of the acceleration vectors is nearly constant for points which are moderately close.

(ii) The direction of the acceleration vector points parallel but in the opposite sense to the radius vector from the center of the circle to the midpoint of the arc. Thus, in the interval AE, the midpoint is C, and the radius vector from the center of the circle to point C makes an angle α 40° to the horizontal, pointing up and to the right, while $\vec{a}_{av}$ is 40° below horizontal to the left. Your bright students may enjoy proving the relationship described in this paragraph.

Notice in particular that the relationship in (ii) above shows, as do the drawings, that as our two points get closer together, the average acceleration over the interval gets closer and closer to perpendicular to the velocity at any point in the interval. Therefore, the instantaneous acceleration is perpendicular to the instantaneous velocity, and "antiparallel" to the radius vector at a given instant.

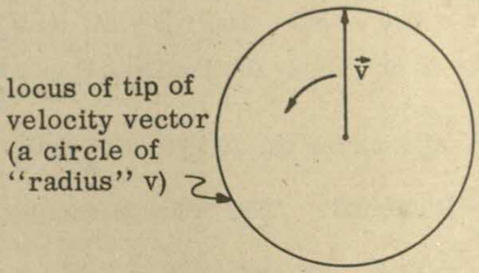
Derivation of v^2/r relation: The previous problems lead naturally to a derivation of the magnitude of the acceleration.

This problem is considered in Part III of the course. Unless you have an unusual amount of time to cover Part I and unless your class has grasped these ideas firmly, you will want to leave this derivation to Part III.



In Example 5 the position of the particle in its circular path could be represented by a radius vector $\vec{R}$ which is uniformly rotating in the circle with its tail at the center of the circle and its head at the particle. The velocity, $\vec{v}$, of the particle is at every instant perpendicular to R, and if the particle makes one revolution in the time T, the magnitude of v is $\frac{2\pi R}{T}$.

Our sequence of velocity vectors $\vec{v}_A$, $\vec{v}_B$, etc. in Example 5 suggests a convenient way of describing the velocity: it changes in time as though pivoted at its tail, turning at the same rate as the radius vector (since $\vec{v}$ is always perpendicular to $\vec{R}$). Because the acceleration is the rate of change of velocity with t it has the same relation to the rotating vector $\vec{v}$ in this circle that the velocity had to the rotating position vector ($\vec{R}$) in the previous circle. Therefore the magnitude is $a = \frac{2\pi v}{T}$.



Eliminating T between the two equations $v = \frac{2\pi r}{T}$ and $a = \frac{2\pi v}{T}$ gives the desired relation that the acceleration everywhere in the circle is: $a = \frac{2\pi v}{T} = \left(\frac{2\pi}{T}\right) v = \left(\frac{v}{r}\right) v = \frac{v^2}{r}$. The direction of $\vec{a}$ must be towards the center of the circle at every instant because, since $\vec{v}$ (the rate of change of $\vec{R}$) is perpendicular to $\vec{R}$, the acceleration $\vec{a}$ (the rate of change of $\vec{v}$) must be perpendicular to $\vec{v}$.

This derivation of centripetal acceleration is the same as that given in Chapter 21 and also in the movie "Deflecting Forces". Since both the textbook and the film derive this equation in relation to forces, you would not want to refer the student to either of these sources if you decide to take up this subject in connection with Chapter 6 of Part I.

APPENDIX

Appendix 2. Supplementary Notes on Nuclear Physics

(Supplement to Teacher's Guide for Chapter 8, Sections 8 and 9)

A. Atomic Masses of Isotopes

The nucleus is composed of Z protons and N neutrons. The main contribution to the mass of an atom is the sum of the masses of protons, neutrons, and electrons. Thus to a first approximation, the

Mass of an atom = Z × mass of proton + N × mass of neutron + Z × mass of electron − (smaller terms discussed below). Or, we could write, mass of an atom = Z (mass of proton + mass of electron) + N (mass of neutron) - (smaller terms).

In atomic mass units:

$$\text{Mass of an atom} = 1.008142\ Z + 1.008982\ N - (\text{small terms})$$

$$\text{Mass of atom} \gtrsim 1.01\ (Z + N) - (\text{small terms})$$

Z + N is called the mass number and is usually given the symbol, A.

Very precise measurements show that the mass of the atom is not 1.01 A as it would be if the small terms mentioned above were absent. Indeed, the atomic masses of isotopes (not elements) are much closer to 1.00 A than to 1.01 A; of course, the above relation is exact for O^{16}, since this is the atom on which the atomic mass scale is based. It was this known fact which made physicists choose O^{16} as 16.00000, the standard, instead of choosing the hydrogen atom or the neutron as the standard.

B. Binding Energy and Mass Defect

The small terms mentioned above are now well understood by physicists. Consider the simplest atom with more than one particle in its nucleus; this is deuterium, or heavy hydrogen. When a light (or ordinary) hydrogen atom and a neutron are near each other (but have not combined to form deuterium), the sum of their masses is exactly 1.008142 + 1.008982 or 2.017124, as would be expected by simply adding masses. However, after they combine, the deuterium mass is only 2.014735. Thus, during the combination, 0.002389 of a mass unit was "lost". This loss is well understood in terms of the equivalence of mass and energy first enunciated by Einstein. The mass difference of 0.002389 atomic mass units equals about 3.94×10^{-30} kg; this mass multiplied by c^2 (i.e., $(3 \times 10^8\ \text{m/sec})^2$) is the energy given off when the neutron and hydrogen atom combine. Thus, the Einstein relation predicts that 3.55×10^{-11} joules of energy must have been given off during the combination. (In more usual nuclear energy units, this is 2.22 mev). If we observe neutrons being captured by hydrogen, we indeed find that, for each combination, a gamma ray of energy 3.55×10^{-11} joules (2.22 mev) is emitted. Thus the equation becomes:

neutron + hydrogen atom → deuterium atom + gamma ray, and the mass-energy balance becomes:

$$1.008982 + 1.008142 \rightarrow 2.014735 + 0.002389$$

The "missing mass" (usually called the mass defect) was carried off as the gamma ray.

The formation of deuterium from a neutron and hydrogen is reversible; a gamma ray whose energy is 3.55×10^{-11} joules (2.22 mev) or more can split deuterium into a neutron and hydrogen. This energy (which was released at the time of formation) is called the binding energy. In order to get a balance of mass-energy, one must remember to include the term Δm = binding energy/c^2 as the "small terms" to be subtracted from the masses of the atomic constituents above.

The presence of "binding energy" explains why the mass of an atom is not the mass of its pieces. If a large atom were formed from individual hydrogen atoms and free neutrons, there would be an energy release (in the form of gamma rays) at the moment of formation. This energy release is analogous to the release of light when a proton captures an electron to form a hydrogen atom.

C. Why Isotopic Masses are Whole Numbers

The reason that O^{16} is a convenient substance on which to base the atomic mass scale is that the binding energy per particle (i.e., per proton or per neutron) in O^{16} is very nearly equal to the binding energy per particle in most nuclei. The total mass conversion (or "mass defect") in O^{16} is $(8 \times (1.008142 - 1.000000) + 8 \times (1.008982 - 1.000000))$ which is 0.143992 mass units; this is an average of 0.008562 mass units per particle (or about 8 mev binding energy per particle).

The fact that the average mass converted on binding is about 0.00856 mass units does not mean that the equivalent energy is released whenever a single proton or neutron is added to a nucleus. Sometimes a smaller mass conversion occurs and sometimes the mass conversion is larger. But these fluctuations which occur for the addition of a single particle tend to average out after several particles have been added. Because this average mass loss per nuclear particle does not change much, isotopic masses expressed in atomic mass units, are very nearly whole numbers. The fluctuations in the binding energy of a single extra particle are relatively unimportant to the value of the total mass – but these same fluctuations are extremely important to the nuclear physicist, who constantly measures and tries to interpret these fine effects.

D. The Neutron

The neutron is a particle which, in a very crude sense, behaves like an extremely condensed hydrogen atom; because of this, the text does not dwell on neutrons but merely lumps them in the term, "hydrogen unit". The neutron is about 10^{-15} m in diameter, whereas atoms, including hydrogen, have a diameter of about 10^{-10} meters.

If the neutron is not inside the nucleus it decays into a proton, an electron, and an antineutrino. This decay is like any other radioactive process in many respects; the half-life of the free neutron is about 12 minutes. When the neutron is inside a nucleus, nuclear binding forces usually prevent it from decaying. There are times when a neutron inside a nucleus changes into a proton, and an electron and an antineutrino are emitted. However, there are other nuclei in which a proton changes into a neutron and simultaneously emits a positron (which is an "anti-electron") and a neutrino. These transmutations are called beta decays or beta radioactivities. When the text refers to neutrons being "made from hydrogen units in the process of condensation" (on page 136) it is a slight oversimplification. The only way a proton and an electron can combine to form a neutron is with the aid of an antineutrino. This process has been observed but it is a very rare process. It is more likely that neutrons are formed in stars by complex nuclear processes in which the principle neutron production comes from radioactive beta decays of nuclei in which protons are converted to neutrons.

E. Isotopes

The simplest family of isotopes are those of hydrogen. In the normal hydrogen atom, the nucleus consists of a single proton. Another isotope of hydrogen, deuterium, has a nucleus consisting of 1 proton and 1 neutron. Deuterium is stable and is found in nature. A third isotope of hydrogen, H^3, is called tritium; it has 1 proton and 2 neutrons but is radioactive. With a half-life of about 12 years, H^3 becomes He^3 by emitting an electron and an antineutrino. (He^3 is a stable isotope of helium: it has 2 protons and 1 neutron).

At least one stable isotope exists for each integral number of mass units from $A = 1$ to $A = 209$ with the exception of $A = 5$ and $A = 8$. Some A values have several stable isotopes, so that there are about 280 altogether. Furthermore, there are many other isotopes which are radioactive, but which have been produced and studied. This list now extends above mass number 250 and elements of 102 protons. The only mass number which does not even have a single radioactive element associated with it is $A = 5$; these nuclei are never formed because they fly apart immediately into two pieces, ($A = 4$ and $A = 1$, or $A = 2$ and $A = 3$).

Most elements have several isotopes stable enough to be found naturally on earth: the exceptions are beryllium ($_4Be^9$), fluorine ($_9F^{19}$), sodium ($_{11}Na^{23}$), aluminum ($_{13}Al^{27}$), phosphorous ($_{15}P^{31}$), scandium ($_{21}Sc^{45}$), manganese ($_{25}Mn^{55}$), cobalt ($_{27}Co^{59}$), arsenic ($_{33}As^{75}$), yttrium ($_{39}Y^{89}$), niobium ($_{41}Nb^{93}$), technetium (43 protons, no stable isotopes), rhodium ($_{45}Rh^{103}$), iodine ($_{53}I^{127}$), cesium ($_{55}Cs^{133}$), praesodymium ($_{59}Pr^{141}$), promethium (61 protons, no stable isotopes), terbium ($_{65}Tb^{159}$), holmium ($_{67}Ho^{165}$), thulium ($_{69}Tm^{169}$), tantalum ($_{73}Ta^{181}$), gold ($_{79}Au^{197}$), and bismuth ($_{83}Bi^{209}$). One interesting and significant fact is that all the above mass numbers are odd; further, except for beryllium, the proton numbers shown as the left subscript are also odd. This fact has been explained within the last ten years on the basis of a nuclear model which exhibits a shell structure, somewhat similar to the shell structure and periodicity of atoms.

The other sixty elements, all have at least two isotopes found in nature; some have many more, tin being the record holder with 10 stable isotopes.

F. Other Fundamental Particles

Physicists, during the last 30 years, have found many particles in addition to the proton, electron, and neutron mentioned here. A complete list would include (in order of increasing mass) the neutrino, the positron, mu mesons, pi mesons, K mesons, antiprotons, antineutrons, lambda particles, sigma particles, and Xi particles. None of these particles exist for very long without interacting with something or decaying. Physicists are still trying to understand the role of these new particles and whether they have any pattern. As far as we know, they do not play a role in determining the chemical behavior of atoms.

TEACHER'S GUIDE FOR LABORATORY EXPERIMENTS

1. The Place of the Laboratory in This Course

Our knowledge of physics is the result of years of experimentation. No student can experience all the discoveries that have been made, but whenever possible we should like him to learn physics in the laboratory. His ability to understand descriptions of the discoveries of others rests on his having real experience himself. He profits most by making his own discoveries.

In this course, the laboratory not only replaces teachers' demonstrations to a large extent, but it also reduces the time spent in classroom explanations. The laboratory work is closely correlated with the text, and it is of equal importance.

The Student's Laboratory Guide provides a minimum set of directions and calls the student's attention to the important points in an experiment by raising questions. Sometimes the answers to these questions may require thought alone; at other times further experimentation is needed. The student must decide what to do. At the beginning of the course, some students may feel a little insecure with this type of laboratory work. They are likely to ask whether they have the right result. You must assure them that nature is not wrong; our job is to understand it by measurement and interpretation. If a student has not measured what he set out to measure, a discussion, rather than a "yes" or "no" answer, is in order.

In most cases we recommend that the experiments be done before their topics are taken up in the text. In this way the students are not told what to expect. As they progress in the course, they learn to enjoy doing experiments whose results they do not know in advance. They like to do research even though they realize that someone has faced the same problem before. Students face questions and look harder for their own answers in this kind of laboratory than they do while reading the text or even during class discussion.

2. Doing an Experiment

When a physicist does an experiment, he has a goal in mind. We suggest that you take time in the class period before the laboratory to discuss with your students the purpose of the experiment. The aims may be very general. For example, in the experiment on the spectra of elements (Experiment I-7), we wish to see whether each kind of atom gives off its own characteristic light, a pattern of light by which we can identify the atomic species. The main purpose of the first ripple-tank experiment (Experiment II-8) is to observe waves and to see for oneself how waves behave (before considering any fancy theory or description by others). In Part III, the experiment on slow collisions (Experiment III-11) is intended to provide only a qualitative study of what happens during a collision.

Other experiments are quantitative in nature. For example, in Experiment I-5 a given motion is investigated in detail. In Experiment II-3 the purpose is to find the relation between the angles of incidence and refraction. In Experiment III-2 we wish to discover how a constant force changes the velocity of a cart. (Notice that the objectives of the last two experiments are not to verify that the ratio of the sines of the two angles is constant, or that a constant force produces a constant acceleration.) In Experiment III-2 some students may tell you that the apparatus is wrong; the cart does not go at a steady speed. Students need only a little encouragement to find their own answers; their questioning and the apparatus will help them to discover Newton's law. Thus, the law becomes more meaningful to them than it would be through memorizing a phrase in a book. In your discussion of an experiment prior to the laboratory period, it is important not to give away the expected results (though you don't have to pretend that they are not known, even to some of the students).

3. Experimental Accuracy

Whenever necessary, we suggest that you have students repeat a measurement several times, take an average, and judge the accuracy of the results from the fluctuations. When this is not possible, have the students try to estimate the error they expect, and express it as a fraction or per cent. For example, the measurement of the time of 30 revolutions of the stroboscope in Experiment I-1 should be repeated three or four times to get an idea of the accuracy. For many experiments, this Teacher's Guide will indicate the accuracy you may reasonably expect.

There seems to be a widespread belief that all measurements must be accurate to three significant figures and that a result with an error of 50% is worthless. This is not true; as an experimenter, the accuracy you demand is determined by the knowledge you have before the experiment and by the purpose of the experiment. For example, suppose a student has no idea whether the size of an oleic acid molecule is 10^{-10} or 10^{-4} m. From Experiment I-8 he can conclude that it is not more than, and very close to, 10^{-9} m. This enriches the student's knowledge, and, what is more, he is likely to get a real thrill. The small sizes of molecules are no longer just words to him; they become part of his world. It is irrelevant at that point to try to push the accuracy further. Similarly, students have no a priori knowledge of the order of magnitude of the mass of the electron. The real achievement in Experiment IV-10 is to see that the mass is of the order of 10^{-30} kg and not 10^{-25} or 10^{-32} kg. In general, the less we know about what we want to measure, the lower the accuracy we require to gain new information. On the other hand, if the student wants to distinguish whether the ratio of angles of incidence and refraction or the ratio of their sines is constant, he must measure the angles accurately enough to tell the difference.

4. Analysis of Results

Setting up apparatus, taking data and analyzing them, and finally drawing conclusions constitute an experiment; just taking data does not.

Very often, experimental results are best summarized in graphs. A graphical presentation is particularly useful when interpolations and extrapolations are required. Make certain your students learn how to draw and read graphs. Experiment I-4 is especially designed to help them in this.

The analysis of the data in many experiments, particularly in the later Parts of the text, is quite time-consuming and should be completed at home. A slide rule is a very useful, timesaving device.. We recommend its use.

Concerning laboratory reports: More often than not, these are a chore for the student who has to write them and for the teacher who must read them. We recommend that each student keep a neat record of what he does at the time he does it. Only occasionally should an experiment be written up in a formal report.

5. Planning

The value of each experiment is greatly increased by doing it at the proper time. The first paragraph of the discussion of each experiment in this Guide will help you to set up your schedule. Also, at the beginning of the discussion of each Part of the course, just after the preface, there is a table which indicates the relation between the experiments and the textbook.

The experiments are flexible in order to fit with laboratory periods of various lengths and the abilities of different students. There is usually enough time in a normal laboratory period for the students to take the basic measurements and do some of the paper work. The rest should be done at home. Not all students should do everything suggested in the Laboratory Guide. Often the last few questions are for the more ambitious students and may involve additional experimental work in the nature of a special project. Such open-ended parts are often indicated in this Guide.

6. Apparatus

The apparatus designed to be used in the experiments is available from a commercial supplier in kit form and can easily be assembled. Students who wish to modify the apparatus, or to build it themselves from materials available in the laboratory or at home, should be encouraged.

7. Acknowledgments

The laboratory part of the Teacher's Guide was prepared by Judson B. Cross, James M. Henry, Ervin H. Hoffart, Guenter Schwarz, James S. Strickland, John H. Walters, and me. Percy Lund made the line drawings. Miss J. Carolyn Safford assisted in the editing and production of the preliminary edition of the Guide. Many useful comments were made by persons working primarily on other parts of the PSSC project and by teachers who corresponded with us.

Uri Haber-Schaim

TEACHER'S GUIDE FOR EXPERIMENTS, PART I

The experiments in this Part, like the text itself, set the stage for the entire course. Their primary purpose is to make the student realize that in order to know the size of things, the time it takes them to move, or their mass, he must make measurements. Unaided, his ability to measure extends only over a very limited range. The instruments he will use will enlarge this range by many orders of magnitude. The particular devices and techniques are of secondary importance.

This Part has only nine experiments, and we recommend that you use all of them. Their relative priority and their relation to the text are shown in the table below.

Figures in the Teacher's Guide are labeled alphabetically; figures labeled numerically are in the Laboratory Guide.

Apparatus lists at the ends of the experiments specify materials for one setup -- which, in this Part, is intended for two students. Materials for general class use appear at the end of the list.

Number	Experiment	Best Time	Priority
I-1	Short Time Intervals	Before discussion of Section 2-5	***
I-2	Large Distances	During Section 3-2	***
I-3	Small Distances	During Section 3-3	**
I-4	Analysis of an Experiment	During Chapter 4	***
I-5	Motion: Speed and Acceleration	After discussion of Section 5-3	***
I-6	Small Masses	After discussion of Section 7-2	**
I-7	The Spectra of Elements	During Section 7-9	**
I-8	Molecular Layers	Any time after Section 7-9	***
I-9	Natural Temperature Scale	Before Section 9-4	**

*** essential
** desirable
* optional

I-1. SHORT TIME INTERVALS

In this experiment the students calibrate a vibrating bell clapper (which performs a repetitive motion with a very short period) against a watch (which has a relatively long period). After the calibration is completed, the clapper is used to measure short times which are not necessarily associated with repetitive motion.

The experiment is best done during the study of Chapter 2, preferably before the discussion of Section 2-5. If the laboratory period is one hour or less, two periods may be needed to complete the whole experiment. This is certainly worth while, considering the importance of the experiment and the slow pace at the beginning of the course. If you prefer to do only part of the experiment, convenient stopping places have been indicated by dashed lines in the Laboratory Guide.

This is the place to start expressing accuracy in terms of per cent. It may be best introduced quantitatively during a follow-up discussion.

The HDL exercises at the end of Chapter 2 include a number of problems relating to the stroboscope which might well be assigned.

Here are some useful technical details. Good lighting is important because of the short observation times through the narrow slits of the stroboscope. One can prevent rapid damping of the motion of the flat steel blade by fastening it with a C clamp as shown in Fig. 2. The flat steel blade should always be set in motion by pulling the C clamp. Pulling the free end of the blade results in a complex motion difficult to observe. A white chalk mark on the free end of the blade makes it easier to see.

The striker block may be removed from the timer before clamping it to the table. When the students clamp the timer to the edge of the table as shown in Fig. 1, be sure they do not bend the frame by tightening the clamp too much.

The disc stroboscope may be held as shown in Fig. 3 or with the handle turned toward the face. In either case, it is best to have the slit approximately parallel to the long dimension of the vibrator when the object is observed.

The reason why the period of vibration of the blade changes when the position of the C clamp is changed has no direct bearing on the experiment and need not be discussed at this time.

To "stop" the motion of the clapper and clothespin with four open slits in the disc, the stroboscope must be turned about 4 or 5 times per second. If a student has difficulty turning the disc at this high rate for any length of time, he may use six open slits.

The period of the timer is about $\frac{1}{40}$ second. It varies with the adjustment of the make-and-break contact and with the voltage of the battery. Only one timer should be connected to a battery at a time.

The most convenient way to calibrate the timer is to pull the paper tape through for 5 or 10 seconds, count the marks, and divide the total time by the number of marks. It is not necessary to pull the tape at constant speed.

Some adjustment of the clapper may be necessary to get it to strike the tape hard enough to leave a visible mark. The circular disc will rotate as the tape is pulled, keeping fresh carbon under the tape.

Answers to Questions

With a wrist watch one can find only the order of magnitude of the time of a single vibration (about 1 sec). The time of ten vibrations can be measured to within 10%.

When the motion of the clamp is "stopped" at one end of its swing by using a stroboscope with one slit open, the time of one rotation is equal to the time of one vibration.

The motion of the clamp will also be stopped at one end if the disc is turned half as fast. Thus, the time of one rotation would be equal to the time of two vibrations. When the C clamp is at the end of the blade, it will be difficult to turn the disc evenly enough to stop the motion at half the normal rate. It is best to check this effect later in the experiment when the times of vibration are shorter.

To make sure that you have measured the period correctly, after finding a rate that stops the motion, increase the rate of rotation continuously until it is doubled. The period is correctly determined by the highest rotation rate that stops the motion.

With two slits open, the time of one rotation of the stroboscope is equal to the time of two vibrations of the blade.

By using a stroboscope with twelve slits, turned directly by hand, students have extended their ability to measure short times by about a factor of forty (about 1 sec to $\frac{1}{40}$ sec).

The timer can be calibrated to an accuracy of about 5% by pulling paper tape through for ten seconds and counting the number of marks. This assumes measuring the ten seconds to an accuracy of half a second.

The time of one vibration is greater (by about 5%) when the clapper vibrates freely instead of striking the tape. This is within the experimental error for a calibration time of 10 sec and therefore may not be detected.

Supplementary Observations

Some students may be interested in making their own stroboscopes or in borrowing one from the laboratory to "stop" the motion of other objects in the laboratory or at home. Some interesting things to look at are neon bulbs, fluorescent bulbs, fan blades, bicycle wheels, and drops of water from the top of a thin jet aimed at an angle with the vertical.

APPARATUS

1 12" flat steel blade
2 4" C clamps
1 2" C clamp
1 disc stroboscope with 12 slits
1 $1\frac{1}{2}$-volt timer
    Paper tape
    Carbon-paper discs
1 $1\frac{1}{2}$-volt dry cell battery
1 Clock or watch with sweep second hand
1 Spring clothespin

Black masking tape

I-2.

LARGE DISTANCES

Two devices for measuring large distances, the range finder and the parallax viewer, are used in this experiment. The fundamental idea in each is triangulation. This fact is somewhat obscured, however, in the range finder by an additional factor, 2, coming from the reflection in the mirror.

The experiment is related to Section 3-2. The parallax viewer should not be omitted, because it clarifies the meaning of parallax which is used repeatedly in Part II and it extends over a larger range of distances than the range finder. The range finder can be deleted if Experiment I-3 is assigned. We recommend that you have the students calibrate the instruments in meters.

The Range Finder

An experimental calibration is adequate, but the mathematical calibration is outlined for those interested. When the arm and movable mirror of the range finder are rotated through an angle θ [Fig. (a)], the line ℓ, representing the line of sight from the movable mirror, moves through the angle 2θ. You can check this experimentally with a plane mirror or prove it by using elementary geometry. In effect, when the mirror rotates through an angle θ the incident ray and reflected ray each rotate through an angle θ. If the path of one remains the same, then the other will rotate through an angle of 2θ.

From Fig. (a), $\tan\theta = \frac{s}{c}$ where s is the distance from the zero mark on the scale and c is the distance from the movable mirror to the scale; $\tan 2\theta = \frac{b}{d}$ where b is the distance between the two mirrors of the range finder and d is the distance to the object. Because the range finder is used to measure distances much greater than the base line, $\frac{b}{d} \ll 1$ and the angles θ and 2θ are always small. Therefore $\tan 2\theta \approx 2\theta$ and $\tan\theta \approx \theta = \frac{s}{c} = \frac{b}{2d}$ or $s = \frac{bc}{2d}$.

Values for s can be calculated for different values of d and the scale marked accordingly to complete the calibration.

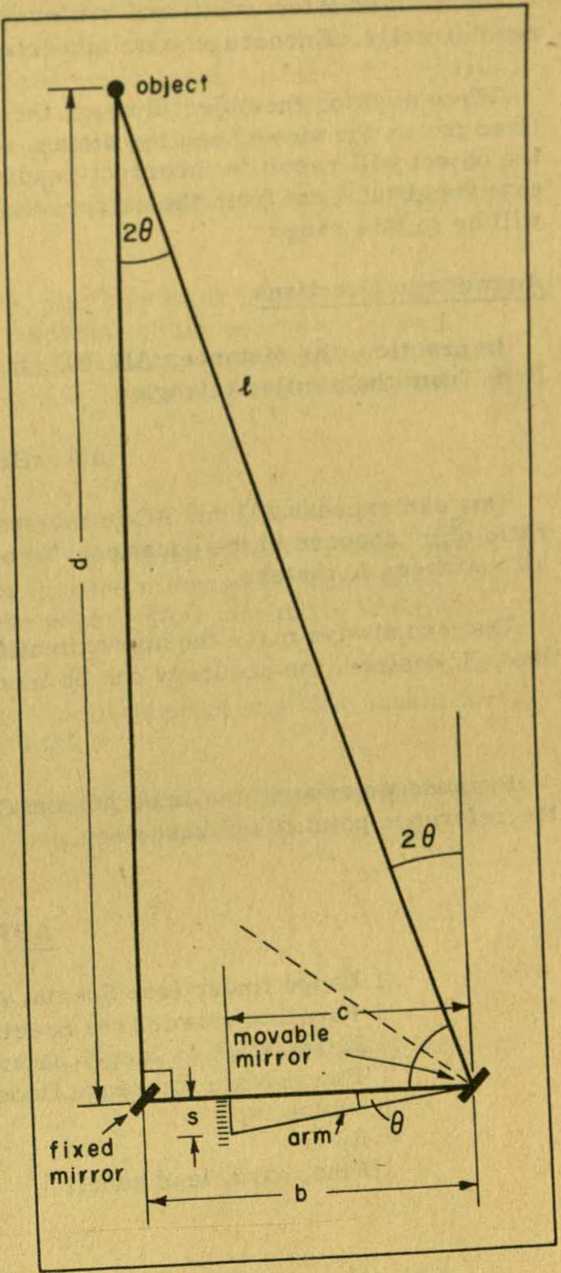


Figure (a)

If the scale is placed in line with the fixed mirror, then b = c and the equation becomes $s = \frac{b^2}{2d}$, which shows that the range of the instrument increases as the square of the base line.

The Parallax Viewer

Unlike the range finder, the parallax viewer does not include the base line. This is an advantage since one can choose a base line of a few meters or several kilometers depending on the distance to be measured. It is convenient, therefore, to mark the scale on the viewer in centimeters and to calculate the desired distance from it. The students can, of course, add a few additional scales for fixed base lines on which the distances can be read directly. Encourage your students to use the viewer over a wide range of distances.

When sighting the object through the pinhole, the eye should be kept in line with the fixed pin on the viewer and the distant reference point. Moving the head while sighting the object will result in incorrect readings. Since, with a pinhole, the range on the scale extends about 4 cm from the center, the reference point must be chosen so the object pin will be in this range.

Answers to Questions

In practice, the distances AD, BD, and CD (in Fig. 4) are practically equal, hence one finds from the similar triangles

$$AD = BD = AC \times \frac{OC}{S_2S_1}$$

One can express AD and AC in meters and OC and S_2 S_1 in centimeters since only the ratio $\frac{OC}{S_2S_1}$ appears in the equation. To avoid confusion, it may be advisable to express all distances in meters.

One can always make the approximation AD = BD by choosing a short enough base line. If desired, the accuracy can be increased by using the relation

$$AD = AC \times \frac{AS_1}{S_2S_1}$$

Because we assume the lines AO and CO to be parallel, the accuracy is highest when the reference point is farthest away.

APPARATUS

1 Range finder (see Special Apparatus Notes for construction)
1 Parallax viewer (see Special Apparatus Notes for construction)
1 Meter stick or long tape measure
1 Paper scale for range finder
Scotch tape
1 Ruler
1 Fine, hard, lead pencil

I-3. SMALL DISTANCES

In this experiment the students use an optical micrometer to measure distances of the order of 10^{-6} m. This experiment relates to Section 3-3 and should certainly be done if the range finder in Experiment I-2 is omitted.

Have the student calibrate the micrometer in the metric system. A calibration by wires of known diameter is more accurate than a calibration with small pieces of paper. However, the calibration with pieces of paper is more fundamental because it requires only a ruler. Furthermore, with pieces of paper, the scale will be marked in multiples of the thickness of one piece of paper. This will facilitate interpolation.

The method of determining the distance between two slits will be useful in Experiment II-14.

If vernier or micrometer calipers are available, you may want to acquaint the students with their use at this time. There is no need for students in this course to learn to read a vernier, but it won't hurt them if it is done incidentally. The students will gain confidence in the accuracy of the optical micrometer by comparing measurements made with it with those made by a micrometer caliper.

Answers to Questions

The purpose of the question is simply to make the student aware of the large magnification of the thickness of the paper. It is evident that the micrometer can register a thickness even a few times smaller than that of the paper. No quantitative statement can be made at this point.

The distance between cuts made with two razor blades pressed together equals the thickness of one razor blade.

The assumptions are that the blades are of equal thickness and that the blade edge is in the center of the blade's thickness. To find the distance between cuts, one has only to measure the thickness of one razor blade.

APPARATUS

1 Optical micrometer
2 Double-edged razor blades
   Assorted materials to measure: aluminum foil, saran wrap, #40 wire, hair, etc.
1 Piece of paper for calibration scale
1 Ruler

   Vernier caliper
   Micrometer caliper
   Tablet or ream of paper
   Small slips of paper of same weight as above

I-4. ANALYSIS OF AN EXPERIMENT

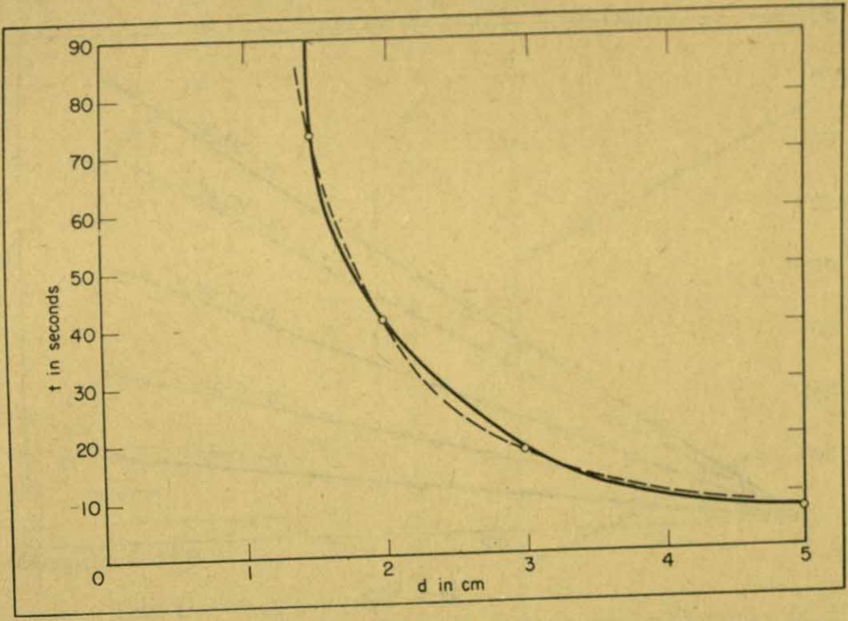


Figure (a)

This experiment is designed to give your students the opportunity to devote a full laboratory period to the analysis of an experiment under your guidance. The experiment has been written to accommodate students with different mathematical backgrounds. It relates to Chapter 4 of the text and should be done after some of the HDL exercises have been assigned.

All students should complete the graphs of t versus d and t versus $\frac{1}{d^2}$ and one graph of t versus h. The better students can be encouraged to proceed to the double interpolation and the investigation of the mathematical relation between t and h. Only students who have studied logarithms in mathematics should attempt to deal with the last two paragraphs.

Millimeter paper is recommended for the graphs. Students can make their own semi-log or log log graph paper by using a plain sheet of paper and the C or B scale on a slide rule.

Answers to Questions

Fig. (a) shows the t versus d plot for $h = 30$ cm on the largest scale. One can clearly see that the curve is not unique even if no unusual variations between measured points are assumed. The x in Fig. (b) shows the interpolated point at $d = 4$ cm. To extrapolate to $d = 8$ cm, the d scale has to be reduced as seen in Fig. (b). The upper curve in Fig. (c) shows that the conjecture $t \propto \frac{1}{d^2}$ is apparently

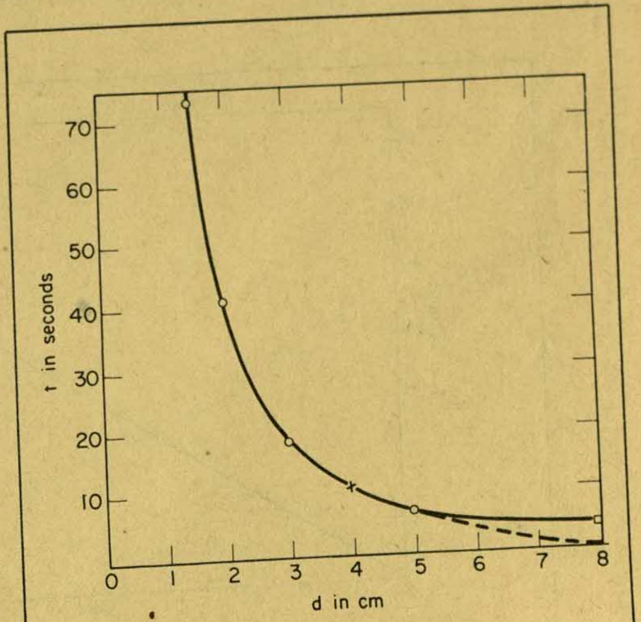


Figure (b)

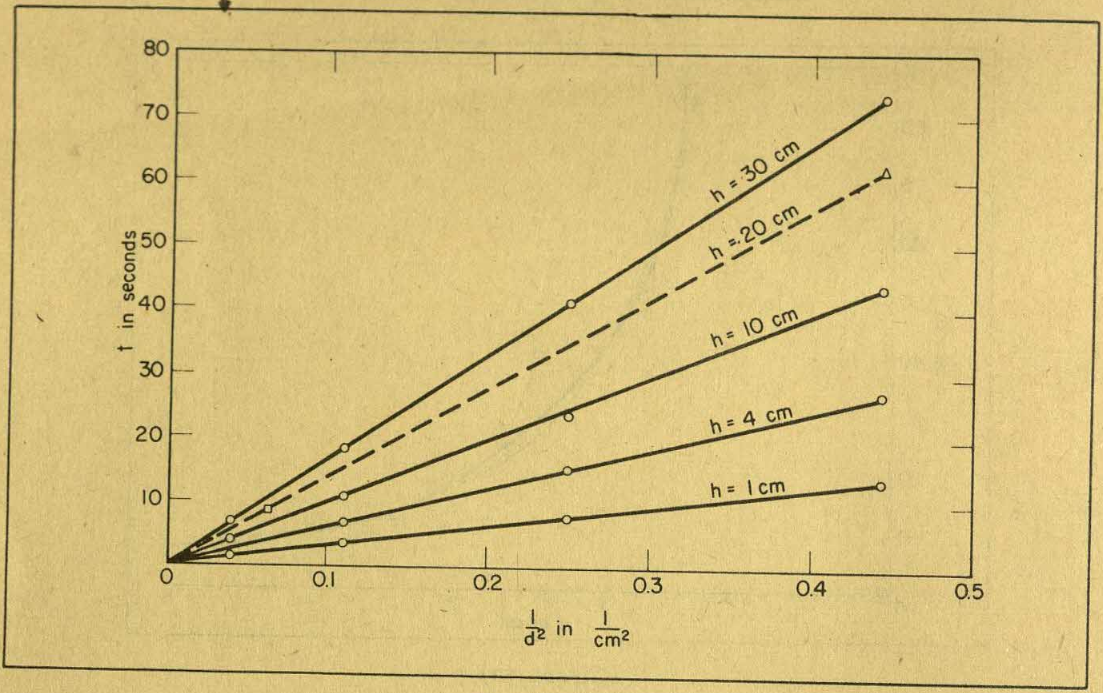


Figure (c)

correct for h = 30 cm. For that height, $t = 165\frac{1}{d^2}$.

The other curves in Fig. (c) show that the conjecture appears to hold also for the other heights.

On the enlarged t scale, [Fig. (d)] one clearly recognizes that the experimental points

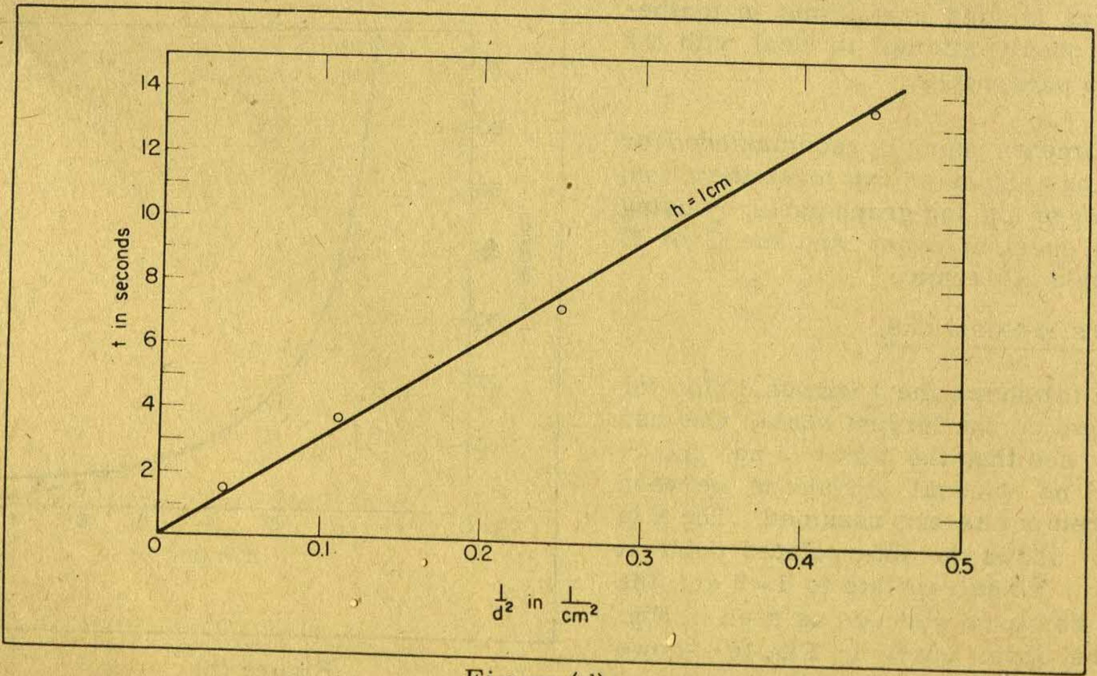


Figure (d)

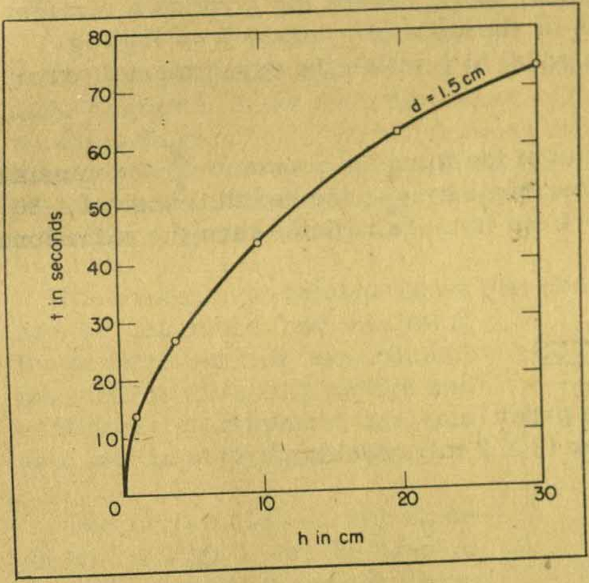


Figure (e)

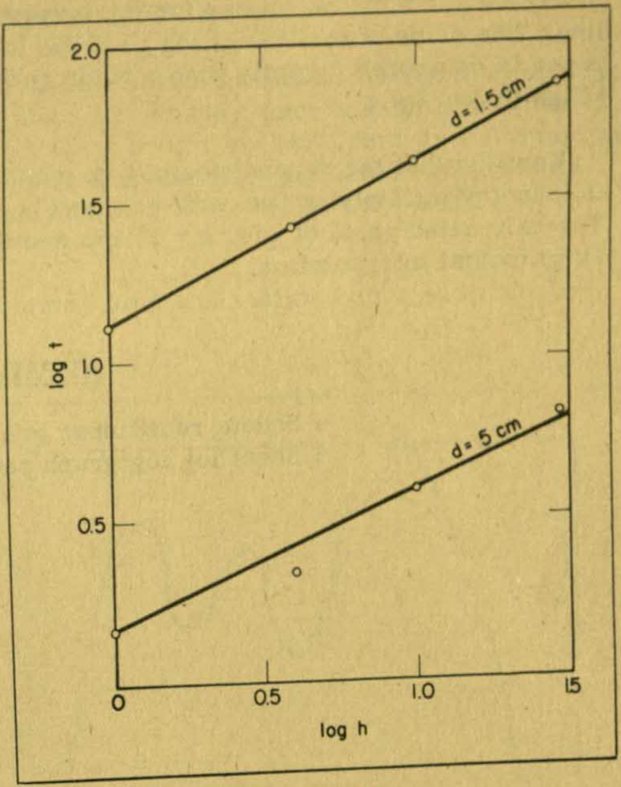


Figure (f)

are not on a straight line but they are all within experimental error from such a line. We conclude that for h = 1 cm, the experiment is consistent with an inverse-square relation $t \propto \frac{1}{d^2}$ but it is not accurate enough to exclude sizable deviations from this relation. In practice, such a situation usually inspires more accurate measurements.

The plot of t versus h [Fig. (e)] should, of course, pass through the origin since, if there is no water in the container, it takes no time to empty it.

To carry out the double interpolation for h = 20 cm and d = 4 cm, we first find t for h = 20 cm and d = 1.5 cm on Fig. (e). Since we have established that $t = \text{constant} \cdot \frac{1}{d^2}$, we can mark the point found in Fig. (e) on Fig. (c) and draw the straight line through the origin. On that line we read the value of t for $\frac{1}{d^2} = \frac{1}{4^2}$. A different method would be to read the values of t in Fig. (c) for all the four values of h and $\frac{1}{d^2} = \frac{1}{4^2}$ and then use these values to draw a new curve of t versus h on which we then interpolate for h = 20 cm. This method would also hold if t were not a linear function of $\frac{1}{d^2}$.

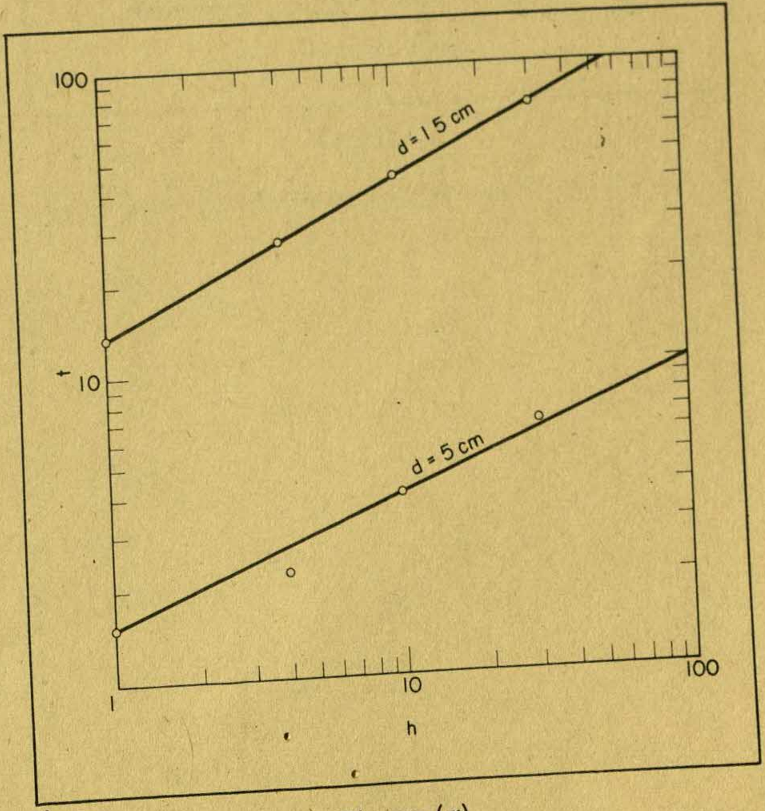


Figure (g)

Students may find by trial and error that $t \propto \sqrt{h}$. This relation does not hold as accurately for $d = 5$ cm as it does for the longer times. Log t versus log h yields a straight line. The slope of the line of log t versus log h or the slope of t versus h on log log paper is measured directly with a ruler and is equal to $\frac{1}{2}$ within the experimental error. [Figs.(f) and (g).]

Knowing that the dependence of t on d and h is of the form $t = \text{constant} \cdot \frac{\sqrt{h}}{d^2}$ the constant is found by multiplying the most accurately known time by $\frac{d^2}{\sqrt{h}}$, the result: constant = 30. The calculated value of t for $h = 20$ cm and $d = 4$ cm is more reliable than the value found by graphical interpolation.

APPARATUS

4 Sheets rectilinear graph paper
1 Sheet log log graph paper (2 X 2 min. cycles)

I-5. MOTION: SPEED AND ACCELERATION

This experiment is essential to the study of kinematics in Chapter 5 and may be done after Section 5-3 has been discussed. (The plot of a versus t may best be done after studying Section 5-6.) Although much of the student's time will be spent in analyzing the information from the tape and in constructing the graphs, as much of this analysis as possible should be done in the laboratory where you can help the student learn how to make the graphical analysis.

The experiment demonstrates that complicated forms of motion occur and that they can be described and analyzed. By performing this experiment, the student will gain practice and confidence in graphical analysis as a tool in studying motion.

One or two timers, set up as shown in Fig. 1, can be used to run off tapes for the whole class. Let each student pull a tape through the timer while walking at a normal rate and then construct his own set of graphs.

About 1.5 meter of tape will be sufficient to get 20 to 25 points on the graph with 5 "ticks" as the unit of time. Make certain that your students begin work on the graphs (v and d versus t) in class. They can then continue this work at home. The plot of acceleration versus time can be done later after studying Section 5-6. A typical analysis is shown in Figs. (a), (b), and (c).

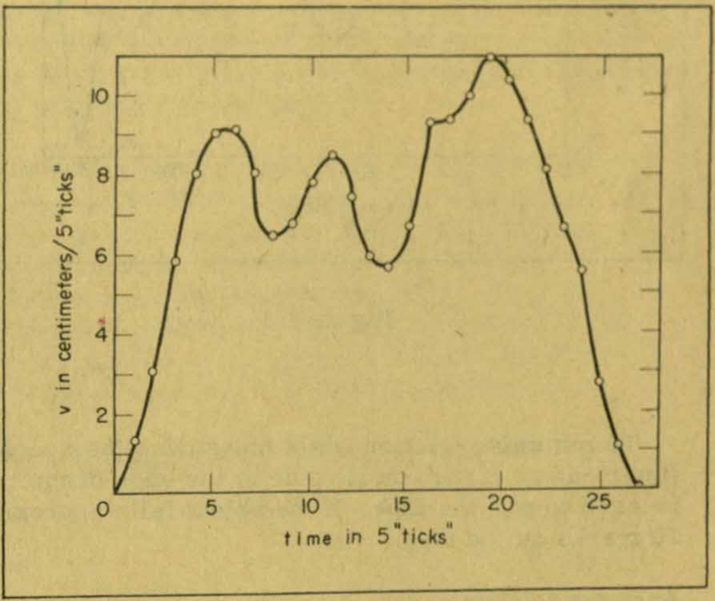


Figure (a)

It is not necessary to know the calibration of the timer for this experiment; any unit of time is good. Taking 5 "ticks" as the unit of time speeds up the analysis.

Students may set up a table of speed as a function of time from which they can plot speed versus time. However, the graphs of distance versus time and acceleration versus time should be plotted from the speed versus time graph.

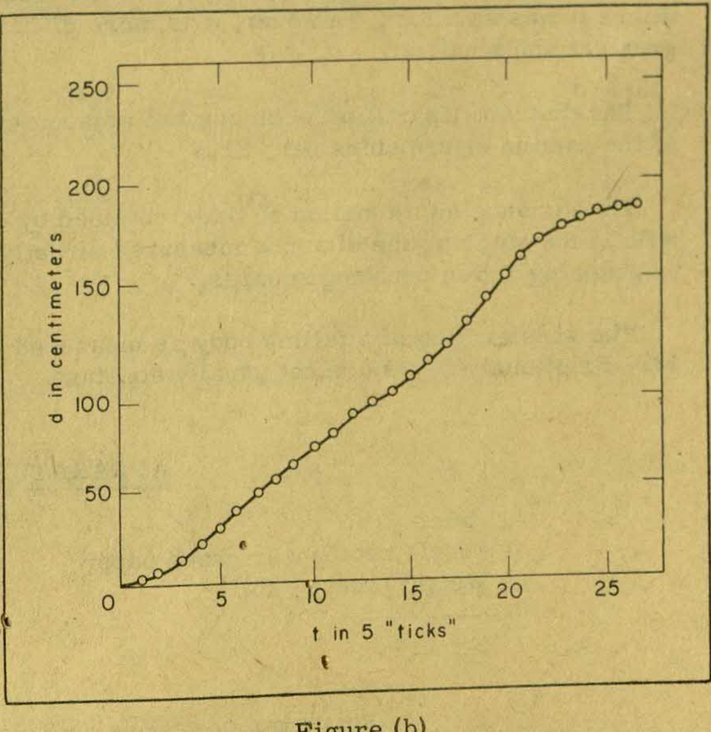


Figure (b)

The average speed is obtained by measuring the distance between the first and last mark on the tape and dividing this by the number of time units (5 "ticks").

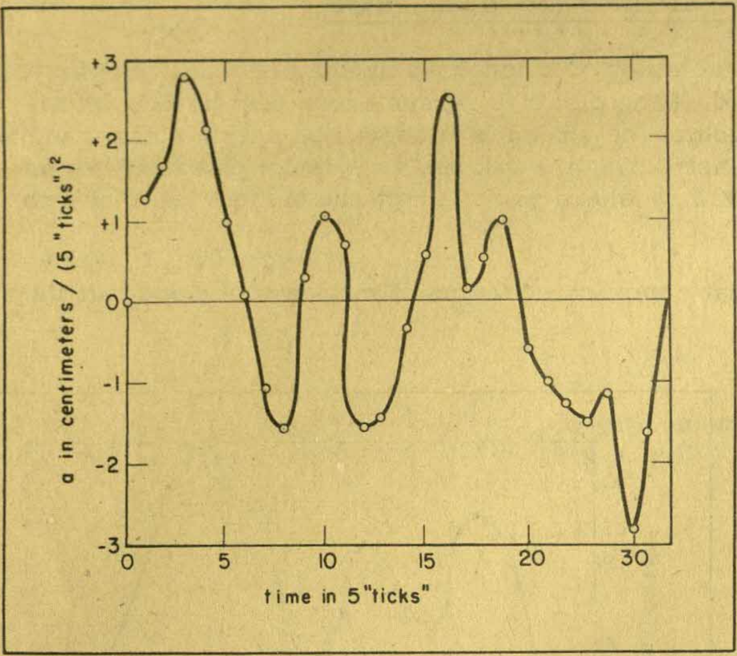


Figure (c)

You can also have your students prepare a speed versus time graph by cutting tape into lengths corresponding to the distance traveled in equal time intervals and pasting these side by side on a common base line. These are then compared with the speed versus time graphs students have plotted. Students can then make an acceleration versus time graph by measuring the successive Δv's on the speed-time graph and cutting tape lengths equal to them. (Just measure and cut the tape by putting it beside the bars on the speed-time graph.) Paste all the Δv's side by side to form the acceleration plot and compare with the graphs the students prepared. This helps students to visualize better what the measurements and the graphs mean.

To minimize friction while measuring the acceleration of a falling object, the bell timer can be turned on its side at the edge of the table and a relatively heavy object can be used to pull the tape. If the object falls approximately one meter, there will be about 20 marks on the tape.

Answers to Questions

It will be possible to see, by inspecting the tape, where the speed was greatest and where it was smallest. However, it is more difficult to see where the acceleration was greatest and smallest.

The distance (in cm) between any two adjacent marks on the tape represents the speed of the hand in centimeters per "tick."

The distance as a function of time, obtained by counting squares under the speed curve, will be the same as the distance measured directly from the tape except for small errors in graphing and in counting squares.

The acceleration of a falling body as measured in this experiment may vary by about 5%. Frictional effects are not usually constant.

APPARATUS

1 Ruler
4 Sheets rectilinear graph paper
1 Weight (about $\frac{1}{4}$ lb.)

Timer*

$1\frac{1}{2}$ volt dry cell*
Ticker tape
Carbon-paper disc
1 C clamp (3" or 4")

* One or two of each is sufficient for entire class.

I-6. SMALL MASSES

An ordinary laboratory balance will measure a mass of several hundred grams with an accuracy of 1% or less, but when it is used to measure the mass of a hair or light object, the balance will not yield even the right order of magnitude. From their study of chemistry, some students may be acquainted with complex analytical balances. This experiment will show the student that a simple soda-straw balance can measure very small masses to much better than an order of magnitude. Each student should calibrate his balance; not for the technique alone, but again for the idea of extending his range of measurements by the use of simple devices. The appropriate time for this experiment is after the discussion of Section 7-2.

The closer the needle is to the center of mass of the straw and bolt, the more sensitive the balance will be. If the needle is below the center of mass, the instrument will be unstable and cannot be balanced. In this case, turning the balance upside down will place the needle above the center of mass and will stabilize the balance.

Considerable time will be saved if several hundred sheets of paper are counted out and weighed on an ordinary laboratory balance before the laboratory period begins. Each student can then calculate the mass of one cm^2 (or less) of one sheet. Small pieces of paper are handled most conveniently with tweezers. The effects of air currents will be minimized by placing the balance in a shoe box or other small container. To avoid making this experiment only a construction and calibration exercise, some use of the balance is essential. Using the balance and optical micrometer to find the mass and volume of small objects will be helpful in the discussion of density in Section 7-4.

See Experiment I-8 for an application of the soda-straw balance to the molecular layers experiment.

Answers to Questions

Most ordinary balances will weigh accurately to 0.1 - 1.0 gram. If the balance weighs accurately to, say, 0.1 gram, the accuracy in weighing a textbook expressed in per cent is $\frac{0.1 \text{ gram}}{\text{mass of textbook}} \times 100$. The accuracy in measuring the mass of different objects depends therefore on the mass of the object.

The mass of a hair cannot be found to within an order of magnitude by using an ordinary laboratory balance; it is impossible to tell whether the mass is 10^{-2} gm or 10^{-4} gm.

To check the equality of the masses of several small pieces of paper, compare the deflection of the arm for each one.

The largest source of error in the calibration is probably due to lack of uniformity in the paper.

Scale readings are interpolated by dividing the space between the marks found by calibration into equal parts.

The range of the soda-straw balance depends on the sensitivity. Since one can make about ten meaningful divisions on the scale, the range is about ten times the smallest measurable mass.

APPARATUS

1 Razor blade or scissors
1 Ruler
1 Clothespin
1 10" soda-straw
1 Machine screw to fit straw as counter-balance ($\approx\frac{1}{8}$")
1 Needle or long pin
A block of wood ($\frac{1}{2}$" x $\frac{1}{2}$" x 3"), two microscope slides, and a rubber band can be used to make a base for the balance, although a variety of other things will serve as well.

Equal arm laboratory balance (range 0 - 2000 gm)
Tablet or ream of paper

I-7.

THE SPECTRA OF ELEMENTS

This experiment demonstrates that atoms of a strongly heated element give off colors characteristic of that element. Since the eye does not resolve mixtures of colors, a spectroscope is used. The theory behind its operation is, however, irrelevant at this time. The appropriate time for doing the experiment is during the study of Section 7-9.

The compounds listed will give good results when used in crystalline or, preferably, powdered form. Some of the nitrates of the metals, such as sodium nitrate, cupric nitrate, etc. should be available to show that the color given off is a characteristic of the metal and not of the chloride. The colors due to chlorine itself are too weak to be seen. The metals mentioned here give off visible light in the temperature range available from the Bunsen burner. If other substances are used, their suitability should be tested in advance. It may be advisable to caution the students against tasting the substances.

The nichrome wire must be new, shiny, and free of impurity. Contamination from sodium salts in perspiration will be prevented if the wire is not handled near the loop. If it is necessary to re-use a wire, it should be cleaned by holding it in the flame for a few minutes, dipping it briefly in a dilute solution of hydrochloric acid, and then reheating the loop. When the clean loops are not in use, they should be stored by wrapping them individually in aluminum foil. A handle for the wire can be made by inserting one end of the wire into a short piece of $\frac{1}{4}$" dowel with pliers.

When the heated loop is dipped into the solid substance, a small amount will melt and stick to the loop. If some substances do not stick well, they may be moistened.

The only critical step in the construction of the spectroscope is to make certain that the slit is parallel to the lines on the diffraction grating. A spectral line, like the yellow line in sodium, may appear doubled. This is not the true doublet of sodium, but a "ghost" doubling due to imperfections in the grating. Eliminate this doubling by moving your head a little in order to use a different portion of the grating. If this does not remove the ghost, replace the grating.

The colors will appear more pronounced if the room is in semi-darkness.

Answers to Questions

The colors are substantially different except for lithium, strontium, and calcium, which are red.

Without the spectroscope, the eye can distinguish the separate colors given off by a fifty-fifty mixture of the copper and lithium chloride because of accidental spatial separations of the luminous regions in the flame. When only a trace (5%) of lithium is in the mixture, it is possible to distinguish the color given off by each substance only by using the spectroscope.

The presence of a trace of lithium mixed with sodium chloride cannot be detected without the spectroscope, but the lithium line is visible through the spectroscope.

The student can convince himself that the colors he observes are characteristic of sodium, calcium, copper, etc. and not their chlorides by observing the colors given off when the nitrates of the same elements are tested in the same way. This also shows that the colors are given off by the individual atoms and not by the element in bulk.

Sodium can be identified in baking soda and calcium in chalk dust.

APPARATUS

The following suitable for entire class:

Small amounts of crystalline or powdered:

lithium chloride
potassium chloride
strontium chloride
sodium chloride
calcium chloride
cupric chloride

One or two nitrates of the above.

baking soda
chalk dust

10	#22 Nichrome (or platinum) wire loops and dowel handles ($\frac{1}{4}$" x 2")
1	Gas burner
1	Spectroscope
1	Ringstand and flask clamp
	Several small watch glasses or similar containers

I-8. MOLECULAR LAYERS

This experiment yields an upper limit for the size of a molecule. It is a major achievement that the student can push this limit down to the order of 10^{-9} m (≈10A). The experiment relates to the content of Section 7-12 but may be done any time after Section 7-9 has been completed.

The following demonstration may help the students understand the indirect method of measurement used in this experiment. Measure out into a graduate the volume of metal shot that will cover the bottom of a circular tray. Find the area of the tray and calculate the thickness (diameter) of the shot. Compare this with the value found by measuring the diameter of one of the shot with a ruler or compass.

Best results are obtained with a 0.5% solution for which a tray with a diameter of at least 15 inches is required. (One per cent requires too large a tray, whereas 0.05% gives very irregular shapes.) If the tray is difficult to clean, it may be lined with aluminum foil. The ripple tank makes a good tray. However, the tanks with wooden frames must be lined to prevent contamination from the putty and wood. Perspiration from the hands can contaminate the water enough to prevent a clear-cut film outline. Barring mistakes on the part of the student, it is not necessary to clean the tray during the experiment. After the experiment, however, the tray must be thoroughly cleaned to remove all the oleic acid.

If the tap water is heavily chlorinated or has a high mineral content, distilled water should be used. An advanced check may be in order. Using warm water will speed up alcohol evaporation. If too much indicator powder is used, it will prevent the oleic acid from forming the largest possible circle and thus the thinnest possible layer.

A mixture of three parts of new crankcase oil and one part of old crankcase oil also makes a good indicator. Use a single drop from a toothpick. Although this indicator works well, it has the disadvantage that two more oils enter the picture to make the situation appear more complicated than it really is.

You may want to mix the solution in advance or have one or two students do it, but each student should calculate the concentration. Following the directions given in the Laboratory Guide results in 50 cm^3 of 0.5% solution. This is more than enough for all of your students.

The first drop of solution from the medicine dropper may take with it some of the solution adhering to the outside of the dropper and thus be larger than others. To avoid this, the first few drops can be returned to the bottle.

If the students do not think to do it themselves, suggest that they try to stretch the film by pulling it with a thin wire to see if they can change its area or only its shape.

If time permits, you may consider an interesting sidelight. Have each group prepare about 5 cm^3 of oleic acid solution by weighing the necessary amount of oleic acid on the soda-straw microbalance and then dissolving it in 5 cm^3 of alcohol. For a 0.5% solution, this will require 22.25 milligrams. A tiny piece of aluminum foil placed in the straw will serve as a container for the oleic acid which is then put with the acid into the alcohol. The amount of oleic acid per drop of solution can be found as before.

Answers to Questions

The thickness of the oleic acid film on the swimming pool can be found by dividing the volume of the drop by the surface area of the pool.

Five cm^3 of oleic acid in 95 cm^3 of alcohol makes a 5% solution. Five cm^3 of the 5% solution contains 0.25 cm^3 of oleic acid and 4.75 cm^3 of alcohol. Mixing this with 45 cm^3 of alcohol results in a solution that contains 0.25 cm^3 of oleic acid in 49.75 cm^3 of alcohol. This is the same as 0.50 cm^3 of oleic acid in 99.50 cm^3 of alcohol. Expressed in per cent, the concentration is 0.5%.

When one drop of alcohol is dropped in the water, the chalk dust moves outward, begins to form a clear area, and then collapses as the alcohol evaporates or goes into solution.

Two drops of oleic acid will form about twice the area of one drop (the radius increases by about 40%). Three drops will form about three times the area of one drop if the tray is large enough for the acid and the indicator to spread out. From this it can be concluded that within the accuracy of the experiment, the film is uniformly thin regardless of whether one or more drops are used.

A sample set of calculations of the thickness of the molecular layer is as follows:

Average diameter of film ring: 20 cm
Area of ring $\pi\frac{d^2}{4}$: 314 cm^2
Concentration of solution: 0.5%
Volume of one drop (50 drops/cm^3): 0.02 cm^3
Volume of oleic acid per drop: 0.5% x 0.02 cm^3 = 1×10^{-4} cm^3
Thickness of the film = $\frac{1 \times 10^{-4} cm^3}{314 cm^2}$ = 3.2×10^{-7} cm

The error in measuring the diameter of the film may be about 15%, therefore, the error in the area is about 30%. The error in the preparation of the solution is about 2%. Variations in the size of the drop may introduce further errors of 5 - 10%. Some error is caused by the indicator even when it is used in small amounts. The students can estimate that their calculation is accurate to within a factor of 2 each way.

If the thickness of the layer were magnified to one centimeter or about 10^7 times, on the same scale a student whose height is of the order of 1 meter would be about 10^4 kilometers tall. He could lie down with his feet in the western U.S. and his head in Europe.

If the layer is one molecule thick and the molecules are assumed to be cubes, 3.1×10^6 molecules could be placed along each dimension of a centimeter cube. Then $(3.1 \times 10^6)^3$ = 3×10^{19} molecules would fill one cubic centimeter. One molecule of oleic acid would have a mass of $\frac{0.89 \text{ gm/cm}^3}{3 \times 10^{19} \text{ cm}^{-3}} = 3 \times 10^{-20}$ gm.

The molecular weight of oleic acid is 282, i.e. one mole is 282 grams. If the mass of one molecule of oleic acid is 3×10^{-20} grams, then there are $\frac{282 \text{ grams}}{3 \times 10^{-20} \text{ grams}} \approx 10^{22}$ molecules in one mole.

The main source of error in this evaluation of Avogadro's number is the assumption that the oleic acid molecule is a cube; in reality it is long and narrow, with a width and thickness each about $\frac{1}{10}$ its length. Therefore, there are about 10 x 10 times the number of molecules, or 10^{24} in one mole.

APPARATUS

1 Large tray (min. 15" x 15")
5 cc of 0.5% solution of oleic acid in isopropyl or methyl alcohol
1 Medicine bottle and dropper
1 Graduated cylinder (sensitive to 0.1 cc)

Fine powder (chalk or lycopodium)
Sandpaper (needed if chalk is used)

Aluminum foil (optional)
Clean water

I-9. NATURAL TEMPERATURE SCALE

The purpose of this experiment is to show that a gas thermometer defines a natural temperature scale. The scale is linearly proportional to gas volume by definition, and it is independent of the nature of the gas. The linearity is not confirmed by checking it against a mercury thermometer. That shows only that the mercury thermometer is calibrated to agree with the gas scale.

The experiment relates to Sections 9-4 and 9-5 and may be done before the sections have been studied.

For best results, the inside of the flask and glass tube must be dry. A small wad of cotton placed over the end of the glass tube will help keep the inside of the flask dry while the flask is immersed in the boiling water. If the humidity is very high, storing the flasks overnight in a box with a small amount of calcium chloride will remove the moisture from the air in the flasks and will give a better final result.

A 250 cc flask is best. Larger flasks require large containers of boiling water and ice water. If the same flask is used for a second run, it should be thoroughly dried. Even one drop of water can throw results off drastically.

The transfer of the flask from the boiling water to the ice water must be done quickly, with a finger placed firmly over the end of the glass tube to prevent outside air from entering the flask during the transfer. The flask must be completely immersed in the ice water, and the mixture stirred gently to insure uniform temperatures.

The volume of the air in the flask is determined by filling the flask with water and measuring the contents in a graduate. This volume is the volume of the gas at the boiling point of water. The difference in this volume and the volume of ice water entering the flask is the volume of the gas at the freezing point of water.

It would be a good idea to have students repeat the experiment two or three times for each different gas used. If time does not permit, the readings from trials by various students can be shared and the average taken to give equally satisfactory results.

To show that the scale is the same for different gases, at least one other gas should be used in addition to air. Oxygen is a good choice since it is relatively easy to prepare in the laboratory. The gas can be dried by passing it through a calcium chloride tube before a run is made. Such gases as helium, propane, and methane will give equally good results. Special care should be taken if combustible gases are used. The solubility of carbon dioxide in water prevents its use with the method outlined in the experiment.

Answers to Questions

The temperature in degrees centigrade found by extrapolating the curve to zero volume will be approximately -273°C.; values ranging from -240°C. to -300°C. will not be uncommon, since an error in measuring the volume of the gas at the freezing point of water will increase approximately ten times when the curve is extrapolated to zero volume.

The curve found by plotting the results using a different gas will be essentially the same as that found for air.

Measuring the temperature of the water will usually change it slightly. When a mercury thermometer is used, the change is negligible. Since a thermometer is in general not at the same temperature as the object whose temperature it is to measure, there will be a small exchange of heat between the two. That is, the thermometer will either warm or cool the object. How noticeable this effect is depends on the relative sizes of the thermometer and the object. This question can serve as an introduction to Chapter 10 of the text.

APPARATUS

3-5	Erlenmeyer flasks, 250 cc
	Rubber stoppers, one-hole, to fit flasks
	Glass tubes, 2"
2	Beakers, 500 cc
	Ice
	Cotton
	Burner
	Graph paper

1	Setup for generating oxygen from the mixture of potassium chlorate and manganese dioxide, and a calcium chloride tube to dry the gas.
	Helium, propane, or methane